A PRACTICAL ALGORITHM TO CORRECT GEOMETRICAL DISTORTION OF IMAGE ACQUISITION CAMERAS

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ABSTRACT

In this paper a new practical and simple method for removing the distortion of image acquisition cameras is described. A pattern image with straight horizontal and vertical lines is used to automatically compute the radial and tangential distortion based upon classical models, relying on the observed apparent distortion of the lines (obtained from a Canny's edge detector and making use of the Hough's Transform). The applied model is very robust and complete. Some practical results with real images are shown.

1. INTRODUCTION

The optics of the image acquisition cameras introduce aberrations which affect the acquired images, showing a geometrical distortion very easily observed in straight lines that become apparently curved. This aberration depends mainly on the lenses quality, and increases with the distance between the observed points and the image's center. In many applications, this distortion is not important because it is almost imperceptible. However in many other cases this aberration is crucial, especially when there is a need to obtain some precise measurements from the detected image points. The algorithm presented in this work corrects this aberration making use of a parametric model of the geometrical distortion, and a line detector for calculating the parameters of the model.

In the next section the classical distortion model and other published approaches are revised. Section 3 explains how to compute the distortion. Experimental results with real images are shown in section 4. Finally, Section 5 provides some concluding remarks.

2. PREVIOUS WORK

The first works about geometrical distortion of lenses were applied in the field of analytical photogrammetry. In 1966, Brown [3] presented a study of decentering distortion of lenses based on classical and well-known theories (the thin prism model and Conrady's model). Later, he proposed a method to measure the total distortion of lenses [4]. Brown relied on the fact that "*in the absence of distortion, the central projection of a straight line is itself a straight line*". So, the deviation of a straight line can be measured to compute the parameters which model the aberration. He used plumb lines for his estimation. His model was the following:

$$\begin{aligned} x' &= x + \delta_x(x, y) \\ y' &= y + \delta_y(x, y) \end{aligned} \qquad (1)$$

where *(x, y)* are the coordinates of a non-distorted point, and *(x', y')* are the coordinates of the distorted one. $\delta_x(x, y)$ and $\delta_y(x, y)$ can be broken down into two terms: the radial (2) and tangential components (3):

$$\begin{aligned} \delta_{rad,x} &= k_1 \cdot x \cdot (x^2 + y^2) + O\left[(x, y)^5\right] \\ \delta_{rad,y} &= k_1 \cdot y \cdot (x^2 + y^2) + O\left[(x, y)^5\right] \end{aligned} \qquad (2)$$

$$\begin{aligned} \delta_{tg,x} &= p_1 \cdot (3x^2 + y^2) + 2 \cdot x \cdot y \cdot p_2 + O\left[(x, y)^4\right] \\ \delta_{tg,y} &= 2 \cdot x \cdot y \cdot p_1 + p_2 \cdot (x^2 + 3y^2) + O\left[(x, y)^4\right] \end{aligned} \qquad (3)$$

where $O\left[(x, y)^n\right]$ are the higher terms of *n* order, k_1 is the radial distortion parameter, and p_1 and p_2 are the tangential ones.

Brown's works have been used by many researchers. In [15], a method for a complete camera calibration is presented in two steps, taking into account a distortion-free camera model (a projection of the 3D world into a 2D world) and the geometrical distortion. The plumb line method is used in [11] too, but only considering radial

[*] Work done with the support of the National Grant PROFIT FIT-070000-2003-30 of the Spanish Science and Technology Ministry.

0-7803-8554-3/04/$20.00 ©2004 IEEE.

distortion and the true image centre. In [1], a closed-form solution of Brown's model is implemented using a non-linear search for estimating the parameters. There are many other ways to correct the geometrical distortion: in [7], a fast and easy algorithm to correct the radial distortion is explained; knowing the real and ideal coordinates of some points, and minimizing their distances [10]; cubic B-spline curves can be used [13] to estimate radial distortion; the distorted lines can be fitted to circles [2]; in [12], a non-parametric method using topological mapping algorithms for regression analysis is used; in [9], circular control points are used for the calibration and in [14] the whole image is considered.

In this work we propose a new method to compute and remove the geometrical distortion of cameras using Brown's model, and the help of a reference image (called the *pattern image*) with some straight lines. We implement our approach in two steps: firstly, we detect and analyse the straight lines associated with our acquired image; and secondly, we calculate the distortion's parameters minimizing the distance between corrected and ideal straight lines.

Our method is faster than [15] because we do not take into account the position of the camera (extrinsic parameters). It is also more precise than [1], [7] and [11] because our distortion model is analytically more complete.

3. SYSTEM DESCRIPTION

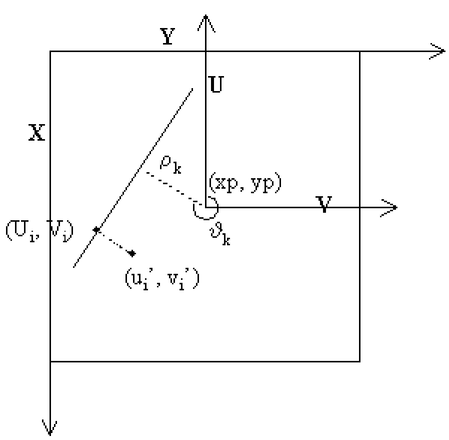


Figure 1: Coordinate system

3.1. Introduction

In [15], it is shown that equations (1) can be rewritten in the following way:

$$u' = u + \delta'_u(u,v)$$
$$v' = v + \delta'_v(u,v) \qquad (4)$$

where *(u, v)* is the distorted point, and *(u', v')* is the corrected one. This replacement can be done according to [15]: "*a) the distortion at the exact image plane projection is approximately equal to that in the actual projection, and b) the actual distortion coefficients will be estimated based on* u *and* v". The complete model of distortion from (1)-(4) result in (5), according to the coordinate system of figure 1.

$$u' = u + k_1 \cdot u \cdot (u^2 + v^2) + p_1 \cdot (3u^2 + v^2) + 2 \cdot u \cdot v \cdot p_2$$
$$v' = v + k_1 \cdot v \cdot (u^2 + v^2) + 2 \cdot u \cdot v \cdot p_1 + p_2 \cdot (u^2 + 3v^2) \qquad (5)$$

3.2. The pattern image

The initial estimate of the radial and tangential distortion coefficients are obtained from a *pattern image*, composed of simple horizontal and vertical lines, as shown in figure 2a. Once this pattern image is acquired, the following steps are applied:

Firstly, the coordinate system is changed from *XY* to *UV* (6) (see figure 1), where (x_i, y_i) is the obtained point in *XY*-system, (u_i, v_i) is this point in *UV*-system and (x_p, y_p) is the true center of the camera. As an initial guess it can be assumed that this point is in the exact center of the image.

$$u_i = -x_i + x_p$$
$$v_i = y_i - y_p \qquad (6)$$

Secondly, the following parameters are estimated: *k*, p_1, p_2, and a refinement of x_p and y_p. For this purpose, the information associated to the lines is extracted.

3.3. Line detection

We need to accurately detect the straight lines. For this purpose, the optimal Canny's edge detector method [5] is applied. The resulting gradient image is used as input to the Hough's Transform in its parametric form (7) (8). The Hough's Transform characterizes each line through the values θ_k and ρ_k[6]. The relation between θ_k and the gradient components g_u and g_v is shown in (8). Only image points with a gradient modulus greater than a predefined threshold are considered.

$$u\cos(\vartheta) + v\sin(\vartheta) = \rho \qquad (7)$$

$$\vartheta = arctg\left(\frac{g_v}{g_u}\right) \qquad (8)$$

The maximum values of θ in the transformed domain determine the existing lines in the image. These maximum values will be our θ_k's. Their corresponding ρ_k's are computed from (7), providing a set of *N* straight lines that apparently fit to the distorted ones.

$$L_k = \{(u_i, v_i) \mid u_i\cos(\vartheta_k) + v_i\sin(\vartheta_k) \approx \rho_k\} \qquad (9)$$

In (9), (u_i, v_i) are the *i*-th point's coordinates belonging to a distorted line in the *UV*-system. The implicit assumption of this procedure is that distortion is not extremely strong (which is the common case in digital imaging) in order to be able to find a good estimation of the straight lines.

3.4. Minimization

When all θ_k and ρ_k associated to the lines have been computed, the next stage is the minimization of a cost function (10). This function is defined as the square sum of the differences between the corrected point coordinates (u_i', v_i') and the ideal ones (U_i, V_i) (see figure 1). Here, we assume we have *N* lines with N_k points in *k*-line.

$$CF = \sum_{k=1}^{N}\sum_{i=1}^{N_k} d_{ik}^2 = \sum_{k=1}^{N}\sum_{i=1}^{N_k} \{(u_i'-U_i)^2 + (v_i'-V_i)^2\}_k \quad (10)$$

For this purpose, the corrected points are linked to the ideal points. If we see figure 1, the next expression can be written:

$$tg(\vartheta_k) = \frac{v_i'-V_i}{u_i'-U_i} \quad (11)$$

Now from (7) and (11), we find that:

$$\begin{aligned} U_i &= u'_i \sin^2(\vartheta_k) - v'_i \cos(\vartheta_k)\sin(\vartheta_k) + \rho_k \cos(\vartheta_k) \\ V_i &= v'_i \cos^2(\vartheta_k) - u'_i \cos(\vartheta_k)\sin(\vartheta_k) + \rho_k \sin(\vartheta_k) \end{aligned} \quad (12)$$

If equations (5) and (12) are introduced in (10), we obtain an expression relating the acquired points, the straight lines parameters and the distortion parameters. Our cost function can then be rewritten in matrix form (13). Now matrix *A* and *B* and vectors *b* and *e* are combined in *Q* and *c* respectively (14). The final expressions can be viewed in (15)-(17).

$$CF = \|A \cdot \overline{p} + b\|^2 + \|D \cdot \overline{p} + e\|^2 \quad (13)$$

$$Q = (A \quad D)^T \; ; \overline{c} = (\overline{b} \quad \overline{e})^T \Rightarrow CF = \|Q \cdot \overline{p} + \overline{c}\|^2 \quad (14)$$

$$Q = \begin{pmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 \\ \vdots & \ddots & \vdots \\ a_{i1}^k & a_{i2}^k & a_{i3}^k \\ \vdots & \ddots & \vdots \\ d_{11}^k & d_{12}^k & d_{13}^k \\ \vdots & \ddots & \vdots \\ d_{i1}^k & d_{i2}^k & d_{i3}^k \\ \vdots & \ddots & \vdots \end{pmatrix} ; \overline{p} = \begin{pmatrix} k \\ p_1 \\ p_2 \end{pmatrix} \quad (15)$$

$$\overline{c} = \left(b_1^1 \cdots b_i^1 \cdots b_1^2 \cdots b_i^k \cdots e_1^1 \cdots e_i^1 \cdots e_1^2 \cdots e_i^k \cdots\right)^T \quad (16)$$

Finally, a closed-form solution for vector *p* is obtained by making null its gradient (18). All the previous procedure provides a good initial estimation of the distortion parameters. However, this estimation has been obtained assuming that the optic center of the camera is in the middle of the image, and this is commonly not true. To solve that, an iterative procedure is started, applying small displacements in horizontal and vertical dimensions to the coordinates (x_p, y_p) in a reduced neighborhood of 10x10 pixels, searching for the minimum value of the cost function. Finally, the vector *p* and the true center (x_p, y_p) which give us the minimum valor of *CF* is accepted.

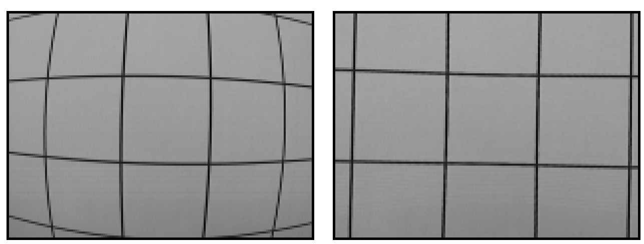
$$\begin{aligned} a_{i1}^k &= (u_i^2 + v_i^2)(u_i \cos^2(\vartheta_N) + v_i \cos(\vartheta_N)\sin(\vartheta_N)) \\ a_{i2}^k &= (3u_i^2 + v_i^2)\cos^2(\vartheta_k) + 2u_i v_i \cos(\vartheta_k)\sin(\vartheta_k) \\ a_{i3}^k &= 2u_i v_i \cos^2(\vartheta_k) + (u_i^2 + 3v_i^2)\cos(\vartheta_k)\sin(\vartheta_k) \\ d_{i1}^k &= (u_i^2 + v_i^2)(u_i \cos(\vartheta_k)\sin(\vartheta_k) + v_i \sin^2(\vartheta_k)) \\ d_{i2}^k &= 2u_i v_i \sin^2(\vartheta_k) + (3u_i^2 + v_i^2)\cos(\vartheta_k)\sin(\vartheta_k) \\ d_{i3}^k &= (u_i^2 + 3v_i^2)\sin^2(\vartheta_k) + 2u_i v_i \cos(\vartheta_k)\sin(\vartheta_k) \\ b_i^k &= u_i \cos^2(\vartheta_k) + v_i \cos(\vartheta_k)\sin(\vartheta_k) - \rho_k \cos(\vartheta_k) \\ e_i^k &= u_i \cos(\vartheta_k)\sin(\vartheta_k) + v_i \sin^2(\vartheta_k) - \rho_k \sin(\vartheta_k) \end{aligned} \quad (17)$$

$$\nabla CF(p) = 0 \Rightarrow \overline{p} = -(Q^T \cdot Q)^{-1} \cdot Q^T \cdot \overline{c} \quad (18)$$

4. RESULTS

We have done some tests to assess our algorithm. A video-camera with a short focal distance of 4.5 mm has been used because this type of cameras shows a lot of geometrical distortion, making easier to apparently see the aberrations.

Figure 2 shows the used *pattern image* (2a) in which the distortion is so strong. After applying our algorithm, we obtain the following parameters: k=1.78x10^{-6}, p_1=-5.28x10^{-6}, p_2=-1.29x10^{-5}, x_p=386, y_p=288, providing the good result shown in figure 2b.



(a) (b)

Figure 2: (a) acquired image; (b) corrected image

Moreover, we have measured the initial error as the sum of the distances between ideal and obtained points, divided by the total number of points of the lines; and the final error as the sum between ideal and corrected points, also divided by the total number of points. The measured errors have been 2.56204 pixels/point and 1.07641 pixels/point for initial and final errors respectively. So, the distortion has been clearly reduced.


(a) (b)

Figure 3: (a) acquired image; (b) corrected image

The previous estimated parameters have been used for correcting another real image (figure 3a) acquired with the same camera. The output of our algorithm is shown in figure 3b, where an important improvement can be noted.

5. CONCLUSIONS

Although an initial assumption of not strong distortion was suggested good results has been obtained with very distorted images. Therefore it is possible to correct the distortion of an image with our simple and effective algorithm. Our approach works with real images and it improves the image's quality. The use of Canny's edge detector and Hough's Transform (with the proper accumulation grid size) guarantee a good result because of their accuracy.

Moreover, even though the method is implemented using a defined pattern image, it should work with any reference image including enough straight lines to establish the appropriate correspondences to compute the defined cost function.

Our current work goes in the line of trying to enhance the estimation of the optic center (x_p, y_p) through the use of sub-pixel precision in the iterative loop.

6. REFERENCES

[1] Ahmed, M., Farag, A., "Non-metric calibration of camera lens distortion", Proc. *ICIP'2001.* Vol. 2, pp. 157 -160, 2001

[2] Brauer-Burchardt, C., Voss, K., "A new algorithm to correct fish-eye- and strong wide-angle-lens-distortion from single images", Proc. *ICIP'2001*, Vol.1, pp. 225 -228, 2001.

[3] Brown, D. C., "Decentering distortion of lenses", *Photogrammetric Eng. Remote Sensing*, pp. 444-462, May 1966.

[4] Brown, D. C., "Close-range camera calibration", Photogrammetric Eng., vol. 37(8) pp. 855-866, August 1971.

[5] Canny J., "A Computational Approach to Edge Detection", *IEEE Trans. PAMI*, vol. 8, no. 6, pp. 679-698, Nov. 1986.

[6] Davies, E. R., *Machine Vision: Theory, Algorithms, Practicalities*, Academic Press, 1990.

[7] Fernandes, J.C.A., Ferreira, M.J.O., Neves, J.A.B.C., Couto, C.A.C., "Fast correction of lens distortion for image applications", Proc. ISIE'1997, Vol.2, pp. 708 –712, 1997.

[8] Grosky, W.I., Tamburino, L.A., "A unified approach to the linear camera calibration problem", IEEE Trans. *PAMI*, Vol.12, N.7, pp. 663 –671, 1990.

[9] Heikkilä, J; '*Geometric Camera Calibration Using Circular Control Points*', IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 22, nº 10, Oct. 2000.

[10] Kyeongtae Hwang, Moon Gi Kang, "Correction of lens distortion using point correspondence", *TENCON 99. Proceedings of the IEEE Region 10 Conference*, Volume: 1, pp. 690 -693, 1999

[11] McLean, G., "Image warping for calibration and removal of lens distortion", *IEEE Pacific Rim Conference on Communications, Computers and Signal Processing, 1993.,* Vol.1, pp. 170 -173, 1993.

[12] NaoLin Qiu, Song De Ma, "The nonparametric approach for camera calibration", *Fifth Int. Conf. on Computer Vision,* pp. 224 –229, 1995.

[13] Remy, S., Dhome, M., Daucher, N., Lapreste, J.T., "Estimating the radial distortion of an optical system; effect on a localization process", Proc. ICIP'1994, Vol.2, pp. 997 -1001, 1994.

[14] Tamaki, T., Yamamura, T., Ohnishi, N., "Unified approach to image distortion", Proc. ICPR'*2002,* Vol.2, pp. 584 -587, 2002

[15] Weng, J., Cohen, P., Herniou, M., "Camera calibration with distortion models and accuracy evaluation", IEEE Trans. PAMI, Vol.14, N.10, pp. 965 –980, Oct 1992.