

# CURVATURE BASED HUMAN FACE RECOGNITION USING DEPTH WEIGHTED HAUSDORFF DISTANCE

Yeung-hak Lee<sup>\*</sup>, Jae-chang Shim<sup>†</sup>

Yeungnam University<sup>\*</sup>  
214-1 Dae-Dong Gyeonsan-Si  
Kyungpook 712-749 South Korea  
annaturu@yumail.ac.kr

Andong National University<sup>†</sup>  
388 Song-Chun Dong Kyungpook  
760-749 South Korea  
jaechang@andong.ac.kr

## ABSTRACT

In this paper, we propose a novel implementation of a person verification system based on depth-weighted Hausdorff distance (DWHD) using the surface curvatures of the human face. This new method incorporates the depth information and curvatures of local facial features. The weighting function used in this paper is based on depth values, which have differential properties of a face according to the people, so that the distance of this extracted edge maps will be emphasized. Experimental results based on combination of the maximum, minimum, and Gaussian curvature according to threshold values show that DWHD achieves recognition rate of 92.8%, 97.6% and 92.8% of the cases for 5 ranked candidates, respectively, and the proposed method of combined recognition rate for each curvature shows the best.

## 1. INTRODUCTION

Recently, range images gained much attention in computer vision area, as the human face recognition. Most human face recognition approaches have centered on feature extraction and using the relationship of these features in face matching process [1-4]. Lapreste etc. [4] presented a system that identifies facial feature points, such as center of eyes, tip of nose, lips, and chin. The identification is based on the local curvature which is determined by means of the range data in a neighborhood. The authors analyzed the curvature on a human face to extract feature points on profile line images. Lee and Milios [2] tried to detect corresponding regions in two range images. They computed an extended Gaussian image (EGI) for extracted the convex regions in range images of human faces. Based on the EGIs similarity measurement is determined between all regions. Gordon [1] presented a detailed study of range images in the human face recognition based on depth and face surface curvature features. The curvatures on a face surface are computed to find face descriptors: nose ridge and eye features. She proposed two recognition strategies based on template matching and on comparing in the feature space, relying on these descriptor.

In recent years, considerable progress has been made in the curved surface analysis with the advances in range finding techniques [3,5]. Surface curvatures, such as Gaussian, mean,

and principal curvatures calculated from range images which are intrinsic surface properties describing local shape have come to play an important role in both characterizing and recognizing free-formed curved surface.

Psychological studies have revealed that human faces can be categorized and human can recognize the line drawings or the edge of objects as quickly and almost as accurately [6]. As it contains important information about its shape and structure, it can be used in face recognition. Moreover Takacs [6] introduced a modified Hausdorff distance, which is a similarity measure derived as a variant of Hausdorff distance, for human face recognition.

In this paper, we propose a new face recognition algorithm using the edge maps which is extracted by the threshold values from each curvature which is analyzed as a curvature-related texture. In order to compare the similarity, this paper is proposed that utilizes a modified type of Hausdorff distance by incorporating the depth information of a human face, as human face has different depth value and different shape for each person.

## 2. SURFACE CURVATURE

For each data point on the facial surface, the principal, Gaussian and mean curvatures are calculated and the sign of those (positive, negative and zero) is used to determine the surface type at every point. The  $Z(x, y)$  image represents a surface where the individual Z-values are surface depth information. Here, x and y is the two spatial coordinates. We now closely follow the formalism introduced by Peet & Sahota [7] and specify any point on the surface by its position vector:

$$R(x, y) = xi + yj + z(x, y)k$$

The first fundamental form of the surface is the expression for the element of arc length of curves on the surface which pass through the point under consideration.

It is given by:

$$I = ds^2 = dR \cdot dR = E dx^2 + 2F dx dy + G dy^2$$

where

$$E = 1 + \left( \frac{\partial z}{\partial x} \right)^2 \quad F = \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} \quad G = 1 + \left( \frac{\partial z}{\partial y} \right)^2$$

The second fundamental form arises from the curvature of these curves at the point of interest and in the given direction:

$$II = edx^2 + 2fdxdy + gdy^2$$

where

$$e = \frac{\partial^2 z}{\partial x^2} \Delta \quad f = \frac{\partial^2 z}{\partial x \partial y} \Delta \quad g = \frac{\partial^2 z}{\partial y^2} \Delta$$

and

$$\Delta = (EG - F^2)^{-1/2}$$

Casting the above expression into matrix form with:

$$V = \begin{pmatrix} dx \\ dy \end{pmatrix} \quad A = \begin{pmatrix} E & F \\ F & G \end{pmatrix} \quad B = \begin{pmatrix} e & f \\ f & g \end{pmatrix}$$

the two fundamental forms become:

$$I = V^T A V \quad II = V^T B V$$

Then the curvature of the surface in the direction defined by  $V$  is given by:

$$k = \frac{V^T B V}{V^T A V}$$

Extreme values of  $k$  are given by the solution to the eigenvalue problem:

$$(B - kA)V = 0$$

or

$$\begin{vmatrix} e - kE & f - kF \\ f - kF & g - kG \end{vmatrix} = 0$$

Which gives the following expressions for  $k_1$  and  $k_2$ , the minimum and maximum curvatures, respectively:

$$k_1 = \frac{\{gE - 2Ff + Ge - [(gE + Ge - 2Ff)^2 - 4(eg - f^2)(EG - F^2)]^{1/2}\}}{2(EG - F^2)}$$

$$k_2 = \frac{\{gE - 2Ff + Ge + [(gE + Ge - 2Ff)^2 - 4(eg - f^2)(EG - F^2)]^{1/2}\}}{2(EG - F^2)}$$

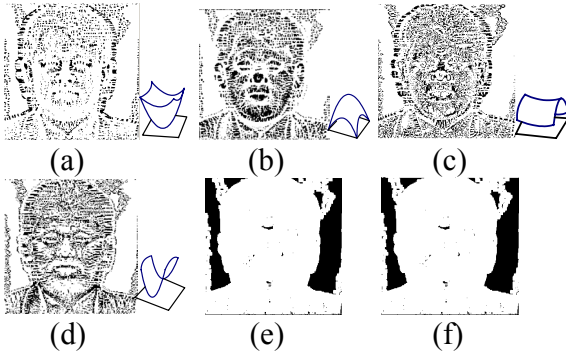


Fig. 1 Six possible surface type according to the sign of principal curvatures for the face surface, (a) concave (pit), (b) convex (peak), (c) convex saddle (d) concave saddle (e) minimal surface, (f) plane..

Here we have ignored the directional information related to  $k_1$  and  $k_2$  and chosen  $k_2$  to be the larger of the two. For the present work, however, this has not been done. The two

quantities,  $k_1$  and  $k_2$ , are invariant under rigid motions of the surface. This is a desirable property for us since the cell nuclei have no predefined orientation on the slide (the  $x - y$  plane). The Gaussian or total curvature and  $K$  mean curvature  $M$  is defined by

$$K = k_1 k_2 \quad M = (k_1 + k_2) / 2$$

which gives  $k_1$  and  $k_2$ , the minimum and maximum curvatures, respectively. It turns out that the principal curvatures,  $k_1$  and  $k_2$ , and Gaussian are best suited to the detailed characterization for the facial surface, as shown Fig. 1. For the simple facet model of second order polynomial of the form, i.e. a 3x3 window implementation in our range images, the local region around of surface is approximated by a quadric

$$z(x, y) = a_{00} + a_{10}x + a_{01}y + a_{01}y + a_{20}x^2 + a_{02}y^2 + a_{11}xy$$

and the practical calculation of principal and Gaussian curvatures is extremely simple.

### 3. CONVENTIONAL HAUSDORFF DISTANCE

The Hausdorff distance measure computes a distance value between two point sets extracted from the object and a test image [6]. The distance between two points  $a$  and  $b$  is defined as  $d(a, b) = \|a - b\|$ , and the distance a point  $a$  and finite point set

$B = \{b_1, \dots, b_{NB}\}$  is commonly defined as

$$d(a, B) = \min_{b \in B} \|a - b\|$$

For the given two finite point sets, the conventional Hausdorff distance measure between two point sets, is defined as

$$H(A, B) = \max(h(A, B), h(B, A))$$

where is  $A = \{a_1, \dots, a_{NA}\}$  and  $B = \{b_1, \dots, b_{NB}\}$ .  $h(A, B)$  and  $h(B, A)$  represent the directed distance between two sets  $A$  and  $B$ . The directed distance  $h(a, B)$  is traditionally defined as

$$h(a, B) = \min_{a \in A} d(a, B)$$

$$= \max_{a \in A} \min_{b \in B} \|a - b\|$$

The function  $h(A, B)$  identifies the point  $a \in A$  that is farthest from any point of  $B$  and measures the distance from  $a$  to its nearest neighbor in  $B$ . Dubussion and Jain[23] presented the modified Hausdorff distance (MHD) measure by employing the summation operator over all distance, rather than the maximum operator:

$$h_{MHD}(A, B) = \frac{1}{N_a} \sum_{a \in A} d(a, B)$$

### 4. DEPTH-WEIGHTED HAUSDORFF DISTANCE

The weighted Hausdorff distance measure proposed in this paper is based on the truth that face has different depth value and surface curvature for each person. A weighted function defined according to the depth value for each points of face is used in this equation. Hence this modified Hausdorff distance is called Depth weighted Hausdorff distance. Given two finite point sets

$Q = \{q_1, \dots, q_{N_Q}\}$  and  $D = \{d_1, \dots, d_{N_D}\}$ , the depth weighted Hausdorff distance measure is defined as follows:

$$H(Q, D) = \max(h(Q, D), h(D, Q))$$

where  $h(Q, D)$  and  $h(D, Q)$  represent the directed distance between two sets  $Q$  (Query) and  $D$  (Database). The definition of depth weighted Hausdorff distance is

$$h_{DWHHD}(Q_C, D_C) = \frac{1}{N_{q \rightarrow Q}} \sum_{q \in Q} d(\text{diff}) \min \|q_C - d_C\|$$

where  $\|\cdot\|$  is some underlying norm on the points of  $Q$  and  $D$ ,  $C$  is curvatures, and  $d(\text{diff})$  is weighted function. This is defined as follows:

$$d(\text{diff}) = \frac{QP(x, y, z) - QD(x, y, z)}{DP(x, y, z) - DD(x, y, z)}$$

where  $QP(x, y, z)$  and  $DP(x, y, z)$  is the depth value for the nose tip point of the face.  $QD(x, y, z)$  and  $DD(x, y, z)$  is the depth value for the minimum distance coordinate point  $(x, y)$  which is calculated by directed Hausdorff distance between query and database image. For another experimental, we proposed general method as the weighting function  $d = 2(\text{diff})$  instead of  $d(\text{diff})$ .

$$d = 2(\text{diff}) = \frac{QP(x, y, z) - QD(x, y, z)}{DP(x, y, z) - DD(x, y, z)}$$

$$Q_{Avg}(x, y, z) = \frac{1}{N} \sum_{x, y, z} (QP(x, y, z) - QD(x, y, z))$$

where  $N$  is window size which is neighbors of the minimum distance coordinate point  $(x, y)$ , 3 (DW1WHD) and 5 (DW2WHD).

## 5. EXPERIMENTAL RESULTS

In this study to evaluate the performance of several Hausdorff distance measure, we used a 3D laser scanner made by a 4D Culture to obtain a 3D face image. First, a laser line beam was used to strip the face for 3 seconds, thereby obtaining a laser profile image that is, 180 pieces. Obtained image, which is extracted by the extraction algorithm of centerline, is 640x480. The calibration processing is done in order to process for the resampling and interpolation. Finally, a 3D face image using in this paper is extracted, at 320x320. The used face database was 84 images (42 person): two pictures available, the second images were taken at a time interval 30 minutes, after the first. And in our system, the person who wears the spectacles was excluded because of noise.

From 3D face image, the first work is finding the nose tip point, because this is fiducial point on the face and it can be used reference point. The image removed background area that has lowest value in 3D image includes noise factors, cloth and hair. To remove these factors we used the Sobel operator, and maximum value points were found by the iterative selection method. Finally we chose the one maximum coordinate.  $k_1$  and  $k_2$

Using the curvature equations previously described, the measured curvature values were displayed between  $-1.0$  and  $1.0$  by experiments. In this paper, used curvatures were principal ( $k_1$  and  $k_2$ ) and Gaussian curvature, and used threshold values were  $k_1$ : 0.3 (TH1), 0.4 (TH2), 0.5 (TH3), 0.6 (TH4), 0.7 (TH5),  $k_2$ : 0.03 (TH1), 0.04 (TH2), 0.05 (TH3), 0.1 (TH4), 0.15 (TH5) and Gaussian: 0.03 (TH1), 0.04 (TH2), 0.05 (TH3), 0.1 (TH4), 0.15 (TH5). Fig. 2 displays each curvature using threshold value and depth contour. In case of mean curvature, in this paper excluded because it is similar to the maximum curvature.

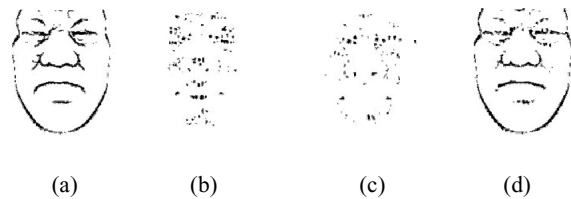


Fig. 2 Extracted curvature images by threshold value (a) maximum curvature, (b) minimum curvature, (c) Gaussian curvature, (d) mean curvature, by each TH3.

### 5.1 Face Recognition

The recognition rate of conventional Hausdorff distance is shown as Fig. 3 (directed Hausdorff distance) and Fig. 4 (modified Hausdorff distance). And the experimental results proposed in this paper are shown as Fig. 5 and Fig. 6. Fig 3 presents the recognition rate for ranked best 5 according to the threshold values. In case of  $k_1$  and TH1 marked highest recognition rate among the curvature. And the combined recognition rate that is the method which rank is selected as the ranked best 1 among each curvature is shown in Fig. 6. As shown in Fig. 3, the more distribution (TH1) of binary pixel the higher recognition rate, but was taken much processing time, vice versa.

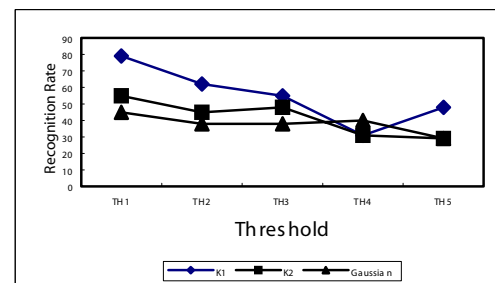


Fig. 3 Recognition rates based on directed Hausdorff distance.

Fig. 4 illustrates the recognition rate based on modified Hausdorff distance method according to the threshold values. In Fig. 4, TH1 having much distribution of black pixel marked lower recognition rate, and it means that the more distribution caused the more confusion for recognition. In case of TH5, it was the lowest recognition rate because of less distribution. And

as shown in Fig. 4, the highest recognition rate was when the threshold value is TH3.

The experimental results proposed methods show in Fig. 5 and Fig. 6. The maximum recognition rates of DWHD using  $k_1$ ,  $k_2$  and Gaussian curvature were 95%, 98% and 95% for ranked best 5, respectively. Fig. 5 illustrates the recognition rates of the each threshold values. In Fig. 5, the recognition rate of  $k_2$  is not changed according to the different threshold value, TH1 to TH3. And TH4 and TH5 are the higher recognition rate than MHD and DHD methods, also. DW1WHD and DW2WHD showed higher recognition rate than DHD, but these methods presented lower recognition rate than DHD and DWHD. Experimental results show that the depth-weighted Hausdorff produces the best performance in the combined recognition rates, as shown Fig. 6.

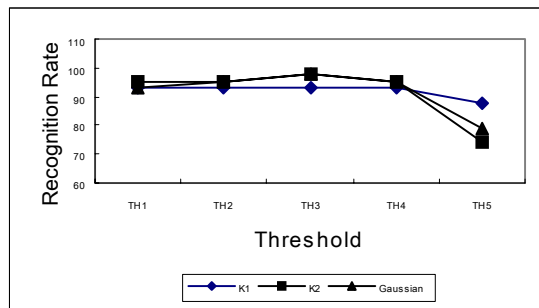


Fig. 4 Recognition rates based on modified Hausdorff distance.

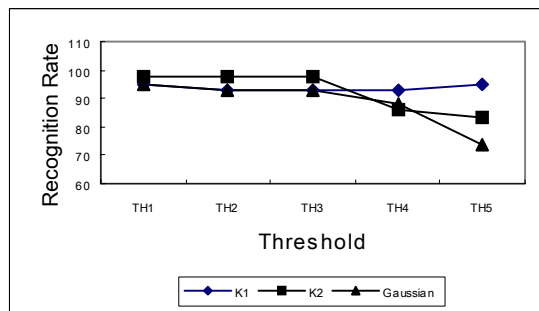


Fig. 5 Recognition rates based on depth-weighted Hausdorff distance.

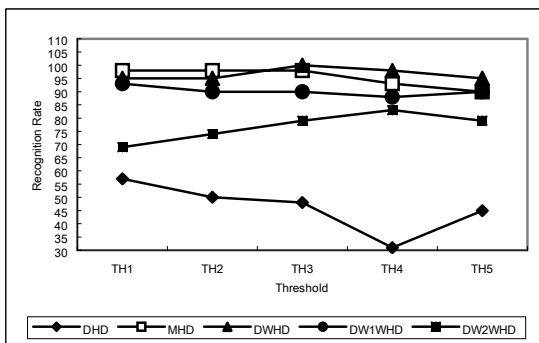


Fig. 6 Combined recognition rate for the ranked best 1 of each curvature

## 6. CONCLUSION

In this paper, a novel Hausdorff distance weighted by a function based on depth value are proposed. Due to different surface curvature and depth for the face of person, the distance measures proposed to emphasize the depth and the shape of face. Proposed a novel implementation of a person verification system based on depth-weighted Hausdorff distance (DWHD) using the surface curvature of the face produced the best performance. Surface curvatures, such as principal, Gaussian and mean curvatures, are desirable intrinsic and extrinsic properties for range image. This new method incorporates the depth information and curvature of local facial features so that the distance of this area will be emphasized. The weighting function used in this paper is based on depth values, which have different values according to the people and important facial features such as nose, mouth, eyes and especially face contour.  $k_1$ ,  $k_2$  and Gaussian curvatures for the DHD, MDH and DWHD were the best recognition, which means the lower black pixel and less processing time, at the TH3. The recognition rates were increased by the lower threshold values, vice versa. Experimental results based on the depth weighting function according to the threshold values show that DWHD achieves the recognition rate of 92.8%, 97.6% and 92.8% for 5 ranked candidates, respectively. In the combined recognition rate according to the threshold values, DWHD produces the best recognition rate than conventional methods.

## 7. REFERENCES

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