COMPARISON OF LINEAR SPECTRAL RECONSTRUCTION METHODS FOR MULTISPECTRAL IMAGING

David Connah, Jon Y. Hardeberg

The Norwegian Color Research Laboratory,
Dept. of Computer Science and Media Technology,
Gjøvik University College

Stephen Westland

School of Design
University of Leeds

ABSTRACT

This paper compares the performance of a number of linear methods for reflectance estimation from digital camera responses. Methods based upon smoothness maximisation, linear models of reflectance and least-squares fitting are compared using two simulated 6-channel camera systems. In our experiments the smoothness methods were generally found to deliver the best performance on test data. Furthermore, they deliver equivalent performance on training data, even compared to those methods that make explicit use of *a priori* knowledge of the training data.

1. INTRODUCTION

A common application of digital cameras is to use them as tools for colour measurement. They offer an advantage over traditional colour measurement instruments, such as spectrophotometers, in that they can measure a large number of colours simultaneously and at a high spatial resolution. Although it is possible to use 3-channel RGB cameras to perform this task, multispectral colour imaging, i.e. using more than three colour channels, is required to produce a colour representation that is accurate and both device and illuminant independent.

A subject of ongoing research in this field is to find an appropriate mathematical method to recover reflectance from camera responses, and a number of different solutions have been proposed and tested in the literature [1, 2, 3, 4, 5, 6, 7, 8]. Some of the more elegant solutions (e.g.[2]) involve computationally intensive procedures, such as linear or quadratic programming. Although these methods are useful for some applications, they can be too demanding for processing entire images. In this paper we will consider only linear methods, which can be implemented by a single matrix multiplication for each pixel and are therefore not computationally intensive. It should be noted however that these methods do not always produce physically realisable estimates (with reflectance values between 0 and 1).

Several recovery methods have been demonstrated and tested in the literature, but there is a need to compare them in order to find out which one, if any, can provide the best performance. This paper will therefore compare the performance of a number of these methods. We will show recovery results for simulated imaging systems with realistic noise parameters, constructed using measurements of real imaging components.

2. BACKGROUND

If we assume a world in which all surfaces are Lambertian and there is no fluorescence, the responses of a camera system to a surface with reflectance $R(\lambda)$ under an illuminant $E(\lambda)$ can be modelled by

$$P_i = \int_\lambda Q_i(\lambda)\, E(\lambda)\, R(\lambda)\, d\lambda + n \qquad (1)$$

where P_i is the response of the i^{th} camera channel, $Q_i(\lambda)$ is the spectral sensitivity of that channel and n is a random value that represents both signal independent and signal dependent measurement noise.

If we assume that all continuous functions of wavelength can be sampled at regular intervals without a loss of information, then the integration in (1) can be re-written as a matrix-vector multiplication thus:

$$\mathbf{p} = \mathbf{Q}_e^T \mathbf{r} + \mathbf{n}, \qquad (2)$$

In this work all functions are sampled at 10nm intervals from 400 to 700 nm therefore, in (2), $\mathbf{r}$ is a 31 element column vector, $\mathbf{Q}_e$ is a $31 \times p$ matrix whose columns contain the wavelength-by-wavelength product of the sensor sensitivity and illuminant, $\mathbf{p}$ is a p-element vector of sensor responses and $\mathbf{n}$ is a p-element sensor noise vector. The reflectance recovery problem is, therefore, to recover the original reflectance $\mathbf{r}$ given a vector of camera responses $\mathbf{p}$.

2.1. Least squares

A common solution is to build a mapping from camera responses to reflectance that minimises the least-squares-error

0-7803-8554-3/04/$20.00 ©2004 IEEE.

for a characterisation or training set of known reflectance functions with known camera responses [2, 3]. Therefore, given k reflectances in the characterisation set we seek a $31 \times p$ matrix $\mathbf{T}$, where p is the number of sensors, that transforms sensor responses to reflectance such that

$$\min_{\mathbf{T}} \sum_{i=1}^{k} \|\mathbf{r}_i - \mathbf{T}\mathbf{p}_i\|^2 \tag{3}$$

where $\mathbf{r}_i$ is reflectance of the i^{th} surface of the characterisation set, $\mathbf{p}_i$ is the corresponding camera response and $\|\mathbf{x}\|$ is the L2-norm of a vector $\mathbf{x}$ defined by $\left(\mathbf{x}^T\mathbf{x}\right)^{\frac{1}{2}}$. If we place the $\mathbf{r}_i$ in the columns of a $31 \times k$ matrix that we will call $\mathbf{R}$, and the $\mathbf{p}_i$ in the columns of a $p \times k$ matrix $\mathbf{P}$ then the solution to (3) is given by

$$\mathbf{T} = \mathbf{R}\mathbf{P}^T \left(\mathbf{P}\mathbf{P}^T\right)^{-1} \tag{4}$$

where $\mathbf{P}^T \left(\mathbf{P}\mathbf{P}^T\right)^{-1}$ is known as the pseudo-inverse of the matrix $\mathbf{P}$. This solution is termed the least-squares (LS) solution. Assuming zero noise in the acquisition we can replace $\mathbf{P}$ in (4) by $\mathbf{Q}_e^T\mathbf{R}$, according to (3), to give a reflectance estimate $\mathbf{r}'$ of

$$\mathbf{r}' = \mathbf{R}\mathbf{R}^T\mathbf{Q}_e \left(\mathbf{Q}_e^T\mathbf{R}\mathbf{R}^T\mathbf{Q}_e\right)^{-1} \mathbf{p} \tag{5}$$

Note that this estimate is dependent upon the properties of the raw-cross-product matrix $\mathbf{R}\mathbf{R}^T$ of the characterisation set [9].

When using this method the solution is strongly tied to the choice of characterisation set, which may not be representative of the reflectances that will be encountered by the system. Unlike scanners, digital cameras are likely to be presented with reflectance spectra whose statistics are unknown, thus it may be advantageous to estimate $\mathbf{R}\mathbf{R}^T$ using assumptions about correlations between infinite sets of reflectance spectra [10]. Examples of such matrices are Toeplitz matrices, which have been applied to many problems including colour correction with trichromatic cameras [10] and spectral recovery with multispectral systems [4]. In this paper we will apply this least-squares- with-Toeplitz matrix approach (LS-TO) using the form of Toeplitz matrix in [4].

2.2. Smoothness

Instead of using a least-squares approach we can find a solution that explicitly maximises the smoothness of the resulting estimate. van Trigt [8] proposed this approach for estimating the continuous reflectance function $R(\lambda)$ from CIE XYZ values by minimising the integral of its squared first derivative, which he did using an analytical method. Li and Luo [5] apply the same constraint by minimising a numerical approximation to the first derivative of $\mathbf{r}$. That is, by minimising $\|\mathbf{G}\mathbf{r}\|^2$ where $\mathbf{G}$ is a "smooth operator" that calculates a numerical estimate of the local first derivative of $\mathbf{r}$. König and Praefcke [4] applied a similar "smoothing inverse" method, which can be formulated in precisely the same way as the first derivative approach but $\mathbf{G}$ now computes the local second derivative of $\mathbf{r}$ instead of the first.

For either method, termed SM-1 and SM-2 for the first and second derivatives respectively, the problem is one of constrained optimisation; an estimate of $\mathbf{r}$ is sought that is both minimal with respect to $\|\mathbf{G}\mathbf{r}\|^2$, and satisfies the image formation equation (2). This problem can be solved using Lagrange multipliers to give

$$\mathbf{r}' = \left(\mathbf{G}^T\mathbf{G}\right)^{-1} \mathbf{Q}_e \left(\mathbf{Q}_e^T \left(\mathbf{G}^T\mathbf{G}\right)^{-1} \mathbf{Q}_e\right)^{-1} \mathbf{p} \tag{6}$$

Note that the matrix $\mathbf{G}^T\mathbf{G}$ does not generally have an inverse and the solution to $\left(\mathbf{G}^T\mathbf{G}\right)^{-1}$ must therefore be regularised [4].

2.3. Linear models

An alternative approach to imposing smoothness on the reflectance estimate is to assume that $\mathbf{r}$ can be approximated by the linear combination of a small number of basis functions [11] i.e.

$$\mathbf{r} \approx \mathbf{B}\mathbf{w} \tag{7}$$

where $\mathbf{B}$ is a $31 \times m$ matrix whose columns contain the m basis functions and $\mathbf{w}$ is an $m \times 1$ vector of basis function weights. The problem of recovering reflectance becomes one of recovering $\mathbf{w}$.

One approach to finding $\mathbf{w}$ is to assume that the approximation in (7) is exact, as employed by Maloney and Wandell in [6]. Therefore, again assuming no noise, (2) becomes

$$\mathbf{p} = \mathbf{Q}_e^T\mathbf{B}\mathbf{w} \tag{8}$$

When $p = m$ then $\mathbf{Q}_e^T\mathbf{B}$ is a square matrix and both sides can be multiplied by its inverse to solve for $\mathbf{w}$. However, for some sensor and basis function combinations this method is not robust [7]. In these cases the number of basis functions in the linear model can be reduced relative to the number of sensors, i.e. we can make $p \geq m$ and solve for $\mathbf{w}$ using the pseudoinverse of $\mathbf{Q}_e^T\mathbf{B}$ instead of the inverse [6, 7]. We term this general approach the LM-MW method. Once $\mathbf{w}$ is known then $\mathbf{R}$ can be recovered directly using (7).

Burns and Berns [1] take a different approach to solving for $\mathbf{w}$. They build a transform that minimises the least squares error between camera responses and basis weights (which are computed by a direct regression for the characterisation set) i.e. they find a transform of sensor responses $\mathbf{T}$ s.t.

$$\min_{\mathbf{T}} \sum_{i=1}^{k} \|\mathbf{w}_i - \mathbf{T}\mathbf{p}_i\|^2 \tag{9}$$

for a set of k characterisation surfaces, where $\mathbf{w}_i$ contains the optimal weights for the i^{th} characterisation surface. This is referred to here as the LM-BB method.

3. EXPERIMENTAL

We would like to objectively assess the recovery performance of the methods outlined in Section 2 for a range of imaging systems. In order to limit the number of variables we will limit the camera sensitivities in this paper to two simulated 6-channel systems. The first uses 6 broad-band filters taken from a set of 139 filters from the Roscolux Supergel range (Rosco Laboratories Ltd., Canada), the second system uses 6 narrow-band liquid crystal filters chosen from 33 individual filter settings in a tunable liquid crystal filter (Varispec, model VIS2, Cambridge Research & Instrumentation, Inc., Boston, MA, USA). In both cases complete channel sensitivities are computed using a wavelength-by-wavelength multiplication of each filter with a conjunction of tungsten halogen illuminant and camera sensitivity (PCO SVGA Sensicam CCD camera) estimated by Hardeberg [3].

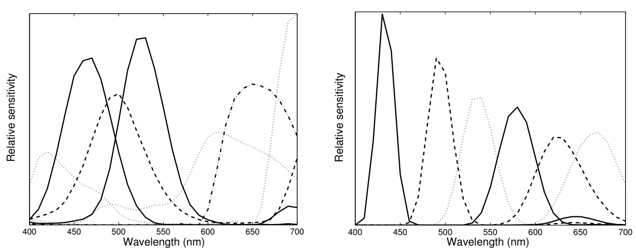

Fig. 1. Normalised spectral sensitivities for the broad-band (left) and narrow-band (right) imaging systems.

The 6 channel sensitivities are chosen from the larger set using a filter choice criterion outlined elsewhere [3]. As a characterisation set we use the 238 surfaces of the Macbeth DC ColorChecker chart and as test sets we use both a set of 1269 Munsell reflectances [12], which are statistically similar to the characterisation set (the ColorChecker uses Munsell surfaces), and a set of 2000 natural reflectances randomly sub-sampled from a set of hyperspectral images [13]. Both 1% image dependent shot noise and 12-bit quantisation noise are added to all simulated responses. The basis functions used for the linear-models methods are the eigenvectors of the raw-cross-product matrix $\mathbf{R}\mathbf{R}^T$. We compute the physical difference between reconstructed and original spectra with RMS error and perceptual difference using a metamerism index (MI), which is the CIELAB ΔE_{ab} error computed under 6 CIE illuminants (A, D50, D65, F2, F7 and F11).

Method	*MI*		*RMS*	
	Mean	*Max*	*Mean*	*Max*
LS	1.3	18.8	0.024	0.10*
LS-TO	1.1*	7.7*	0.020	0.10*
SM-1	1.1*	7.7*	0.019*	0.10*
SM-2	1.5	28.6	0.019*	0.12
LM-MW	1.5	14.7	0.026	0.11
LM-BB	1.3	16.3	0.026	0.11

Table 1. Results for natural reflectance data using narrow-band filters

Method	*MI*		*RMS*	
	Mean	*Max*	*Mean*	*Max*
LS	1.3	25.0	0.016	0.05*
LS-TO	1.2*	10.9	0.014*	0.06
SM-1	1.2*	10.8	0.014*	0.07
SM-2	1.7	17.9	0.018	0.07
LM-MW	1.3	10.3*	0.017	0.07
LM-BB	1.3	16.4	0.017	0.07

Table 2. Results for Munsell reflectance data using narrow-band filters

4. RESULTS & DISCUSSION

The results are shown in Table 1 through to Table 4 with the best results in each column indicated by a *. The meaning of the abbreviation for each method is outlined in Section 2. Results for testing data in the narrow-band system (Table 1) show that SM-1 and LS-TO methods give lowest mean and maximum perceptual error (1.1 MI and 7.7 MI respectively) as well as good RMS performance. In the broad-band system (Table 3) the SM-1, SM-2 and LS-TO methods show similar performance to each other, but are superior to the other methods. Note that all methods show larger error for this system due to the decreased noise robustness of the sensor set. We can also see that in Tables 1 and 3 the LS-TO method is always superior to the LS method, where the latter uses *a priori* statistical information of the Munsell dataset. However, in Tables 2 and 4 we can see that this *a priori* information does not always result in lower error scores for the LS compared to the LS-TO method. In Tables 2 and 4 we again see that the SM-1 method is comparatively good.

5. CONCLUSIONS

We investigated the relative performance of a range of reflectance recovery algorithms. In general, methods that maximise smoothness were found to deliver the best performance on test datasets. These smoothness based methods assume that the estimated reflectance has a minimal squared first or second derivative. By assuming this they therefore require

Method	*MI*		*RMS*	
	Mean	*Max*	*Mean*	*Max*
LS	2.4	46.9	0.025	0.11
LS-TO	2.0	17.7	0.021*	0.09*
SM-1	1.9*	16.5	0.021*	0.09*
SM-2	1.9*	16.0*	0.021*	0.11
LM-MW	4.3	49.1	0.030	0.12
LM-BB	2.4	56.0	0.027	0.12

Table 3. Results for natural reflectance data using broad-band filters

Method	*MI*		*RMS*	
	Mean	*Max*	*Mean*	*Max*
LS	1.7*	29.9	0.018	0.05*
LS-TO	1.9	13.4*	0.017*	0.08
SM-1	1.9	13.4*	0.017*	0.08
SM-2	2.0	13.5	0.022	0.10
LM-MW	2.0	49.9	0.019	0.09
LM-BB	1.7*	20.5	0.018	0.08

Table 4. Results for Munsell reflectance data using narrow-band filters

no explicit *a priori* knowledge of the reflectance data to produce a solution. The least-squares and linear models methods, however, exploit knowledge of the raw-cross-product matrix of a set of training data to produce a solution. Surprisingly, the smoothness methods perform just as well as least-squares and linear models methods on training data for which the latter methods are optimised.

Here we have used simulated imaging systems to test the performance of the different linear methods. Although our models are widely used, and are believed to hold under controlled experimental conditions, they can only be approximations. Therefore, in future work, we intend to validate these results by applying the methods to real camera responses.

6. REFERENCES

[1] P. D. Burns and R.S. Berns, "Analysis multispectral image capture," in *Proceedings of IS&T/SID Fourth Color Imaging Conference*, Scottsdale, Arizona, November 1996, IS&T/SID, pp. 234–241.

[2] G. D. Finlayson and P. M. Morovic, "Metamer constrained colour correction," in *IS&T/SID 7th Color Imaging Conference*, Scottsdale, Arizona, 1999, IS&T/SID, pp. 26–31.

[3] J. Y. Hardeberg, *Acquisition and reproduction of colour images: colorimetric and multispectral approaches*, Ph.D dissertation, Ecole Nationale Supériure des Télécommunications, Paris, France, 1999.

[4] F. König and W. Praefcke, "A multispectral scanner," in *Colour Imaging: Vision and Technology*, L. W. MacDonald and M. R. Luo, Eds., pp. 129–144. John Wiley and Sons Ltd, 1999.

[5] C. Li and M. R. Luo, "The estimation of spectral reflectances using the smoothness constraint condition," in *Proceedings of IS&T/SID Ninth Color Imaging Conference*, Scottsdale, Arizona, November 2001, IS&T/SID, pp. 62–67.

[6] L. T. Maloney and B. A. Wandell, "Color constancy: A method for recovering surface spectral reflectance," *Journal of the Optical Society of America - A*, vol. 3, no. 1, pp. 29–33, 1986.

[7] S. Tominaga, "Multichannel vision system for estimating surface and illuminant functions," *Journal of the Optical Society of America - A*, vol. 13, no. 6, pp. 2163–2173, 1996.

[8] C. van Trigt, "Smoothest reflectance functions. 1. definition and main results," *Journal of the Optical Society of America - A*, vol. 7, no. 12, pp. 1891–1904, 1990.

[9] G. D. Finlayson and M. S. Drew, "Constrained least-squares regression in color spaces," *Journal of Electronic Imaging*, vol. 6, no. 4, pp. 484–493, 1997.

[10] J. A. S. Viggiano, "Minimial-knowledge assumptions in digital still camera characterization i: Uniform distribution, toeplitz correlation," in *Proceedings of IS&T/SID Ninth Color Imaging Conference*, Scottsdale, Arizona, November 2001, IS&T/SID, pp. 332–336.

[11] L. T. Maloney, "Evaluation of linear models of surface spectral reflectance with a small number of parameters," *Journal of the Optical Society of America - A*, vol. 3, no. 10, pp. 1673–1683, 1986.

[12] J. Parkkinen, J. Hallikainen, and T. Jaaskelainen, "Characteristic spectra of Munsell colors," *Journal of the Optical Society of America - A*, vol. 6, pp. 318–322, 1989, See *http://cs.joensuu.fi/ spectral/*.

[13] S. M. C. Nascimento, F. Ferreira, and D. H. Foster, "Statistics of spatial cone-excitation ratios in natural scenes," *Journal of the Optical Society of America - A*, vol. 19, no. 8, pp. 1484–1490, 2002.