SURFACE RADIANCE: EMPIRICAL DATA AGAINST MODEL PREDICTIONS

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ABSTRACT

In this paper we report an empirical study of departures from Lambert's law for rough and shiny surfaces. We use a recently reported method to recover estimates of the surface radiance function for objects illuminated in the viewer direction. We compare the radiance data with a number of phenomenological and physics-based reflectance models. The models studied include those of Oren-Nayar, Wolff and different variants of the Beckmann model. Our experiments show that among these models the best fit to the empirical data is found to be the Wolff model for smooth objects and the modified Beckmann model for rough objects.

1. INTRODUCTION

The modelling of rough surface reflectance is important in both computer vision and computer graphics, and has been the subject of sustained research activity for some four decades. In fact the quest for a reflectance model that can accurately account for the observed surface radiance under a variety of roughness conditions, and a variety of viewing geometries has proved to be an elusive one [2]. Surface roughness can be characterized in a number of ways. For very-rough surfaces, one approach is to use a model which describes the distribution of surface wall cavity angles [3]. For slightly-rough surfaces, roughness can be modelled using the angular distribution of microfacets [6]. An alternative that can capture both effects is to describe the roughness phenomenon using the variance and the correlation length of variations in the surface height distribution [1].

In computer vision there have been a number of recent attempts to model rough surface reflectance in a phenomenological way. These models attempt to account for roughness as departures from Lambert's cosine law. For instance, Oren and Nayar [3] use the surface cavity picture to model very-rough surfaces. The departures from Lambert's law are greatest at large light scattering angles, and are accounted for by adding a sine-squared term. For smooth surfaces, on the other hand, Wolff [8] has a model motivated by physics. Here refractive attenuation in the surface-air layer is modelled by multiplying Lambert's cosine law by two Fresnel terms, one for incident light and one for reflected light. For surfaces of medium roughness, Wolff et al. [9] combine the two models, and have an additive roughness term and a multiplicative Fresnel term.

Some of the earliest work in the area was undertaken by Beckmann [1], who used the Kirchhoff integral to model the wave scattering of light from surfaces characterized using the variance and correlation length of the surface relief distribution. The model is mathematically quite complex, and is hence not well suited for analysis tasks of the type encountered in computer vision. In particular, it is not a simple modification of Lambert's law. However, He et al. [2] exploited an improved representation of the Kirchhoff integral for surface synthesis in computer graphics. They proposed a comprehensive model that incorporates complex factors including surface statistics, sub-layer scattering, and polarization. Unfortunately, the Beckmann model fails to account for the observed radiance at large scatter angles due to self shadowing and multiple scattering effects. Some of the problems have recently been overcome by Vernold and Harvey [7] who have used a simple Lambertian form-factor to modify Beckmann's predictions.

Although there are clearly a number of reflectance models that can be used to describe reflectance from real-world surfaces, there has been relatively little effort aimed at comparing their predictions against empirical reflectance data. One of the reasons for this is that performing measurements of the BRDF (bidirectional reflectance distribution function) is a time consuming and laborious process which the four degrees of freedom of the BRDF to be systematically varied. Recently, Robles-Kelly and Hancock [5] have reported a method that can be used to estimate the surface radiance function from complex surfaces without the need to accurately measure the four angular variables, and without the need for accurate light-source or camera calibration. The method can be applied to surfaces with isotropic and homogeneous reflectance properties. The method applies when the viewer and light-source directions are almost identical, and makes use of the cumulative distribution of image intensity gradients to estimate the surface radiance function. The aim in this paper is to use this method and to describe how the surface radiance estimates can be used to test the various reflectance models described earlier. We experiment with both rough and shiny surfaces. The main conclusions of our study are that in the case of rough surfaces, the best fit to the empirical data are given by the modified version of the Beckmann model. For smooth surfaces, on the other hand, the Wolff model gives the best possible fit.

2. BECKMANN-KIRCHHOFF MODEL

The Beckmann-Kirchhoff (B-K) theory attempts to account for the wave interactions of light with rough surfaces. Beckmann's contribution was to show how to apply the Kirchhoff theory to rough surfaces, and how to obtain simplifications to the Kirchhoff integral under different roughness conditions [1]. By requiring that the surface has both a Gaussian height distribution and a Gaussian correlation function, according to the B-K model the total scattered intensity is

$$I(\theta_i, \theta_r, \phi_r; \sigma, T) = I_0 e^{-g} + [\pi T^2 F^2(\theta_i, \theta_r, \phi_r) e^{-g}/A] \times \sum_{n=1}^{\infty} [(g^n/n!n) \exp(-v_{xy}^2 T^2/4n)] \quad (1)$$

The incident beam has zenith and azimuth angles θ_i and ϕ_i, and the reflected beam has zenith and azimuth angles θ_r and ϕ_r (on local tangent planes). From trigonometry it follows that $v_x = k(\sin\theta_i - \sin\theta_r \cos\phi_r)$, $v_y = -k(\sin\theta_r \sin\phi_r)$, $v_z = -k(\cos\theta_i + \cos\theta_r)$, $v_{xy}^2 = v_x^2 + v_y^2$, $g = \sigma^2 v_z^2$ and $k = 2\pi/\lambda$, where λ is the wavelength. The parameter σ is the root-mean-square (RMS) height deviation of the topographic surface features about the mean surface level. The correlation length T is the lag-length at which either the Gaussian or the exponential correlation function drops to $1/e$ of its maximum. The geometrical term $F(\theta_i, \theta_r, \phi_r)$ is given in [1]. However, in this paper we use some alternative terms to improve the model.

The quantity g has been used in the literature to divide surfaces into three broad categories, i.e. slightly-rough ($g \ll 1$), moderately-rough ($g \approx 1$) and very-rough ($g \gg 1$) surfaces. The parameter A is the area of a plane sheet on which the scattering coefficient I_0 is defined [1]. The first term in Eq. (1) determines the specular component. The scattering coefficient I_0 vanishes everywhere except near the direction of specular reflection. The second term, i.e. the infinite series, determines the diffuse component. The **first** interesting case arises when the surface is **slightly rough**, and so the series converges rapidly. In practice, only the first term needs to be considered and the diffuse intensity is

$$I_d(\theta_i, \theta_r, \phi_r) \approx (\pi g T^2 F^2/A) \exp[-(g + T^2 v_{xy}^2/4)] \quad (2)$$

When we consider the specular direction ($\theta_i = \theta_r$ and $\phi_r - \phi_i = \pi$), then both v_x and v_y vanish and $F = 1$. Hence the optical intensity is proportional to $\exp(-g)$. This simple equation may be used for surface roughness estimation [4].

The **second** interesting case arises when the surface is **very rough** when compared to the test wavelength. Under these conditions the specular term becomes insignificant and the equations for total intensity and diffuse intensity are identical. When the surface correlation function is assumed to be exponential, the total (or diffuse) intensity is

$$I(\theta_i, \theta_r, \phi_r) \approx (2\pi F^2 T^2/Ag)(1 + v_{xy}^2 T^2/g)^{-3/2} \quad (3)$$

Although the use of the Gaussian correlation function is more common in literature, here we confine our attention to the exponential correlation function since it gives a better fit to measured surface data for the surfaces under our study.

Vernold-Harvey Modification: Vernold and Harvey [7] have recently modified the B-K model by replacing the geometrical factor F^2 used in the original model with a Lambertian term, i.e. $\cos(\theta_i)$. This modification gives reasonable experimental agreement with scattering data for rough surfaces at large angles of incidence and large scattering angles. Note that the modification relies on empirical and phenomenological arguments. However, this does not diminish its pragmatic use since it compensates for some effects (due to non-paraxial angles) which are not included in the original B-K model. The B-K model depends on both the incidence and the reflectance angles. However, in this paper it is mainly the incidence angle behavior of the reflectance models that is of interest. Hence, here we derive a formula for the case of $\vec{L} \approx \vec{V}$, where $\theta_i = \theta_r = \theta$ and $\phi_r = \phi_i$. Hence, the Vernold-Harvey (V-H) modification to the variant for very-rough surfaces, given by Eq. (3), is

$$I(\theta) \approx 1/\{\cos(\theta)[1 + (\tan^2(\theta)/m^2)]^{3/2}\} \quad (4)$$

Note that the physical properties of the surface are captured by the surface slope $m = \sigma/T$. We also apply the V-H modification to the variant for slightly-rough surfaces. Hence the modified B-K model for slightly-rough variant, given by Eq. (2), for the case of $\vec{L} \approx \vec{V}$ is

$$I_d(\theta) \approx \cos^3\theta \exp\{-k^2[(T\sin\theta)^2 + (2\sigma\cos\theta)^2]\} \quad (5)$$

Fresnel Correction: One of the problems with the B-K model [1] is that although it provides specific equations for slightly-rough and very-rough surfaces, this is not the case for moderately-rough surfaces. To address this omission, here we examine the effect of combining the Fresnel coefficient and the B-K model to develop a model that is applicable to surfaces of intermediate roughness. The Fresnel coefficient has been widely used for reflectance modelling. For instance, Wolff [8] used it to develop a model for smooth surfaces by correcting the Lambertian model. Torrance and Sparrow [6] included the Fresnel coefficient in their specular intensity model. It is also used in the complex reflectance model of He et al. [2] which attempts to account for a number of effects including subsurface scattering.

Our approach is similar to that followed by Wolff et al. [9] which combines the Fresnel term with the Oren-Nayar model to develop a model for moderately-rough surfaces. We exploit the Fresnel model used by Wolff which includes the effects of both incidence and reflectance angles. Here the geometrical term F^2 in the B-K model given by Eq. (3) is replaced by a correction term to produce a new variant. Whereas Vernold and Harvey [7] have replaced F^2 by $\cos\theta_i$, we replace it by the correction term

$$\mu^2 = \{1 - f(\theta_i, n)\}\{1 - f(\sin^{-1}[(\sin\theta_r)/n], 1/n)\}\cos^2\theta_i \quad (6)$$

The correction hence combines the advantages delivered by the V-H modification [7] with some additional ones too. For instance, the conditions for switching from one equation to another in the B-K model is ambiguous and causes practical problems, specifically when the surfaces under study are of the medium or unknown roughness. This problem may be overcome if the Fresnel corrected form of the B-K model is used instead over a wider range. When $\vec{L} \approx \vec{V}$, then the Fresnel corrected variant is given by

$$I_d(\theta) \approx [1 - f(\theta, n)]^2 / [1 + (\tan^2(\theta)/m^2)]^{3/2} \quad (7)$$

3. OREN-NAYAR MODEL

According to the Oren-Nayar reflectance model [3], the rough surface is composed of extended symmetric V-shaped cavities. Each cavity consists of two planar facets. The roughness of the surface is specified using a probability distribution function for the facet slopes. Finally, each facet is assumed to exhibit a Lambertian behavior. In this paper, we use their qualitative model in which inter-reflections are ignored. Also, we rewrite the model for the case where $\vec{L} \approx \vec{V}$. Hence, for a surface with a roughness parameter σ_α and incidence angle θ the reflected radiance is

$$L_r(\theta, \sigma_\alpha) = A \cos\theta + B \sin^2\theta \quad (8)$$

The dimensionless parameters A and B are only dependent on the surface roughness σ_α which measures the distribution of cavity wall slope angles, and is hence measured in degrees or radians. The correction to Lambert's law, which is additive and proportional to $\sin^2\theta$, is greatest at the occluding boundary, and hence results in limb brightening.

4. WOLFF MODEL

Wolff has developed a physically motivated model for diffuse reflectance from smooth surfaces [8]. The model accounts for subsurface refraction using a Fresnel attenuation factor. Again, we rewrite the Wolff model to the case where $\vec{L} \approx \vec{V}$. Under these conditions the reflected radiance is

$$L_r(\theta, n) = \cos\theta[1 - f(\theta, n)]^2 \quad (9)$$

The Fresnel coefficient ($0 \leq f \leq 1$) is given by

$$f(\alpha_i, r) = \frac{1}{2}\frac{\sin^2(\alpha_i - \alpha_t)}{\sin^2(\alpha_i + \alpha_t)}\left[1 + \frac{\cos^2(\alpha_i + \alpha_t)}{\cos^2(\alpha_i - \alpha_t)}\right] \quad (10)$$

According to Snell's law, the transmission angle of light into the surface α_t is given by

$$r = (\sin\alpha_i)/(\sin\alpha_t) \Rightarrow \quad \alpha_t = \sin^{-1}[(\sin\alpha_i)/r] \quad (11)$$

Almost for all commonly found dielectric materials, the index of refraction n is in the range $[1.4, 2.0]$, and so, the Fresnel coefficient is weakly dependent upon n. When light is transmitted from air into a dielectric $r = n$ and $\alpha_i = \theta_i$. However, when transmission is from a dielectric into air, then $r = 1/n$ and $\alpha_i = \sin^{-1}[(\sin\theta_r)/n]$.

5. EXPERIMENTS

In this section we experiment with terracotta and porcelain objects. These objects have been illuminated using a single collimated tungsten light-source with a parallel beam. We have used a red filter which provides a single wavelength of $\lambda = 0.65\mu m$. Both the camera (viewing) direction $\vec{V}$ and the light-source direction $\vec{L}$ are aligned with the z axis. In practice, the light-source and camera are slightly displaced from the ideal configuration in the vertical plane.

In what follows, we compare the radiance data extracted from real-world images with the corresponding reflectance models. The empirical radiance data is extracted from real-world images using the Gauss map method described in [5]. We use four matte terracotta objects, which have rough surfaces, and four shiny porcelain objects, with smooth surfaces. For each model, we need to estimate the values of its parameters so that it can be fitted to the data. Specifically, for the B-K model, one way is to use our recently developed method [4] and estimate different roughness parameters for the different model variants. However, here we use empirical values for each parameter so that the best possible fit is obtained for the different models to the data. It is interesting to note that for the B-K model, we found that the empirical roughness values were very close to those estimated using the method described in [4].

Terracotta Objects: The top row of Fig. 1 shows the images of terracotta objects used in our study, while in the bottom row we compare the normalized radiance data extracted from the terracotta objects with the V-H modification variant for very-rough surfaces (Eq. 4, m=1.5) and the Oren-Nayar model (σ_α = 0.2rd). The values of the parameters are chosen to give the best fit to the data. Here, both models fit the radiance data for most incidence angles. However, the Oren-Nayar model fails at large angles due to the limb brightening effect. Finally, the poorest agreement with the radiance data is given by Lambert's law.

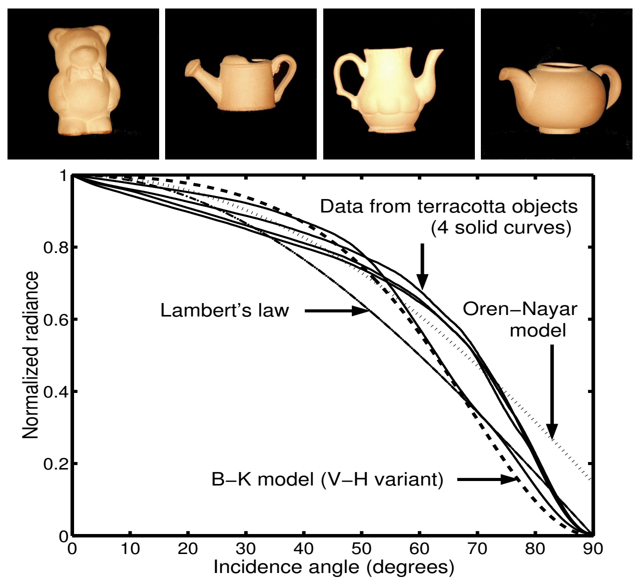

Fig. 1. (1) Images of terracotta objects; (2) radiance versus θ_i compared to the Oren-Nayar model and modified B-K model.

Porcelain Objects: In Fig. 2 we show the porcelain objects used in our study. Because these images contain specular highlights, we perform the comparisons in two ways as follows. **First**, we compare the total radiance including both diffuse and specular components with the modified B-K model. In the second row of Fig. 2 we show the normalized total radiance from porcelain objects versus incidence angles as solid curves. The prediction of total radiance using the variant of the B-K model that applies to slightly-rough surfaces is shown as a dashed curve. Here, the diffuse component is computed using the model of Eq. (5). Also, the specular component is computed using the first term of Eq. (1) which is proportional to $\exp(-g)$. We have used $\sigma = 0.065\mu m$ and $T = 0.1\mu m$ as the values of the roughness parameters. There is a good agreement between the model and the data for most incidence angles.

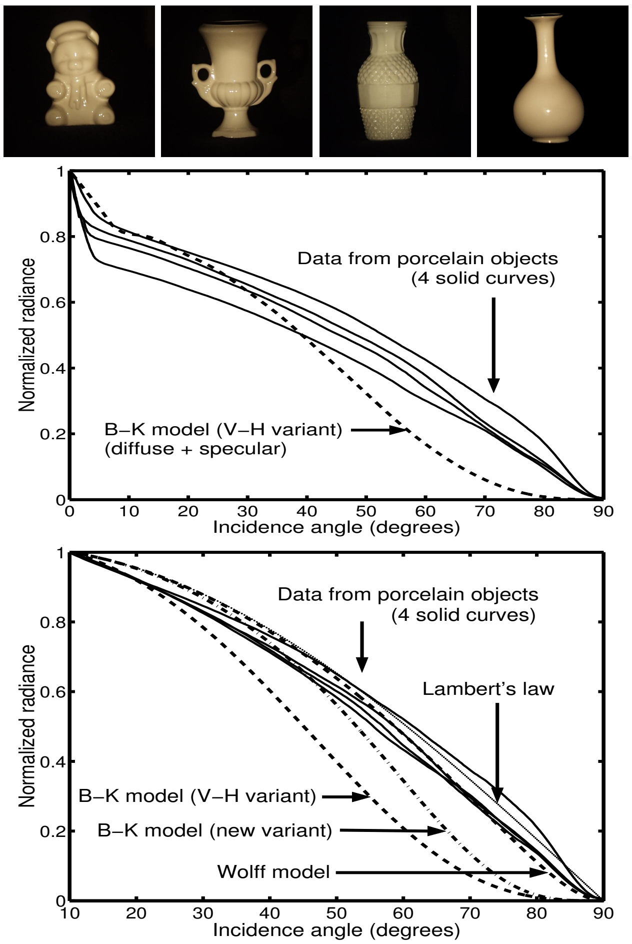

Fig. 2. (1) Images of porcelain objects; (2) total radiance versus θ_i compared to the modified B-K model; (3) radiance versus θ_i for $\theta_i > 10^\circ$ compared to the Wolff model, the modified B-K model (Eq. 5) and the new variant of the B-K model (Eq. 7).

Second, we assume that for off-normal incidence angles the diffuse component is dominant and the specular component is negligible. Hence, we compare the diffuse radiance with the modified B-K model and the Wolff model. In the bottom row of Fig. 2 the solid curves show the radiance data extracted from the porcelain objects using the Gaussian map method. The dashed curves show the V-H modification of the B-K model (Eq. 5, $\sigma = 0.065\mu m$, $T = 0.1\mu m$) and the Wolff model ($n = 1.7$). The dash-dot curve shows our new Fresnel correction variant of the B-K model (Eq. 7, $m = 0.65$). The slope parameter value of this new variant, under the assumption of a moderately-rough surface, is found using the same values of σ and T that is used for the slightly-rough surface variant, i.e. $m = \sigma/T$. Although the Lambertian curve is close to the radiance data curves, the Wolff model gives the best agreement with the data. Note that none of these two models account for surface roughness variations. The modified B-K model for slightly-rough surfaces cannot be fitted to the data. The main reason for this is that this variant can only be used where $g << 1$. As σ is increased, g is increased too. This means that we have to use the variant that applies to moderately-rough surfaces. Since there is no such variant available, we use our Fresnel correction variant (Eq. 7) instead. Fig. 2 shows that the new variant for moderately-rough surfaces provides a better fit to the data than the existing variants of the B-K model.

6. CONCLUSIONS

This paper has described a study aimed at testing a number of phenomenological and physics-bases reflectance models against empirical data. We use estimates of the required radiance distribution using a novel method (described in [5]) that makes use of the cumulative distribution of image gradients. For rough surfaces, the best fit to the scattering data is given by the Vernold-Harvey modification of the Beckmann model. For smooth surfaces, we have improved the results by modifying the Beckmann model using a Fresnel correction term. However, the best fit to the radiance data for off-normal angles are given by the Wolff model.

7. REFERENCES

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