AN IMPROVED EZW ALGORITHM BASED ON SET PARTITIONING IN HIERARCHICAL TREES USING WAVELET REGULARITY

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ABSTRACT

This paper presents an improved embedded zerotree wavelet (EZW) coding algorithm, which makes use of the wavelet regularity to derive a classification criterion of wavelet coefficients in spatial-orientation hierarchical trees. Variations of the EZW algorithm discussed in the open literature have proposed some modifications in the process of exploiting the similarity of coefficients through the scales, however, not defining a figure of merit to measure such a similarity. Simulation results achieved from the coding of well-known images in the literature, for several bit rates, show a better performance of the proposed algorithm in both PSNR and subjective terms, as compared with EZW and SPIHT algorithms.

1. INTRODUCTION

For a long time, image coding algorithms (particularly lossy ones) have become more complex as their efficiency increases. However, the image compression technique named embedded zerotree wavelet (EZW) [1] has interrupted this process. The EZW algorithm is based on three main concepts:

i) Partial ordering of transformed image coefficients, with transmission of the order of those coefficients through a set-partitioning algorithm. That algorithm compares the magnitude of the transformed coefficients with a set of thresholds, classifying such coefficients as significant or insignificant;
ii) Ordered transmission of bit planes;
iii) Exploiting the wavelet transform similarity across scales.

EZW approach is a competitive technique for performance as compared with other coding techniques [1], giving rise to a bit stream whose decoding process can be stopped at any place, without preventing the image reconstruction. However, such a technique does not present a particular figure of merit to classify the coefficient set.

This paper presents a new insight for the EZW algorithm implementation based on the use of wavelet regularity, defining a figure of merit for partitioning the coefficients in hierarchical trees. The proposed approach outperforms other modified EZW algorithms [2]-[4], achieving better results in both numerical and subjective terms.

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2. HÖLDER REGULARITY

The wavelet transform permits to quantify the smoothness of a signal. In a mathematical way, the smoothness is represented by a decay rate of the wavelet coefficients in successive scales of its transform, and described by the Hölder regularity (general case of the Lipschitz continuity property [5]).

2.1. Wavelet regularity

The local regularity of a function $f(x)$ in a point a can be analyzed under the Lipschitz continuity concept [5]: a function $f(x)$ is called r-Lipschitz in a, $0 \le r < 1$, if

$$| f(x) - f(a) | \le K \, | x - a |^r , \qquad (1)$$

where K is an arbitrary constant. For instance, a step discontinuity has Lipschitz exponent $r = 0$. The parameter $r > 1$ is possible if (1) is satisfied for higher order derivatives. The following condition is valid for the local behavior of wavelet coefficients in the vicinity of an r-Lipschitz point [6]: if $f(x)$ is r-Lipschitz in $x = a$, $r < N$, and $\psi(x)$ has at least N vanishing moments, then

$$\max\{ | \psi_{j,k}(x) |_{(j,k)\in A} \} \le K \, 2^{-j(r+0.5)} , \qquad (2)$$

where A contains the (j,k) pairs for which a belongs to the support of $\psi_{j,k}(x)$.

An extension of the Lipschitz continuity concept ($r > 1$) requires that (1) is satisfied for $f(x)$ and its n-th derivatives. A function $f : \mathbb{R} \to \mathbb{R}$ has Hölder regularity (or Hölder exponent) defined as $\alpha = n + r$, with $n \in \mathbb{N}$ and $0 \le r < 1$, if there is a finite constant such that

$$| f^{<n>}(y) - f^{<n>}(x) | \le K \, | y - x |^r , \quad \forall x, y \in \mathbb{R} , \qquad (3)$$

where $f^{<n>}(\cdot)$ denotes the n-th derivative of $f(\cdot)$, and r corresponds to the Lipschitz exponent [5].

The Hölder exponent defines the continuous derivative number of a function. Functions with larger Hölder exponent values are mathematically and visually smoother [7].

2.2. Estimation of the Hölder regularity

The undecimated dyadic wavelet transform is a way to measure the smoothness of a function $f(x)$, synthesizing it by translations and dilatations of a mother wavelet function $\psi(x)$. This wavelet family is given by

$$\psi_{j,k}(x) = \psi(2^j x - k) \ . \quad (4)$$

A signal has Hölder exponent *r* if there is a constant *K* such that the coefficients of its wavelet transform $\Psi_{j,k} = \langle f, \psi_{j,k} \rangle$ satisfy

$$|\Psi_{j,k}| \leq K 2^{-k(r+0.5)} \ , \quad (5)$$

for all $j \in \mathbb{Z}^+$, $k \in \mathbb{R}$ [6], [8]-[10]. This theorem characterizes the regularity of a signal by the magnitude decay through the scales of its wavelet transform, defining the similarity between scales.

2.3. Correlation between scales

Research in multiresolution analysis have pointed out the presence of characteristics similar in form, but different in spatial support for several scales [5], [7], [11]. Smoother functions show more similarity through scales, mathematically defined by the decay theorem (5). The correlation between wavelet subbands s_m and s_n, for the scales 2^m and 2^n, is limited by

$$|< s_m, s_n >| \leq K 2^{-(m+n)(\alpha+0.5)} \ , \quad (6)$$

where *K* is a constant and α is the Hölder exponent. Such an expression suggests that the similarity between scales decreases exponentially as the regularity increases [5], [7]-[9].

The Hölder regularity is a useful tool to analyze correlated signals. Therefore, it can also be used to define a criterion for classifying coefficients in progressive coding algorithms, since such algorithms use the correlation measure between scales to obtain the threshold values.

3. EZW ALGORITHM

The EZW algorithm [1] is a wavelet progressive image coding algorithm, based on two rules:

i) Larger wavelet coefficients contain more information. Thereby, they should be coded first;
ii) The maximum- and mean-absolute coefficient values growing from high to low frequency subbands.

The EZW output is progressive, i.e., it contains more details as more data are added to the decoded sequence. The next sections summarize the main points of the EZW algorithm [1].

3.1. Hierarchical tree structure

A hierarchical tree structure consists of a tree whose offspring has four descendants. Each of them forms a new offspring, and so on. In a hierarchical tree, the coefficient absolute value decreases vertically; thus, if a coefficient is classified as insignificant, the next ones will also be insignificant (not containing any notable information). The coefficients from lower frequency subbands generate four new descendants for the higher subband, giving rise to four new offspring, and so forth.

3.2. EZW algorithm operation

After computing the image discrete wavelet transform (DWT) $\Psi_{j,k}$, an initial threshold is obtained by

$$T_{\text{init}} = \max\{|\Psi_{j,k}|\} \ . \quad (7)$$

The current threshold T corresponds to the highest power of 2 that is less than the maximum coefficient absolute value (Fig. 1). Next, a procedure called **dominant pass** identifies which of those coefficients are significant (higher in absolute value than the initial threshold). Completing the procedure, the current threshold value is halved, and a second procedure called **subordinate pass** is performed, coding progressively the significant coefficients. This process repeats until the minimum threshold value (usually equal to unity) is obtained.

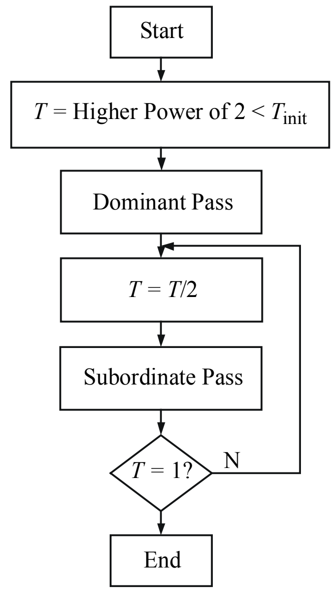

Fig. 1. Block diagram of the EZW image coding algorithm.

3.3. EZW algorithm steps

The dominant pass discards the coefficients which are smaller than the current threshold, identifying the insignificant coefficients, so that no output algorithm is produced. If a given coefficient value is significant for the current threshold value the difference between these values is put in a subordinate list. The subordinate pass refines the value of each significant coefficient, comparing it with the current threshold value and assigning a '1' or '0' bit to the output whether the threshold level is attained or not, respectively. After completing the subordinate pass, the subordinate list is classified in increasing order, so that, after several iterations, higher coefficients are placed at the beginning of the list.

In the EZW decoding task, the dominant pass extracts the original coefficient values from the coded information, while the subordinate pass either sums or subtracts from those coefficients the current threshold value according to the signs of the coded coefficients.

4. PROPOSED EZW ALGORITHM

4.1. Classification by set partitioning of coefficients

Modified EZW algorithms found in the open literature [4], [12] use a set partitioning criterion based on a fixed threshold value (generally as a power of 2). The dominant pass defines coefficient sets classifying them as significant under the following test:

$$\max\{|\Psi_{j,k}|\} \overset{?}{\geq} 2^{\lfloor \log_2 \max\{|\Psi_{j,k}|\} \rfloor}. \quad (8)$$

According to this criterion, each coefficient set classified as significant is partitioned in new subsets until all coefficients are considered. Thus for each new coefficient set $\mathbf{B}$, a significance function $S_n(\mathbf{B})$ is defined as

$$S_n(\mathbf{B}) = \begin{cases} 1, & \max\{|\Psi_{j,k}|\}_{(j,k)\in\mathbf{B}} \geq 2^{\lfloor \log_2 \max\{|\Psi_{j,k}|\} \rfloor} \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

However, we notice that the threshold value used to classify a coefficient set as significant does not provide an exact value for such coefficients, since they are classified as insignificant for the next threshold. Then, we propose to consider a criterion for refining the threshold value, taking into account the wavelet regularity between scales. This approach therefore leads to better results in comparison with the ones obtained by the original EZW algorithm [1] and the modified EZW algorithms [4], [12]; the former does not use any coefficient partitioning scheme.

4.2. Threshold values using Hölder regularity

Usually, the most energy portion in an image is concentrated at a low-frequency range. In this way, the variance of the wavelet transformed coefficients decreases from higher to lower spatial resolution levels as well as the spatial similarity between subbands (characterized by Hölder exponent). Thus, we can exploit the correlation between scales to propose a new set partitioning criterion for the EZW algorithm. Since for the *n*-th decomposition level we have $2^n \leq |\Psi_{j,k}| \leq 2^{n+1}$, the coefficients within this interval can cause a reconstruction error. For this reason, the following steps are proposed (Fig. 2):

i) For each coefficient set an estimate of Hölder exponent r is obtained by extrapolating three levels of wavelet decomposition from the given coefficient set (5);
ii) A new higher threshold value T is computed for each new coefficient set, such that $T \leq 2^{n+1}$. This new threshold value is sent to the decoder;
iii) The coefficient-set partitioning is performed from step (i), until all coefficients have been tested.

5. SIMULATION RESULTS

The proposed algorithm is assessed by using the biorthogonal 9/7 Daubechies wavelet family [12], applied to both the Lena and Goldhill images, originally coded at 8 bpp. The obtained results are compared with the EZW [1] and SPIHT [4] algorithms. Tables 1 and 2 show the PSNR values for coding the images at 1 bpp. Figs. 3 and 4 depict the reconstructed images (1 bpp) for perceptual comparison, and Figs. 5 and 6 illustrate the PSNR curves for several bit rates. We can verify that the proposed algorithm characterizes more properly the high frequency components as compared with the EZW [1] and SPIHT [4] algorithms.

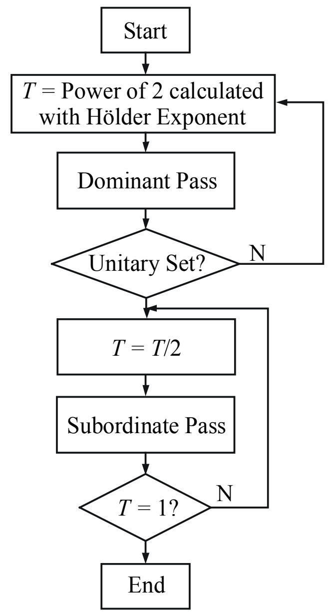

Fig. 2. Block diagram of the proposed EZW algorithm.

TABLE 1
PSNR VALUES FOR THE LENA IMAGE

Algorithms	PSNR (dB)
EZW [1]	39,11
SPIHT [4]	39,77
Proposed EZW	40,89

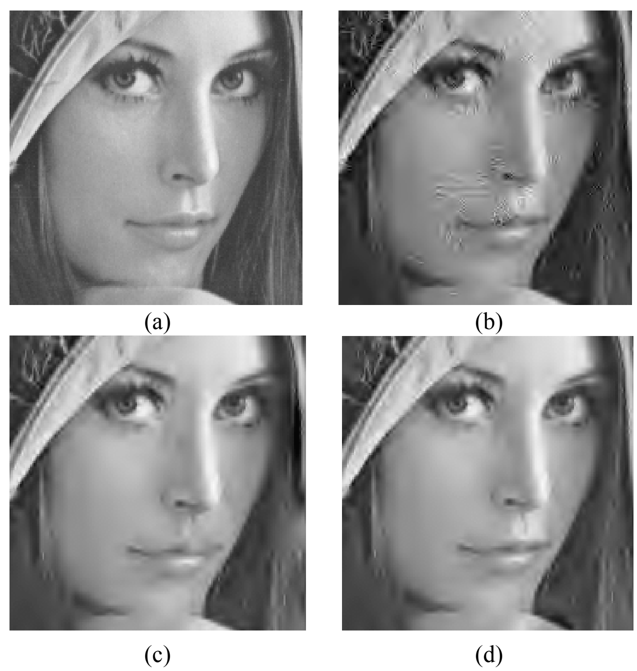

Fig. 3. (a) Original Lena image; (b) image reconstructed by the EZW algorithm [1]; (c) image reconstructed by the SPIHT algorithm [4]; (d) image reconstructed by the proposed algorithm.

TABLE 2
PSNR VALUES FOR THE GOLDHILL IMAGE

Algorithms	PSNR (dB)
EZW [1]	35,11
SPIHT [4]	35,60
Proposed EZW	36,31

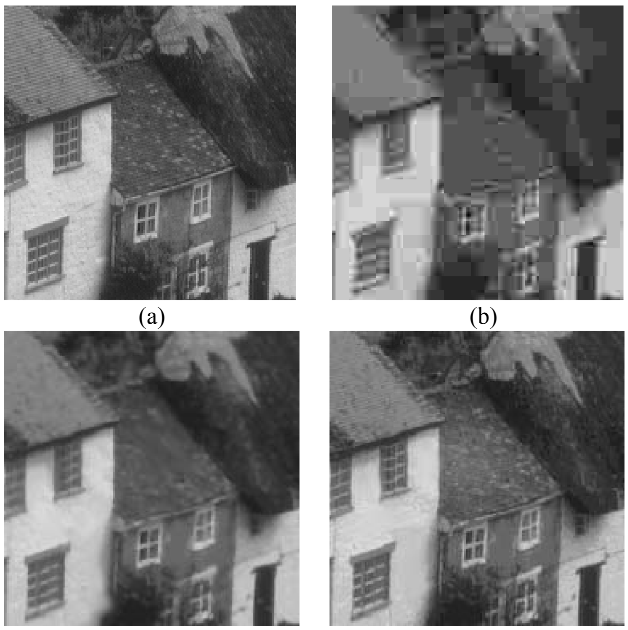

Fig. 4. (a) Original Goldhill image; (b) image reconstructed by the EZW algorithm [1]; (c) image reconstructed by the SPIHT algorithm [4]; (d) image reconstructed by the proposed algorithm.

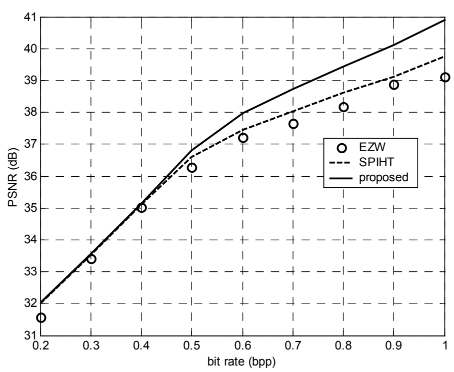

Fig. 5. Curves of PSNR *versus* bit rate for the Lena image.

6. CONCLUSIONS AND REMARKS

The proposed coding method exploits the wavelet regularity property, preserving adequately the information concerning high-frequency components, which are severely affected by the progressive coding algorithms. By defining a new set partitioning criterion based on the correlation between scales, we obtain results that surpass the ones of the EZW [1] and SPIHT [4] algorithms in both the PSNR values and perceptual terms, since our approach exploits more properly the correlation between scales than the referenced algorithms do.

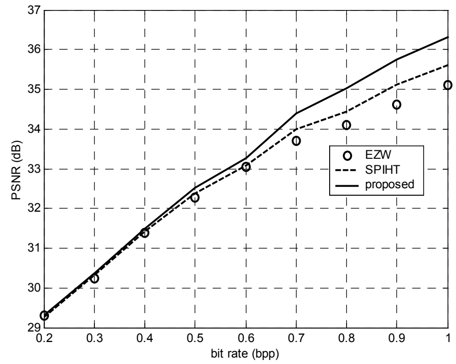

Fig. 6. Curves of PSNR *versus* bit rate for the Goldhill image.

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