MMSE RECONSTRUCTION FOR 3D ULTRASOUND IMAGES

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ABSTRACT

The reconstruction of 3D ultrasound (US) images from a set of irregularly spaced B-scans is of great interest in clinic procedures. Due to the presence of speckle, a form of multiplicative noise, 3D US reconstruction for high image quality data is a difficult task. Conventional reconstruction algorithms have low computational complexity but the reconstructed volume suffers from severe speckle contamination. In this paper, we propose a novel reconstruction method, which is optimal in minimum mean-squared error (MMSE) sense with a multiplicative noise model. Simulation results for synthetic US images, as well as experimental data are presented.

1. INTRODUCTION

Ultrasound (US) is an important medical imaging modality because of its portable, non-invasive, real-time, low-cost nature However, compared with CT and MRI, ultrasound has poor image quality due to the presence of speckle. Speckle is a type of random multiplicative noise and a common phenomenon in all coherent imaging systems, such as laser and SAR imagery. 3D US is anticipated to improve upon conventional 2D US by increasing diagnostic information dramatically and facilitating visualization [1,2]. There are several methods of acquiring 3D ultrasound images, one of which is freehand imaging [3]. In freehand imaging, a series of irregularly spaced 2D B-scans, along with their 3D spatial coordinates, are recorded into a computer. These 2D slices are usually reconstructed onto a cubic grid for visualization and data analysis. A lot of 3D interpolation algorithms have been proposed to estimate the missing points in 3D volume. These include the voxel nearest neighbor (VNN) [4], pixel nearest neighbor (PNN) [5], bilinear and distance-weighted (DW) interpolation [6]. These algorithms are rather simple because they are designed to have low computational complexity. The missing point is estimated by either the nearest measurement, or by the weighted average of the several pixels in the neighborhood. Unfortunately, speckle artifacts still remain significantly for these methods.

An efficient way for speckle reduction is adaptive filters. The most often used are Frost [9], Lee [10], Kuan [11], and the modified Frost and modified Kuan [12]. All these filters are formulated for simplified multiplicative, fully developed speckle model. Taking advantage of local image coefficients which measure the scene homogeneity, these filters can suppress speckle to some degree while preserving edge information. However, all these filters are designed for uniform sampling points. For 3D freehand imaging, US slices are acquired with arbitrary positions and angles. The sampling points are non-uniformly scattered in the 3D space. So, adaptive speckle filters cannot be applied to 3D US reconstruction directly.

In this paper, we propose using the minimum mean-squared error (MMSE) principle to optimally combine the acquired 2D data. Data redundancies due to overlapping samples as well as correlation of the target and speckle are naturally accounted for in our algorithm. Thus, it is a novel filtering method for irregular sampling points in a multiplicative noise environment.

2. MMSE RECONSTRUCTION IN MULTIPLICATIVE NOISE

In this section, we derive the MMSE reconstruction algorithm based on statistical models of the image and multiplicative speckle noise. Let $g(\mathbf{x})$ be the US signal at a spatial point $\mathbf{x} = (x, y, z)$. Then $g(\mathbf{x})$ is modeled as

$$g(\mathbf{x}) = s(\mathbf{x}) \cdot n(\mathbf{x}) \tag{1}$$

where $s(\mathbf{x})$ is the noiseless object image, and $n(\mathbf{x})$ is a multiplicative noise. It is reasonable to assume that noise is uncorrelated with signal

$$\mathrm{E}\{s(\mathbf{x})n(\mathbf{x}')\} = 0 \tag{2}$$

Let $\mathbf{g} = [g(\mathbf{x}_1), g(\mathbf{x}_2), ..., g(\mathbf{x}_m)]^{\mathrm{T}}$ be the vector of (irregularly sampled) signal at spatial points $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_m$. Our goal is to construct an estimator $\hat{s}(\mathbf{x})$ which is function of $\mathbf{g}$. Deriving the maximum likelihood estimator is difficult because of the multiplicative noise. Therefore we restrict ourselves to the class of estimators that are linear functions of $\mathbf{g}$, and seek the optimal (in the MMSE sense) estimator within the class. That is, we seek an estimator of the form $\hat{s}(\mathbf{x}) = \mathbf{g}^{\mathrm{T}}\mathbf{k}(\mathbf{x})$, where $\mathbf{k}(\mathbf{x})$ is the reconstruction basis function to be found, such that

$\mathbf{E}\left\{\left|s(\mathbf{x})-\hat{s}(\mathbf{x})\right|^2\right\}$ is minimized. Standard estimation theory [13] gives the following orthogonality condition:

$$\mathrm{E}\{\mathbf{g}(s(\mathbf{x})-\mathbf{g}^{\mathrm{T}}\mathbf{k}(\mathbf{x}))\}=0 \tag{3}$$

which can be expanded and rewritten as

$$\mathbf{r}_{gs}(\mathbf{x})-\mathbf{R}_{gg}\mathbf{k}(\mathbf{x})=0 \tag{4}$$

where

$$\mathbf{r}_{gs}(\mathbf{x})=\begin{bmatrix}\vdots\\ \mathbf{E}\{s(\mathbf{x})g_m\}\\ \vdots\end{bmatrix}=\begin{bmatrix}\vdots\\ \mathbf{E}\{s(\mathbf{x})s_m\}\mathbf{E}\{n_m\}\\ \vdots\end{bmatrix} \tag{5}$$

and $\mathbf{R}_{gg}$ is the measurement covariance matrix

$$\begin{aligned}\mathbf{R}_{gg}&=\begin{bmatrix} & \vdots & \\ \cdots & \mathbf{E}\{g(\mathbf{x}_m)g(\mathbf{x}_{m'})\} & \cdots\\ & \vdots & \end{bmatrix}\\ &=\begin{bmatrix} & \vdots & \\ \cdots & \mathbf{E}\{s(\mathbf{x}_m)s(\mathbf{x}_{m'})\}\mathbf{E}\{n(\mathbf{x}_m)n(\mathbf{x}_{m'})\} & \cdots\\ & \vdots & \end{bmatrix}\end{aligned} \tag{6}$$

We further assume that ultrasound signal has an exponential autocorrelation function. So $s(\mathbf{x})$ can be modeled as an autoregressive process with autocorrelation function of the form:

$$\mathbf{R}_{ss}(\mathbf{x},\mathbf{x}')=\mathbf{E}\{s(\mathbf{x})s(\mathbf{x}')\}=\sigma_s^2e^{-\alpha\|\mathbf{x}-\mathbf{x}'\|} \tag{7}$$

where σ_s^2 and α are parameters, $\|\bullet\|$ refers to the L_2 (Euclidian) distance. On the other hand, we model the autocorrelation function of the noise $n(\mathbf{x})$ as

$$\mathbf{R}_{nn}(\mathbf{x},\mathbf{x}')=\mathbf{E}\{n(\mathbf{x})n(\mathbf{x}')\}=\sigma_n^2\delta(\mathbf{x}-\mathbf{x}')+\bar{n}^2 \tag{8}$$

where σ_n^2 and $\bar{n}$ are the noise variance and mean respectively. Substituting (7), (8) into (5), (6), we obtain:

$$\mathbf{r}_{gs}(\mathbf{x})=\bar{n}\sigma_s^2\begin{bmatrix}\vdots\\ e^{-\alpha\|\mathbf{x}-\mathbf{x}_m\|}\\ \vdots\end{bmatrix} \tag{9}$$

and

$$\mathbf{R}_{gg}=\begin{bmatrix} & \vdots & \\ \cdots & \bar{n}^2\sigma_s^2e^{-\alpha\|\mathbf{x}_m-\mathbf{x}_{m'}\|} & \cdots\\ & \vdots & \end{bmatrix}+\sigma_n^2\sigma_s^2\mathbf{I} \tag{10}$$

The solution for $\mathbf{k}$ of equation (4) is given by

$$\mathbf{k}(\mathbf{x})=\mathbf{R}_{gg}^{-1}\mathbf{r}_{gs}(\mathbf{x}) \tag{11}$$

Finally, the MMSE estimator $\hat{s}(\mathbf{x})$ is given by

$$\hat{s}(\mathbf{x})=\mathbf{g}^{\mathrm{T}}\mathbf{R}_{gg}^{-1}\mathbf{r}_{gs}(\mathbf{x})=\mathbf{r}_{gs}^{\mathrm{T}}(\mathbf{x})\mathbf{R}_{gg}^{-1}\mathbf{g} \tag{12}$$

3. MMSE RECONSTRUCTION FOR 3D ULTRASOUND

The computational complexity of reconstruction process is determined by the number of measurements *N*. To reconstruct one point, N^2+N multiplications and N^2-1 additions are required. Furthermore, a $N\times N$ matrix inversion is required in the process. The total complexity is $O(N^3)$. Therefore, MMSE reconstruction using (12) is not practical in real US reconstruction due to the large number of sampling points.

The inversion of measurement covariance matrix $\mathbf{R}_{gg}$ is the most time-consuming step in reconstruction process. For the matrix $\mathbf{R}_{gg}$, if the signal $s(\mathbf{x})$ has short correlation length, for example, if $\alpha\geq 0.3/$ millimeter, it is observed that the off-diagonal element $\mathbf{R}_{gg}(i,j)=\bar{n}^2\sigma_s^2e^{-\alpha|\mathbf{x}_i-\mathbf{x}_j|}$ for $i\neq j$ is far less than the diagonal element $\bar{n}^2\sigma_s^2+\sigma_s^2\sigma_n^2$. So the inversion of $\mathbf{R}_{gg}$ can be approximated by the inversion of its diagonal elements.

Appling the matrix inversion approximation to (11), we obtain the approximate MMSE reconstructor $\mathbf{k}'(\mathbf{x})$. Its *i*th component is given as

$$\mathbf{k}_i'(\mathbf{x})=\frac{\bar{n}}{\bar{n}^2+\sigma_n^2}e^{-\alpha\|\mathbf{x}-\mathbf{x}_i\|} \tag{13}$$

Note that $\bar{n}/(\bar{n}^2+\sigma_n^2)$ is the speckle noise's mean divided by the sum of its mean square and variance in the sampling point $\mathbf{x}_i$. It can be regarded as the index of texture information which measures scene homogeneity. It is large in homogeneous areas, and small in noisy areas. The term $e^{-\alpha\|\mathbf{x}-\mathbf{x}_i\|}$ reflects the correlation between the reconstruction point $\mathbf{x}$ and the *i*th sampling point $\mathbf{x}_i$. So, the reconstructor $\mathbf{k}'$ balances between the speckle noise's strength and signal correlation at the reconstruction point.

4. MMSE RECONSTRUCTION RESULT

In this section, we validate the MMSE reconstruction method using both synthetic and experimental ultrasound data. The synthetic simulation allows the computation of ground truth information and thus quantification of algorithm performance. The method to synthesize ultrasound data is described in [14].

In our experiments, we quantify the algorithm performance in the following terms:

(1) Mean to Standard Deviation Ratio in Speckle Region: the signal-noise ratio in the speckle region is defined as the ratio of the mean to standard deviation as given by

$$\mathbf{SNR}=\frac{\bar{E}}{\sqrt{\overline{E^2}-\bar{E}^2}} \tag{14}$$

For a fully developed speckle ultrasound image, the theoretical SNR is 1.91. A successful ultrasound reconstruction method will suppress speckle in homogeneous regions. Therefore, it should increase SNR.

(2) Contrast in Cystic Region: This is defined as

$$C = \overline{E}_s / \overline{E}_t \tag{15}$$

where $\overline{E}_s$ and $\overline{E}_t$ are the means of the speckle region and cystic target respectively. The ultrasound image is dark in the cystic target region and bright in the speckle region. A large C represents good contrast in the image.

4.1. 3D reconstruction simulation study

In this study we reconstruct a spherical object. We synthesize 71 parallel slices. In each slice, the target is a dark circular disk with varying radii. From these 71 slices, 20 slices are randomly selected. We then try to reconstruct the original 3D data from these 20 slices.

To test the performance of MMSE reconstruction method, we compared its result against the most frequently used interpolation method – bilinear interpolation method. In bilinear method, each reconstruction point is estimated by the nearest two samples in the *B*-scans that fall on either side of it. The point is then set to the inverse distance-weighted average of the two contributing samples. In our algorithm, not only two nearest samples, but all samples within a window are also used to estimate the reconstruction point. Different window sizes may be used according to the image content. Here, we select a 5×5 window. σ_n^2 and $\overline{n}$ are calculated by the local mean and local variance within the window. $\alpha = 0.5/$ millimeter Figure 1 shows the reconstruction result. All the images are log-compressed for display.

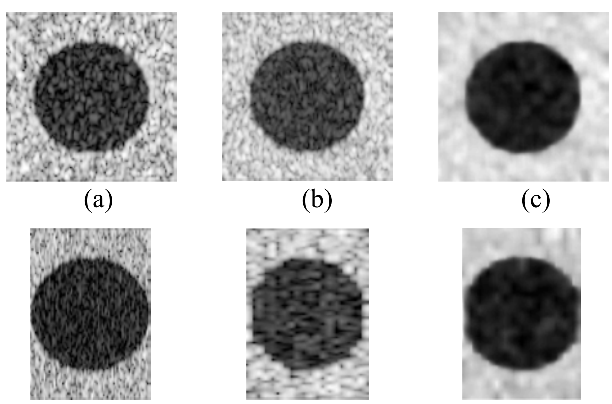


Figure 1. Reconstruction result for a spherical target: (a) a slice of simulation data; (b)-(c) a slice of reconstruction result from Bilinear and MMSE method; (d) sagittal view of simulation data; (e)-(f) sagittal view from bilinear and MMSE reconstruction result.

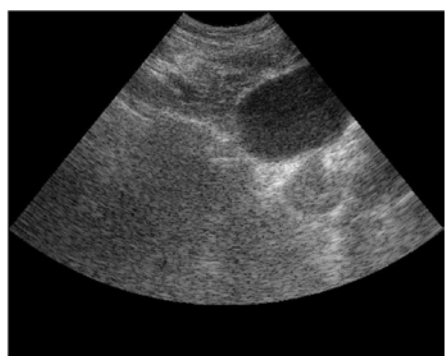
Table 1: Quantitative performances for synthetic data

	Contrast	*M/Std*
Raw data	5.6937	1.8907
Bilinear reconstruction	5.7209	2.5149
MMSE reconstruction	8.6114	5.9863

From Figure 1, it can be observed that speckle remains significant for the bilinear reconstruction, while speckle is much reduced and contrast of the target is enhanced for the MMSE reconstruction. Table 1 shows the quantitative reconstruction performance by the two criteria introduced earlier in this section. These quantitative results show that MMSE reconstruction obtain a high quality target image while suppressing speckle noise. The MMSE reconstruction outperforms the bilinear reconstruction in each criterion.

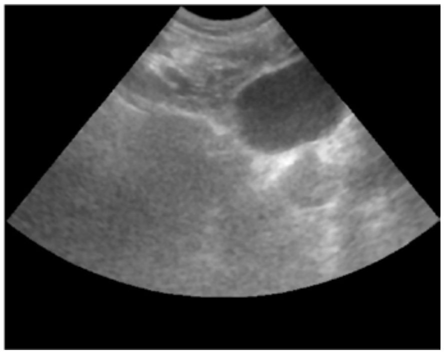
4.2. 3D reconstruction for clinical images

Figure 2 (a) shows a slice of US B-scan image of a human prostate. We acquired another two adjacent B-scans slices, which fall on the two sides of this slice, and use them to reconstruct this slice. Figure 2 (b) and (c) show the reconstruction results by the bilinear method and MMSE method respectively.



(a)

(b)



(c)

Figure 2. (a) A slice of US B-scan of prostate; (b) Bilinear reconstruction; (c) MMSE reconstruction.

For clinical images, the ground truth is unknown. However, judging from the visual appearances, one may reasonably say that our MMSE reconstruction produces less noisy and higher quality image.

5. CONCLUSIONS

We apply the MMSE principle to 3D US reconstruction and present a novel reconstruction. The MMSE reconstruction outperforms bilinear reconstruction in terms of speckle signal-to-noise ratio and contrast between the target and homogeneous regions.

Another advantage of MMSE reconstruction method is that it can unify spatial compounding and interpolation during the reconstruction process. Spatial compounding is a method of reducing speckle in US image [7]. It averages a series of B-mode images, each with independent speckle patterns, thereby enhancing the target and smoothing the speckle. Spatial compounding can also be applied to 3D US reconstruction to form higher quality data [8]. First, two or more 3D US datasets are interpolated from 2D scattered US slices, acquired from different angles. Second, The corresponding intensity values of the acquisitions are combined by their mean. From a statistical signal processing perspective, both interpolation and spatial compounding are estimations of an underlying signal in the presence of speckle noise. Interpolation estimates the signal at an unmeasured point, while compounding estimates the signal at a redundantly measured point. Thus, the two processes can be unified with a single objective of minimizing the estimation error. Our MMSE reconstruction method provides such a unifying approach. This is in contrast to the conventional ultrasound reconstruction methods, where interpolation and spatial compounding are regarded as two separate processes.

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