

# COPYRIGHT TRACING USING INVARIANTS OF CONTENTS

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## ABSTRACT

In this paper, a method of copyright monitoring or tracing using invariants of contents is proposed. We first define certain feature parameters of color images which are invariant under arbitrary bi-continuous or smooth transforms, called topological or differentially topological invariants. To extract the topological invariants robustly, we introduce scale-space of invariant features. By selecting topologically stable features with respect to scale transform, noise-robust invariants of images are obtained. These invariants can be applied to copyright tracing or monitoring and protection due to their robustness against various deformation attacks. <sup>1</sup>

## 1. INTRODUCTION

In recent years, distribution of digital contents over internet has attracted serious attention of content holders on copyright protection. Although encryption and authentication are important and effective for secure delivery of contents to legitimate users, other technologies are obviously necessary to protect copyright of the contents from potentially illegal copy once after the encryption is decrypted[1].

Currently the major technology used in copyright protection is watermarking, which is basically to embed certain foreign information (the watermarks) into the contents so that they can be detected later. These watermarks could become undetectable under attacks include geometric such as Euclidean and Affine transforms or other nonlinear deformations. Other image processings such as compression, filtering, gray scaling or histogram transformations or other linear or nonlinear transformations could also significantly reduce the detectability of watermarks. In order to guarantee the stable detectability of watermarks, one usually has to embed strong enough signals so that they could not be easily erased. However, this caused deterioration of quality of original contents.

An interesting way is, under name of soft authentication, image hashing or passive watermarkings, to identify contents without explicit embedding of foreign information,

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but based on intrinsic features of contents, e.g.[2]. Advantages of this method include no quality deterioration of contents, and the intrinsic features of contents will not be easily erased as presently used watermarks.

In this paper, we propose a method to identify contents under attacks using invariants under bi-continuous or smooth deformations, called topological or differentially topological invariants. We define several invariants of color images by using the homological invariants of tri-stimuli or their Gaussian curvature surfaces and homotopical invariants of the intersection curves between these tri-stimuli surfaces or between zero-crossing of their Gauss curvatures.

Another important problem in implementation is how to extract robustly invariants from noisy contaminated or attacked images. We introduce a scale-space representation of the invariants to overcome the problem. In particular, by selection of only topologically stable invariants with respect to the scale transform, noise, error and/or attack robust invariants can be obtained. Furthermore, global invariants of the scale-space is also defined, e.g. based on Morse theory. Reeb graphs are then used to register the invariants of the continuum of the features in the scale-space.

Simulations shown that the proposed approaches are able to extract robust invariants from images. Using the Gaussian curvature as primal sketches, Morse theoretical homological data and Reeb graphs are obtained. Besides the Jone polynomials for knots between zero-crossing of the Gaussian curvature surface are also obtained.

## 2. INVARIANTS OF CONTENTS

An important difference between image retrieving and copyright tracing is that the latter is always subjected to various deliberate and malevolent deformation attacks. The problem in watermarking is that it is difficult to detect embedded watermarks correctly under such attacks without a priori knowledge of the deformation.

Below, we show a general framework of copyright tracing based on invariants of contents under action of transformations.

To resist an attack to a content  $C(x)$ ,  $x \in \mathbf{R}^n$  is generally very hard since, in the first place, the attack is always unknown and hard to estimate, and in the second place, the number of possible attacks are usually very large and the exhaustive search is too expensive.

In order to trace up a content  $C$  which is subjected to every possible and unknown attacks, one could define the attacks as a transformation group  $T$  (or a monoid including noninvertable transformations) acting on the set of contents, and calculate the invariants. In particular, the orbit of  $C$  under action of the transformation group  $T$  is defined as  $C^T := \{\tau(C), \forall \tau \in T\}$ . The set of all possible orbits under action of  $T$  is called the orbit space. Apparently, an orbit space is a division of the space of all possible contents. If one could distinguish orbits of any two different contents under action of  $T$ , then the contents can be traced correctly. Invariant under action of  $T$  is defined as i.e. functions which are invariant under action of  $T$  or take a constant value on each orbit.

$$\mathcal{I} := \{\iota : C \mapsto \iota(C), \text{ s.t. } \iota(\tau(C)) = \text{const}, \forall \tau \in T\}$$

Thus, once one chose a class of attacks as a transformation group, contents under such attacks can be traced out by using invariants with respect to this group action. However, in practise it is difficult to restrict such deformations into any prespecified and known subclasses. Thus, in order to defend these deformation attacks, it is reasonable to use feature parameters of images which are intrinsic in the sense that they are not effected by any continuous or smooth deformation attacks.

It is known that topological (or differentially topological) properties of an object are those properties which are invariant under bi-continuous (or bi-smooth) maps, i.e. continuous (or smooth) maps whose inverse exists and is also continuous (or smooth). Such mapps are called homeomorphisms (or diffeomorphisms). The topological (or differentially topological) properties are then preserved by homeomorphisms (or diffeomorphisms)[9]. Thus, it seems that the topological or differentially topological properties of images should be used as basic invariants of images.

Below, we show examples for meaningful features of contents.

A color image is represented by three coordinates such as RGB etc., as their tri-stimuli according to the chosen color space. Denote for a 2D color image the tri-stimuli as  $u = u(x, y)$ ,  $v = v(x, y)$ ,  $w = w(x, y)$  which are functions of the position in the frame coordinates  $(x, y)$ . Then tri-stimuli of each pixel can be denoted as

$$I(x, y) = (u(x, y), v(x, y), w(x, y))$$

and the whole image can be denoted as  $I = (U, V, W)$ , where  $U, V, W$  are the graphs of the tri-stimuli which define a triple of surfaces.

Since deformations observed in attacks i.e. those near to the identity and usually local maps will not change the spatial relation between the distributions of tri-stimuli of the image, below, we consider topological or differentially topological invariants among these surfaces such as

1. Homological invariants of intersection between the tri-stimulus surfaces  $U, V, W$ , e.g. pairwise or triple-wise intersections between them;
2. Homological invariants of intersection between Gaussian curvatures of the tri-stimulus surfaces.
3. Homotopical invariants of intersection curves between tri-stimulus surfaces  $U, V, W$ .
4. Homotopical invariants of between zero-crossings of Gaussian curvature of tri-stimulus surfaces  $U, V, W$ .

As in 1-2, homological invariants, whose simplest example is the Euler characteristic numbers, are used to compare two topological manifolds. Regarding the tri-stimuli surfaces of color images as simplicial complex, one can define cycles, boundaries and boundary operation, then calculate the homological groups using simplex or cubic simplex [9][5] [3][7].

In 3-4, we consider spatical entanglements of curves. Invariants of knots and links between these curves can be extracted by various polynomial invariants e.g. Jones polynomials [6]. More precise invariants for spatial entanglement between higher genus curves such as bouquets can be obtained using e.g. the hierachical representation for 3D spatial topology[4][3][7].

### 3. ROBUST EXTRACTION OF INVARIANTS

#### Scale-space of invariants

Computation of invariants could be sensitive to noise or numerical error, or a minor deformation attack, e.g. when a content has two different parts close to each other they may be misjudged as one connected part. In such cases, stable extraction of the invariants becomes difficult.

In fact, meaningful differential and topological properties should be stable or unchanged under assumption of general positions, or when a small perturbation does not cause change of these poperties (also called structural stability).

To overcome this problem, we use the scale-space representation [8]. A scale space of an image  $x$  is defined as a 1-parameter smooth family  $L(x, t)$  such that  $L(x, 0) = x$ . Convolution by a Gaussian filter or a smoothing operator is used as scale transformation with the covariant as scale  $t$ .

In our application, the scale-space is used to obtain robust or topologically stable invariants. In fact, one knows that topologically unstable phenomina will generally become stable when it is embedded as a continuum in a 1-dimension-higher space. In fact,  $L(\cdot, \cdot)$  forms a

1-dimension-higher object (than original contents) in the scale-space. Its topology in scale space will remain intact, even its crosssection  $L(\cdot, t)$  at a particular  $t$  may be effected by perturbation. In other words, when scale increased, perturbations will only quicken or delay the change of its topology. we call a feature of a content stable if it is stable with respect to the smoothing or scale-transform. More precisely, we only adopt features which can be observed constantly over an interval of a certain length on the scale axis. In this way, one can extract topologically stable invariants.

### Global invariants of scale-space

In fact, instead of looking at each smoothed content or its features at a particular scale  $t$ , one can go further to regard the whole 1-dimension-higher object  $L(\cdot, \cdot)$  in the scale-space as a single manifold, and use its invariants as features of the original content.

There are various ways to describe such a manifold, here we adopt the so-called Morse theory and its homological invariants[9]. It is known that these homological data determine a smooth manifold up to bi-smooth maps.

Reeb Graphes are known as labeled spatial graphes which provide efficient representation of the cell decomposition of a smooth manifold according to the Morse theory[9]. A Reeb graph is derived by contracting all connected components of level sets of a Morse function on a smooth manifold into a point. Each vertex corresponds to a critical point and information on topological change before and after this critical point is labeled on the vertex( fig.1).

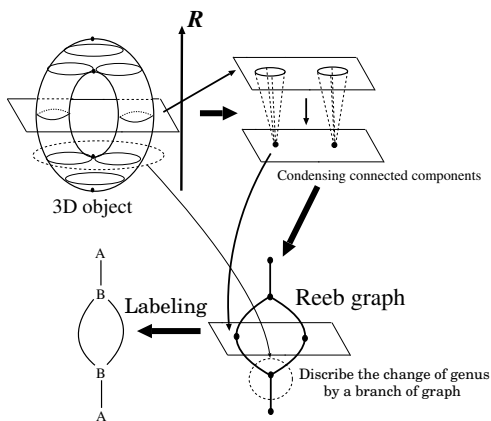


Fig. 1. Reeb Graph

## 4. SIMULATION RESULTS

Using “Girl” in simulations, we show, as invariants of the image, the Jones polynomials of entanglements between zero-crossings of Gaussian curvatures of tri-stimulus and Reeb graphs of scale-space representations of the Gaussian

curvatures.

### Invariants between zero-crossing of Gauss curvatures

The surfaces  $K_R, K_B, K_G$  of Gaussian curvatures of the tri-stimulus  $R, G, B$  are computed. For the 9 types of Jones polynomials which represent different spatial entanglements listed below, the number of them detected between zero-crossings of Gauss curvatures  $K_R - K_G, K_G - K_B, K_B - K_R$  (fig.2,3,4) are shown in the Table.

$$\begin{aligned}
 H_1 &= -t^{-\frac{1}{2}} - t^{-\frac{1}{2}} \\
 H_2 &= -t^{-\frac{1}{2}} - t^{-\frac{5}{2}} \\
 H_3 &= -t^{\frac{5}{2}} - t^{\frac{1}{2}} \\
 H_4 &= -t^{-\frac{3}{2}} - t^{-\frac{7}{2}} + t^{-\frac{9}{2}} - t^{-\frac{11}{2}} \\
 H_5 &= -t^{\frac{11}{2}} + t^{\frac{9}{2}} - t^{\frac{7}{2}} - t^{\frac{3}{2}} \\
 H_6 &= -t^{-\frac{5}{2}} - t^{-\frac{9}{2}} + t^{-\frac{11}{2}} - t^{-\frac{13}{2}} + t^{-\frac{15}{2}} - t^{-\frac{17}{2}} \\
 H_7 &= -t^{\frac{17}{2}} + t^{\frac{15}{2}} - t^{\frac{13}{2}} + t^{\frac{11}{2}} - t^{\frac{9}{2}} - t^{\frac{5}{2}} \\
 H_8 &= -t^{-\frac{7}{2}} - t^{-\frac{11}{2}} + t^{-\frac{13}{2}} - t^{-\frac{15}{2}} + t^{-\frac{17}{2}} - t^{-\frac{19}{2}} + t^{-\frac{21}{2}} - t^{-\frac{23}{2}} \\
 H_9 &= -t^{\frac{23}{2}} + t^{\frac{21}{2}} - t^{\frac{19}{2}} + t^{\frac{17}{2}} - t^{\frac{15}{2}} + t^{\frac{13}{2}} - t^{\frac{11}{2}} - t^{\frac{7}{2}}
 \end{aligned}$$

Jones	#(R-G)	#(G-B)	#(B-R)
$H_1$	20	20	25
$H_2$	5	8	1
$H_3$	5	4	3
$H_4$	2	1	0
$H_5$	1	1	1
$H_6$	1	0	2
$H_7$	0	0	1
$H_8$	0	1	0
$H_9$	0	1	0

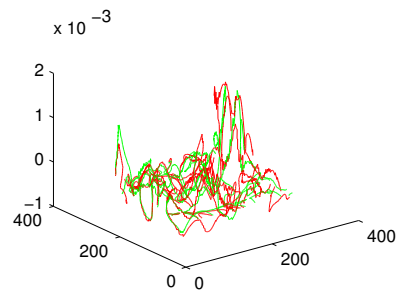


Fig. 2. Entanglement between curvatures of R and G

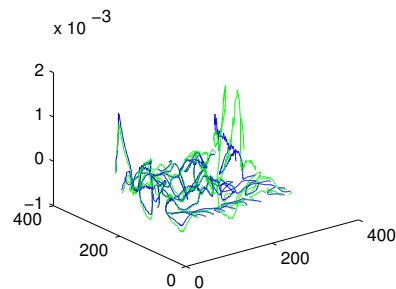


Fig. 3. Entanglement between curvatures of G and B

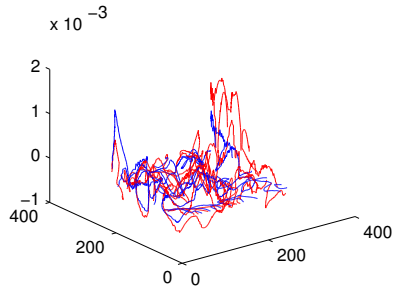


Fig. 4. Entanglement between curvatures of B and R

### Scale-space of zero-crossing of Gaussian curvature

The scale-space representations of surfaces  $\{L_{K_R}(\mathbf{x}, t)\}$ ,  $\{L_{K_G}(\mathbf{x}, t)\}$ ,  $\{L_{K_B}(\mathbf{x}, t)\}$  are produced by Gaussian filtering of  $3 \times 3$  neighborhood. The blob is chosen as the negative part of the Gaussian curvatures, then transformed to binary images. Homological data of these surfaces are obtained using simplicial homology groups. Fig.5,6 and 7 shown the Reeb graphs of  $\{L_{K_R}(\mathbf{x}, t)\}$ ,  $\{L_{K_G}(\mathbf{x}, t)\}$ ,  $\{L_{K_B}(\mathbf{x}, t)\}$ . These Reeb graphs then can be used in matching.

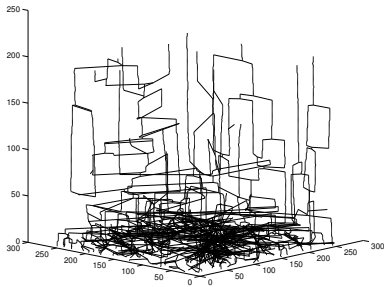


Fig. 5. Reeb Graph of curvature of Red

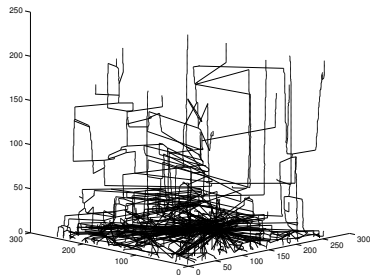


Fig. 6. Reeb Graph of curvature of Green

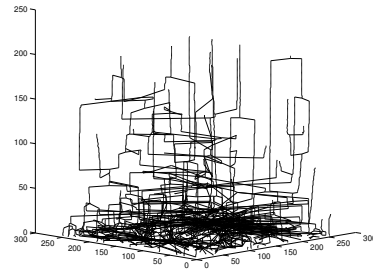


Fig. 7. Reeb Graph of curvature of Blue

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