PERFORMANCE ANALYSIS OF AN IMPROVED TENSOR BASED CORRESPONDENCE ALGORITHM FOR AUTOMATIC 3D MODELING

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ABSTRACT

3D modeling of a free-form object involves the acquisition of multiple views (range images) of the object to cover its entire surface. These views are then registered in a common coordinate basis by establishing correspondence between them. Our tensor based correspondence algorithm can automatically establish correspondence between overlapping view pairs and register them. In this paper we present an improved version of our tensor based automatic correspondence and registration algorithm and integrate it with a global registration and a surface reconstruction algorithm to make an efficient, automatic and complete 3D modeling framework. We also present a quantitative analysis of this automatic correspondence algorithm based on our results according to the following criteria: robustness to resolution, efficiency, robustness to the amount of overlap and noise.

1. INTRODUCTION

3D models are required by a number of applications including reverse engineering, surgery and computer animation. In order to construct a 3D model of a free-form object, multiple range images from different viewpoints of the object are acquired by 3D scanners to cover its entire surface. These views must then be registered in a common coordinate basis. This is done by keeping sufficient overlap between adjacent views and establishing correspondence between them. Previous correspondence algorithms were manual, inefficient, based on various assumptions or they required initial estimates of registration. We proposed a fully automatic tensor based representation and correspondence algorithm[1] which overcomes all these limitations [1]. In this paper we present an improved and optimized version of this algorithm and its analysis. This version utilizes much less memory than its predecessor and the execution time is also improved many folds. We integrate the improved algorithm with a global registration, volumetric integration and reconstruction algorithm to form a fully automatic, efficient and complete 3D modeling system. We then present the results of our 3D modeling experiments when performed on real and synthetic data. We also present quantitative analysis of our improved algorithm according to the criteria given in the abstract.

2. AUTOMATIC 3D MODELING

Fig. 1 shows the block diagram of our fully automatic 3D modeling algorithm. It takes an ordered set of input views (in the form of point clouds) of an object and converts them into triangular meshes. The approximate x,y and z dimensions $\mathbf{D}$ of the object are estimated from the input views according to Eqn. 1 ($\mathbf{P}_i$ is the rotation matrix that aligns the ith mesh $\mathbf{M}_i$ on its principle axis). All the remaining parameters (including the ones required for the calculation of tensors) are derived from $\mathbf{D}$. The meshes are first reduced using Garland's algorithm [2] to form $\mathbf{M}'_i$. Next, third order tensors (using surface area only) are calculated for the meshes [1]. Mesh reduction is added in this version of the algorithm to achieve computational and memory efficiency by reducing the number of points per view and hence the number of tensors required to represent them. Memory utilization is further improved by reducing the tensors to sparse arrays. Since most elements of the tensors are zeros, reducing the tensors to sparse arrays reduces the memory utilization by 85%. The total number of tensors that are generated to represent a view is limited to $3n$ (where n is the number of points per view) by allowing a point to participate in the calculation of a maximum of three tensors. Moreover, tensors that have less than 5% of the entries as non-zero are discarded because they are unlikely to give correct matches.

$$\mathbf{D} = \max_{\mathrm{xyz}}(\max_{\mathrm{xyz}}(\mathbf{M}_i\mathbf{P}_i) - \min_{\mathrm{xyz}}(\mathbf{M}_i\mathbf{P}_i)) \quad (1)$$

The three tuple ($\mathbf{M}_i, \mathbf{M}'_i$, tensor representations) and parameters are then fed to our tensor based automatic correspondence and registration algorithm (pseudo-code given in Fig. 2). For each pair of overlapping meshes say $\mathbf{M}_m$ and $\mathbf{M}_s$, each tensor $\mathbf{T}_m$ of $\mathbf{M}'_m$ is matched with each tensor $\mathbf{T}_s$ of $\mathbf{M}'_s$. The matching proceeds as follows. First, the overlap ratio R_O of the two tensors is calculated by dividing the intersection of the occupied bins of $\mathbf{T}_m$ and $\mathbf{T}_s$ by the union of their occupied bins [1]. If R_O is greater than

[1] Australian Patent Application No. 2004902436

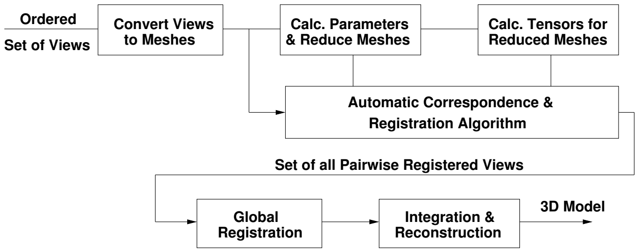

Fig. 1. The fully automatic 3D modeling system.

a threshold t_r (set to 0.5 in our experiments) the correlation coefficient C_c of the two tensors is calculated in their region of overlap. If C_c is greater than a threshold t_c (also set to 0.5), the tensors are considered as matching and the $\mathbf{M}'_s$ is transformed to the coordinates of $\mathbf{M}'_m$ by transforming the coordinate basis of $\mathbf{T}_s$ to $\mathbf{T}_m$ [1]. Next, the match is *locally verified* as follows. The bounding box of the combined meshes $\mathbf{M}'_{ms}$ is calculated (Eqn. 1) and compared to $\mathbf{D}$ of the object. If the maximum difference between the two is less than a tolerance t_D ($t_D = \text{mean}(\mathbf{D})/10$), $\mathbf{M}_m$ and $\mathbf{M}_s$ are also aligned and points on the two meshes that are within a distance $2d_{res}$ (where d_{res} is the resolution of the fine meshes) are turned into correspondences. If the number of correspondences found are more than a threshold n_c (n_c=min(number of points of $\mathbf{M}_m$ and $\mathbf{M}_s$)/4) the transformation is refined with a variant of the ICP [3]. Correspondences are established once again between the points of the two views that are within a distance of d_{res}. If the total number of correspondences are more than $2n_c$, the dimensions of the registered meshes are calculated (Eqn. 1) and compared with $\mathbf{D}$. If the maximum difference between the two dimensions is less than a tolerance $2d_{res}$ the algorithm proceeds to the `global-verification` step. If any of the `local-verification` steps fails another set of tensors is considered for matching.

Using the above procedure and by concatenating transformations all the meshes are registered in the coordinates of a reference mesh. Each time a new mesh is added to the registered meshes, the registration is verified globally. `Global-verification` is performed by calculating the combined dimensions (Eqn. 1) of all the meshes registered so far and comparing them with $\mathbf{D}$. If the maximum difference between the two is less than $4d_{res}$, the newly added mesh is accepted. If `global-verification` fails, the automatic correspondence algorithm is repeated and the next pair of tensors is matched. Once all the meshes are registered, correspondences are established between all pairs of overlapping meshes. This exhaustive list of correspondences is fed to a global registration algorithm [4] which registers the models globally, distributing the registration errors evenly over the 3D model. Finally, the meshes are integrated and reconstructed using VripPack [5] which uses an integration algorithm by Curless and Levoy [6] and the marching cubes algorithm [7] for reconstruction.

The Automatic Correspondence and Registration Algorithm

Input : $\mathbf{M}_i, \mathbf{M}'_i$ *(where* $i = 1, 2, \ldots, N$*), Tensors*

Output : *Transformations, Correspondences*

for each pair of overlapping views $\mathbf{M}_m$ and $\mathbf{M}_s$
  for each tensor $\mathbf{T}_m$ of $\mathbf{M}_m$ and $\mathbf{T}_s$ of $\mathbf{M}_s$
    if $R_O \geq t_r$ **AND** $C_c \geq t_c$
      Transform $\mathbf{M}'_s$ to $\mathbf{M}'_m$
      Calculate bounding dimensions $\mathbf{D}'_{ms}$ of $\mathbf{M}'_{ms}$
      if max $(\mathbf{D}'_{ms} - \mathbf{D}) \leq t_D$
        Transform $\mathbf{M}_s$ to $\mathbf{M}_m$
        Establish correspondences between $\mathbf{M}_m$ and $\mathbf{M}_s$
        if the number of correspondences $\geq n_c$
          Refine registration with ICP
          Establish correspondences between $\mathbf{M}_m$ and $\mathbf{M}_s$
          if the number of correspondences $\geq 2n_c$
            Calculate bounding dimensions $\mathbf{D}_{ms}$ of $\mathbf{M}_{ms}$
            if max $(\mathbf{D}_{ms} - \mathbf{D}) \leq 2d_{res}$
              Find dimensions $\mathbf{D}_L$ of the L views registered so far
              if max $(\mathbf{D}_L - \mathbf{D}) \leq 4d_{res}$
                RETURN Transformation and Correspondences

Fig. 2. Pseudo-code of the automatic correspondence & registration algorithm.

3. 3D MODELING RESULTS

We performed our experiments on seven real objects and three synthetic models. For the real objects, 20 to 26 views (in the form of point clouds from [8]) were taken and 3D models were built from them using our automatic 3D modeling algorithm. Fig. 3 shows the results of real objects as well as synthetic models. Since the ground truth data was not available in the case of real objects, the resultant 3D models were analyzed qualitatively. The registered views were observed closely but no misalignments nor seams could be noticed. The models were also analyzed after reconstruction for surface irregularities and no visually noticeable defects were found. In the case of synthetic data, existing 3D models namely, Happy Buddha, Stanford Bunny and Armadillo, were taken from [9] and 26 synthetic views of each model were generated from different viewpoints (30^o apart) using a z-buffer. The ground truth rotation matrix and translation vector ($\mathbf{R_{iGT}}$ and $\mathbf{t_{iGT}}$) were recorded for each view i. Next, the views were registered using our automatic algorithm and the resultant transformations ($\mathbf{R_i}$ and $\mathbf{t_i}$) were compared to the ground truth transformations using Eqn. 2 and Eqn. 3. Eqn. 2 is derived from Rodrigue's formula. θ_{ie} and t_{ie} are the rotation (about a single axis) and translation errors of view i respectively. t_{ie} is normalized with mesh resolution to make it scale independent. Fig. 4 shows the histograms of the rotation and translation errors of the complete synthetic data set.

$$\theta_{ie} = \cos^{-1}\left(\frac{trace(\mathbf{R_i}\mathbf{R_{iGT}^{-1}}) - 1}{2}\right)\frac{180}{\pi} \quad (2)$$

$$t_{ie} = \frac{||\mathbf{t_i} - \mathbf{t_{iGT}}||}{\text{mesh resolution}} \quad (3)$$

4. ANALYSIS

We performed extensive tests of our automatic correspondence and registration algorithm according to the following criteria: robustness to resolution, efficiency, robustness to the required amount of overlap and robustness to noise. In the first test, we checked the performance of our algorithm with varying mesh resolution of the data set of seven real objects. Fig. 5 (left column) shows the number of correctly and incorrectly matched view pairs at different resolutions (number of faces, points and tensors). Fig. 5 (right column) shows the %age of correct matches with varying resolution. 80% correct matches are found at a very low resolution of 200 faces (corresponding to 175 points and 300 tensors).

Since a very small number of tensors is sufficient to find a correct match, one can conclude that the matching process is very efficient. Fig. 6 shows the histogram of the number of correct matches found (for the data set of seven real objects) verses the number of view 1 tensors matched. Most of the matches are found while matching the first 50 tensors. The median number of tensors that are matched is 26.

The overlap test was performed on the data set of four real objects namely, the bone, the dinosaur, the dog and the biplane. Fig. 7 shows the number of correctly and incorrectly matched view pairs with different amount of overlaps. The results vary with the type of object but generally a 50% overlap ensures a correct match.

The last test of our experiments checks the robustness of our algorithm to noise. During this test we injected Gaussian noise with different standard deviations into the data set of the real objects and applied our automatic algorithm to register them. We found that our algorithm can correctly register view pairs containing noise with standard deviations as high as four times the resolution of the meshes (note that sensors generally contain noise less than their resolution). Fig. 8 shows the registration of two views injected with Gaussian noise. The robustness of our algorithm to noise is contributed to two main reasons. First, our algorithm has a mesh reduction step right at the beginning of the process. During this step the original mesh is approximated by a reduced mesh which smooths the surface to a great extent (Fig. 8 second column). Second, our algorithm uses a correlation coefficient to match tensors. Correlation coefficient being a statistical measure is less sensitive to noise.

5. CONCLUSION

We presented a fully automatic 3D modeling framework using an improved version of our tensor based automatic correspondence and registration algorithm [1]. We also presented qualitative and quantitative analysis of the results of our 3D modeling technique. We performed extensive testing of the improved automatic correspondence and registration algorithm according to four important criteria. Quantitative results of the tests were presented. They show that

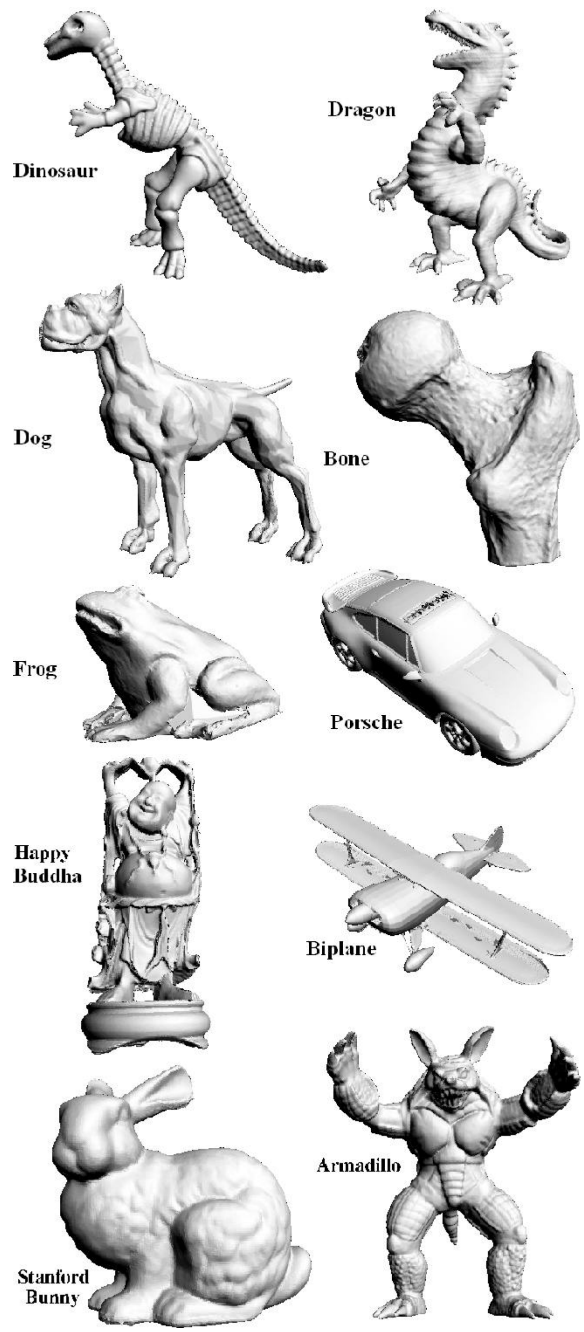

Fig. 3. 3D modeling results.

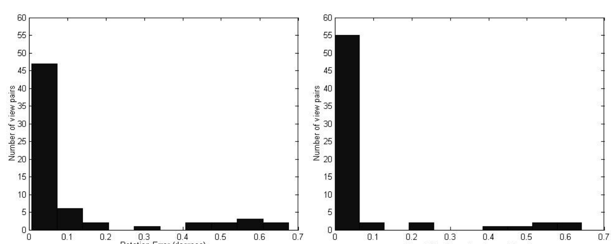

Fig. 4. Transformation errors of the synthetic data set.

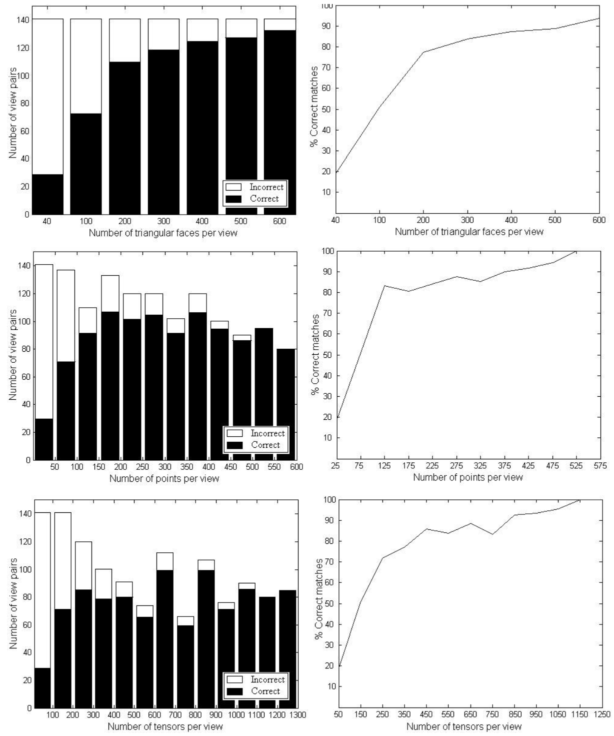

Fig. 5. Performance of the automatic correspondence and registration algorithm with varying resolution.

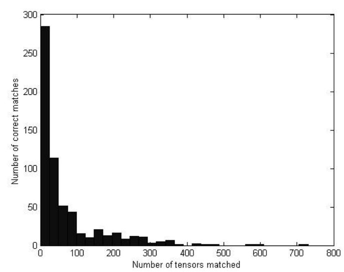

Fig. 6. Histogram of correct matches verses the number of tensors matched.

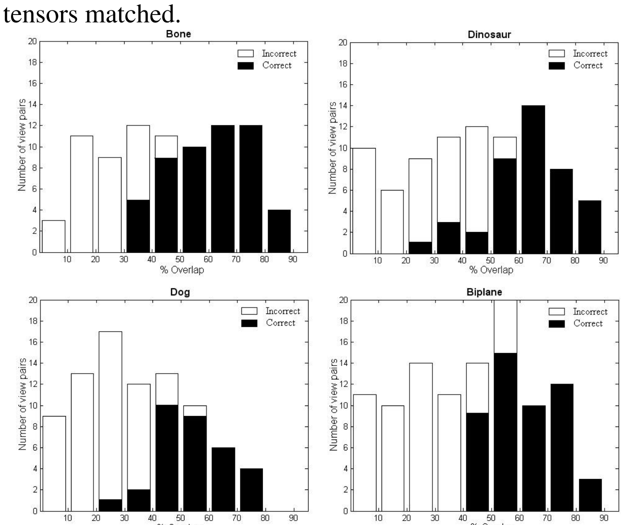

Fig. 7. Performance of the automatic correspondence and registration algorithm as a function of overlap.

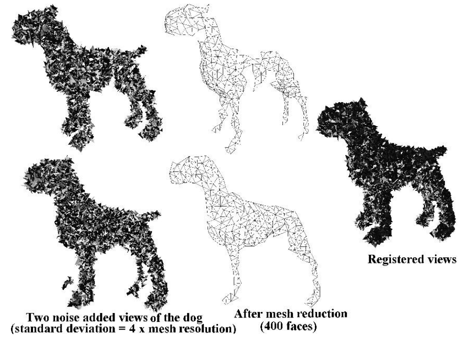

Fig. 8. Registration of noisy views.

our algorithm is independent to the resolution of the views, efficient, requires only 50% overlap and is robust to noise.

6. ACKNOWLEDGMENTS

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7. REFERENCES

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