EXTRACTING PROJECTIVE INVARIANT OF 3D LINE SET FROM A SINGLE VIEW

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ABSTRACT

Although projective invariants do not exist for general 3D objects in a single view, they do exist for some special 3D constrained structures. So far, several kinds of structure and their invariants have been proposed, such as six points on two adjacent planes, six lines on three planes and so on. But the sorts of them are still limited, which constrains their wide use to 3D object recognition. In this paper, a novel non-coplanar line structure and its projective invariant were derived. This kind of structure can be easily found in the objects formed by intersected polyhedrons. The experimental results showed that the values of the proposed invariant remain stable in the perspective projection between 3D rigid object and its perspective images, and are slightly different over a range of viewpoints.

1. INTRODUCTION

It is well known that the geometric invariant is an effective tool for the 3D object recognition from a single view. By using it, some difficult problems such as the object appearance depending on viewpoints and complicated computing of object pose can be avoided completely in the object model indexing procedure. But research has shown that there is no geometric invariant for a 3D object in general configuration from a single view [1]. Fortunately, Rothwell et al demonstrated that the projective invariants can be extracted from some special constrained 3D structures [2]. Therefore, the study on exploring constrained 3D structures which have projective invariants has received considerable interests in the recent years.

In the beginning, researchers focused their attention on the geometric invariants of coplanar features, such as coplanar five points, two conics and so on [3, 4]. But these invariants are only suitable for recognizing planar objects in 3D space. For complicated 3D objects, they are not sufficient.

Recently, some promising efforts for finding 3D constrained structures which have projective invariants have been made by several authors. Rothwell et al derived a projective invariant for seven points on three adjacent planes [2]. Zhu et al proposed an invariant for the structure which consists of six points on two adjacent planes [5]. Except these point structures, Sugimoto gave a projective invariant of the line structure which is composed of six lines on three planes [6]. Song et al also derived two invariants from two 3D line structures: five lines on two adjacent planes and six lines on four planes [7].

In this paper, we focus on the study of finding new projective invariant for 3D line structures. A novel projective invariant of 3D line structure composed of five lines on three adjacent planes, which can be easily found in the objects made by intersected polyhedrons, was proposed. In comparison to six lines structure, this line structure needs fewer elements, so more combination of lines could be obtained, which can increase the accuracy of recognition. Experimental results showed that the values of the proposed invariant are stable in the perspective projection and are slightly different over a range of viewpoints.

2. PERSPECTIVE TRANSFORMATION OF LINE

2.1. Notation

We use capital letters $X,Y,Z,\ldots$ for the coordinates of points in 3D space, the low case letters $x, y,\ldots$ for the coordinates of image points, and the bold low case letters $\boldsymbol{m}, \boldsymbol{n},\ldots$ for the column vectors.

2.2. Perspective transformation of line

Here we adopt a pinhole projective model, in which the origin of camera coordinate system is at the optical center, and the Z axis is aligned with the optical axis. The image

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plane is at $Z = f$, f is the focal length of lens. The image coordinates axes x and y are coincide with the X and Y axes, and the origin of the image coordinates system is at the intersection of Z axis and the image plane (see FIG.1).

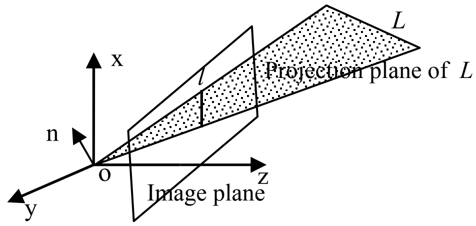

FIG.1 Perspective transformation of line

Consider one point $P(X, Y, Z)$ in the 3D space, and its corresponding image point is $p(x, y)$, if represented in normalized image coordinates, then

$$\begin{cases} x = X / Z, \\ y = Y / Z. \end{cases} \tag{2.1}$$

For a line L in 3D space, its image l could be represented as the intersection of projection plane (plane which passes through the origin and the line L) and the image plane. Therefore, the equation of the image line l can be written as follow,

$$\begin{cases} AX + BY + CZ = 0 \\ Z = f \end{cases} \tag{2.2}$$

where $\boldsymbol{n} = (A, B, C)$ is the normal vector of the projection plane of L. In the image plane, if we still use normalized image coordinates, substituting equation (2.1) into (2.2), we will have,

$$ax + by + c = 0 \tag{2.3}$$

where $\boldsymbol{m} = (a, b, c)$ is also the normal vector of the projection plane, but there is a scaling factor between $\boldsymbol{m}$ and $\boldsymbol{n}$.

In practice, we have to be confronted with the pixel image coordinates. There

$$\begin{cases} u = f \dfrac{x}{\Delta x} + u_0 \\ v = f \dfrac{y}{\Delta y} + v_0 \end{cases} \tag{2.4}$$

here (u, v) is the pixel coordinates of image point, (u_0, v_0) is the pixel coordinates of principle point, $\Delta x, \Delta y$ is the physical width and height of pixel. So if we get a line equation from pixel coordinates as follow,

$$\tilde{a}u + \tilde{b}v + \tilde{c} = 0 \tag{2.5}$$

there would be a relation between the vector $\boldsymbol{m}$ and the coefficients $\tilde{\boldsymbol{m}} = (\tilde{a}, \tilde{b}, \tilde{c})$ as follow,

$$\boldsymbol{m} = \begin{pmatrix} f/\Delta x & 0 & 0 \\ 0 & f/\Delta y & 0 \\ u_0 & v_0 & 1 \end{pmatrix} \tilde{\boldsymbol{m}} \tag{2.6}$$

3. CONSTRAINED STRUCTURE AND ITS PROJECTIVE INVARIANT

In this section, we will derive a new kind of constrained line structure and its projective invariant. First of all, we will rewrite the fundamental theorem given by Sugimoto [6] as the following Lemma 1.

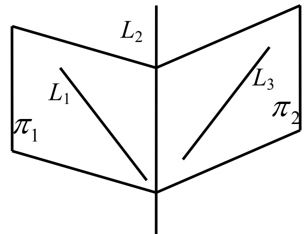

FIG.2 Three lines on two adjacent planes

Lemma 1. For a structure composed of three lines on two adjacent planes π_1 and π_2 (see FIG.2) in 3D space, require these three lines do not intersect at same point and could not be parallel to each other.

Let $\boldsymbol{n}_i$ (i=1,2,3) be the normal vector of projection plane corresponding to each of the three lines L_i(i=1,2,3).

Set

$$\boldsymbol{N}_{123} = |\boldsymbol{n}_1 \quad \boldsymbol{n}_2 \quad \boldsymbol{n}_3| \tag{3.1}$$

be a determinant constructed by $\boldsymbol{n}_i$(i=1,2,3). If the line structure undergoes a rigid motion in 3D space, then, L_i becomes L'_i, and $\boldsymbol{n}_i$ becomes $\boldsymbol{n}'_i$, therefore N_{123} becomes N'_{123},

$$\boldsymbol{N}'_{123} = |\boldsymbol{n}'_1 \quad \boldsymbol{n}'_2 \quad \boldsymbol{n}'_3|. \tag{3.2}$$

We will have the following relation between N_{123} and N'_{123},

$$\frac{N'_{123}}{N_{123}} = \frac{d'_1 d'_2}{d_1 d_2} \tag{3.3}$$

where d_i and d'_i are the negative dot product of the normal vector of the planes π_i and π'_i with any coordinate vectors of points on the corresponding planes.

Lemma 2. Let l_i be the image of 3D line L_i, $\boldsymbol{m}_i$ is the normal vector of projection plane of l_i, which is derived from the normalized image coordinates of l_i, M_{123} is a determinant which is formed by $\boldsymbol{m}_i$(i=1,2,3), e.g.

$$M_{123} = |\boldsymbol{m}_1 \quad \boldsymbol{m}_2 \quad \boldsymbol{m}_3| \tag{3.4}$$

according to the description of subsection 2, there exists

$$\boldsymbol{m}_i = \rho_i \boldsymbol{n}_i \tag{3.5}$$

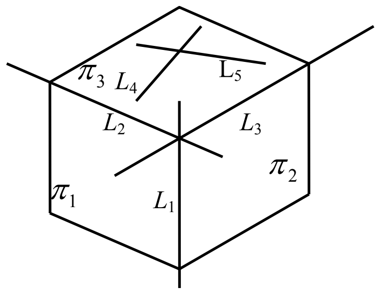

FIG.3 Five lines on three adjacent planes

ρ_i is a scaling factor, by substituting (3.5) into (3.4), we will get

$$M_{123} = \rho_1 \rho_2 \rho_3 N_{123} \tag{3.6}$$

Next, we will put forward a new constrained structure consisting of five lines on three adjacent planes (see FIG.3). This type of structure often emerges on the objects formed by intersected polyhedrons. So it is useful in the practice.

We first give the definition of the structure of five lines on three adjacent planes.

Definition 1. For five lines L_i(i = 1~5) and three adjacent planes π_i (i = 1~3), in which planes π_1 and π_2 intersected at line L_1, π_1 and π_3 intersected at line L_2, π_2 and π_3 intersected at line L_3, L_4 and L_5 on π_3. The line set {L_1, L_2, L_4},{L_1, L_2, L_5},{L_1, L_3, L_4} and {L_1, L_3, L_5} satisfy:

a) the three lines in each line set do not share a common point;
b) they are not parallel to each other.

Based on the lemmas above, we propose a new projective invariant for the structure of five lines on three adjacent planes.

Theorem 1. For the structure of five lines on three adjacent planes there exists one projective invariant I

$$I = \frac{N_{124} N_{135}}{N_{125} N_{134}} \tag{3.7}$$

Proof: Set

$$\begin{cases} \alpha = d_1' d_3' / d_1 d_3 \\ \beta = d_2' d_3' / d_2 d_3 \end{cases} \tag{3.8}$$

according to Lemma 1,

$$\begin{cases} \boldsymbol{N}_{124}' = \alpha \boldsymbol{N}_{124}, \boldsymbol{N}_{134}' = \beta \boldsymbol{N}_{134} \\ \boldsymbol{N}_{125}' = \alpha \boldsymbol{N}_{125}, \boldsymbol{N}_{135}' = \beta \boldsymbol{N}_{135} \end{cases} \tag{3.9}$$

then,

$$\frac{N_{124}' N_{135}'}{N_{125}' N_{134}'} = \frac{\alpha N_{124} \beta N_{135}}{\alpha N_{125} \beta N_{134}} = \frac{N_{124} N_{135}}{N_{125} N_{134}} \tag{3.10}$$

and according to Lemma 2,

$$\frac{M_{124} M_{135}}{M_{125} M_{134}} = \frac{\rho_1 \rho_2 \rho_4 N_{124} \rho_1 \rho_3 \rho_5 N_{135}}{\rho_1 \rho_2 \rho_5 N_{125} \rho_1 \rho_3 \rho_4 N_{134}} = \frac{N_{124} N_{135}}{N_{125} N_{134}} \tag{3.11}$$

equations (3.10) and (3.11) demonstrate that I is a projective invariant and does not vary with the viewpoints.

4. EXPERIMENTS AND RESULTS

Here, we will verify the statement that the proposed projective invariants keep unchanged in the projective transformation between a rigid 3D objects and its perspective views. Firstly, by using a fixed camera, several perspective images of a polyhedral from different viewpoints were obtained (see FIG.4). And then, by applying Canny Operator, the edge of the polyhedral was extracted in each image (see FIG.5). Finally, through the least squares method, the equations of these lines were acquired.

In FIG.5, we will find that the line set $\boldsymbol{S}_1$ containing lines 1, 4, 5, 6, 7 and the line set $\boldsymbol{S}_2$ containing lines 2, 3, 4, 6, 7 can construct the line structure of five lines on three adjacent planes. So, the invariant I can be extracted from these two structures accordingly. The calculation results were listed in table 1.

Table 1. Values of invariant of line structure

	I of $\boldsymbol{S}_1$	I of $\boldsymbol{S}_2$
Model	1.0020	1.0003
(a)	1.0064	0.8506
(b)	1.0421	1.1086
(c)	1.0646	1.0404
(d)	0.9915	0.9906
(e)	0.9340	0.9681
(f)	0.9720	0.9665
(g)	1.0873	0.9432
(h)	1.0323	0.9711
m	1.0163	0.9799
σ	0.0503	0.0745

In table 1, the perspective invariant extracted from 3D model and each image are showed in columns. And we also give the mean m and standard deviation σ for each column.

These data shows that, for the five lines structure, the values of I are slightly different from each other between 3D object and its perspective views. It can be said that the I is a perspective invariant, and does not change over a range of viewpoints.

5. CONCLUSION

In this paper, a new invariant for the structure of five lines on three adjacent planes was proposed. Such invariant often exists in the object formed by intersected polyhedrons. In comparison to six lines on three planes, this structure needs fewer elements which could increase the robustness and accuracy of the recognition. Experimental results showed that the proposed invariant slightly changed in the projective transformation between 3D rigid object and its perspective images from different viewpoints. So, it is valuable in the 3D object recognition from a single view.

6. ACKNOWLEDGEMENTS

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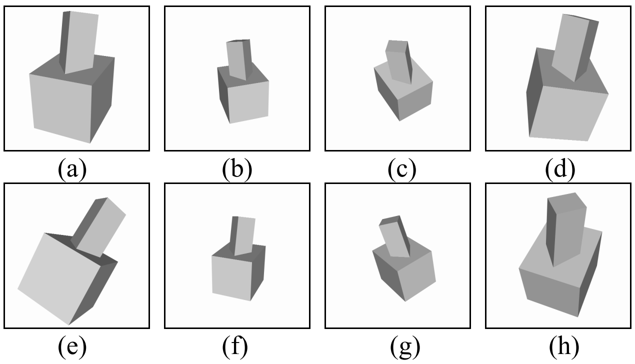
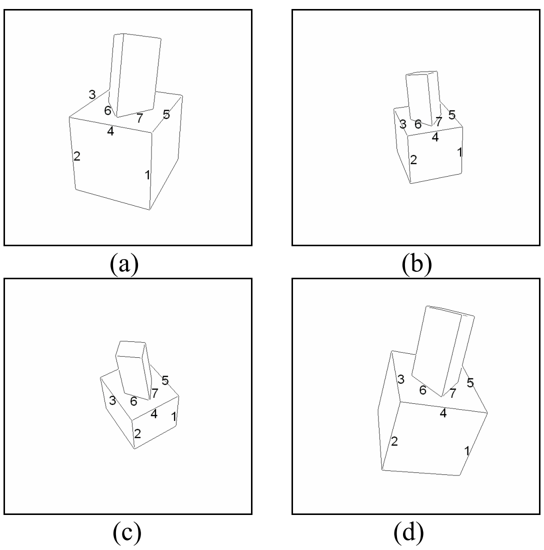


FIG.4 Images of the same object from different viewpoints



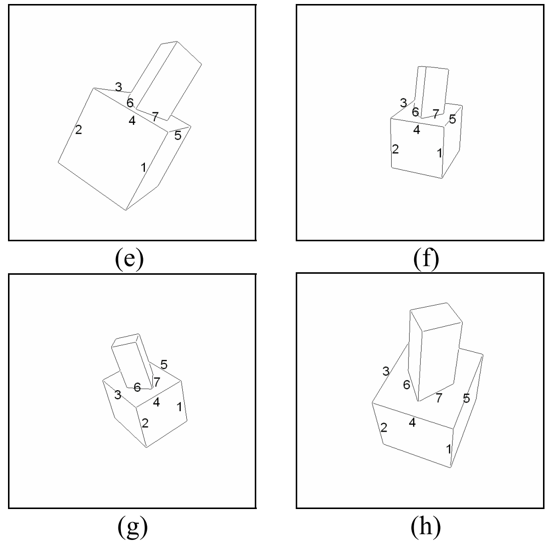


FIG.5 Lines extracted from images

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