

3D-COLOR-STRUCTURE-CODE – SEGMENTATION BY USING A NEW NON-PLAINNESS ISLAND HIERARCHY

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ABSTRACT

The Color Structure Code (CSC) [RE98] is a very fast and robust hierarchical region growing technique for segmentation of color or gray-value images. It works on a hierarchical hexagonal grid structure of the 2d space that fulfills several interesting topological properties. It is known that not all of these properties can be fulfilled together in 3d space. Some of the properties are more important than others. Here we introduce a new 3d hierarchical grid structure based on the orthogonal grid that fulfills the most important properties.

1. INTRODUCTION

In image analysis segmentation is considered to be an essential step. The aim of segmentation is to divide an image into possibly large, pairwise non-overlapping segments [HP76]. Segments are connected sets of pixels that fulfill some homogeneity criterion which is dependent on the application. Generally, segments could be considered to be homogeneous in gray value, color or texture. Segmentation is used to get an abstract, compact and symbolic representation of an image. Nowadays 3-dimensional images play an important role especially in medical imaging. They are generated, e.g., by diagnostic methods like Computer Tomography (CT), Magnet Resonance Tomography (MRT) and Positron Emission Tomography (PET). Thus, it is a valuable task to generalize the very successful 2d CSC-segmentation technique ([PR93], [RE98]) to 3d images. However, this approach leads to some surprising difficulties. In [ST03] we showed one solution which leads unfortunately to new difficulties.

2. THE HEXAGONAL 2D-CSC

The CSC follows a hierarchical region growing on a special hierarchical hexagonal topology that was firstly introduced by Hartmann [HA87]. This hierarchical topology (see fig. 1a) is formed by so-called *islands* of different levels. One island of level 0 consists of seven pixels (one center pixel

and its 6 neighbors) in the hexagonal topology. The partition of the image is organized in such a way that the islands are overlapping (each second pixel of each second row is a center of an island of level 0). One island of level $n+1$ consists of seven overlapping islands of level n (s. fig. 1a). Repeating this until one island covers the whole image the number of islands decreases from level to level by factor 4.

The segmentation algorithm operates essentially in three phases: *Initialization*, *Linking* and *Splitting*. In the *initialization phase* color homogeneous regions (initial segments) in level 0 islands of at most seven pixels are detected independently within each island of level 0. Such an (initial) segment is represented by a *code* c^0 of level 0 which stores the mean color and all pixels ($Sub(c^0)$) of that segment. Two codes c_1^0, c_2^0 are called connected iff they are representing segments possessing a common pixel, i.e. $Sub(c_1^0) \cap Sub(c_2^0) \neq \emptyset$. The result of the initialization phase is a set of codes, each one describing an initial segment.

In the linking phase these initial segments grow hierarchically to complete color segments. For that purpose in each island I^{n+1} codes of level n in the sub-islands I_1^n, \dots, I_7^n of I^{n+1} are linked to new codes of level $n+1$. Codes will be linked if the segments represented by them are connected and similar in color. The connectivity of codes can easily be determined within the hexagonal island structure: two codes c_1^n, c_2^n are connected if they share a common sub-segment in their common sub-island of level $n-1$. On level 1 this simply means that they possess a common pixel. The linking operations are repeated for all islands on each level. They start from level 1 and stop on the topmost level of the island hierarchy where only one island exists that covers the whole image. The codes c_1^n, \dots, c_m^n of level n that are linked to a new code c^{n+1} of level $n+1$ are so called sub-codes of c^{n+1} ($Sub(c^{n+1})$). Both the mean color of the sub-codes and pointers to them are stored together with the new code c^{n+1} . Note, codes may have more than seven sub-codes ($m > 7$) in the CSC (see example in figure 1c). The linking of color similar and connected codes builds up a code tree. A code forms a root of such a code tree if no link partner could be found. Each code tree represents a

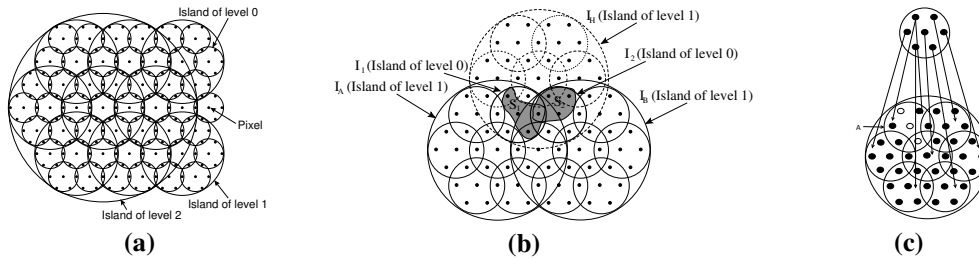


Fig. 1. a The Hexagonal Island Hierarchy. **b** Island I_H covers both islands I_1 and I_2 . **c** Code with more than seven sub-codes.

complete color segment whose pixels can be computed by descending the code tree starting with its root. We denote $S(c^n)$ for the pixels of a segment that is represented by a code tree with root c^n .

Local region growing methods often link differently colored regions due to a chain of connected pixels with smoothly changing colors (chaining mismatches). In the CSC this problem is solved by additional color checks between connected codes on every linking level. If the color of two connected codes c_1^n, c_2^n are not similar the two codes won't be linked although they are connected by a common sub-code. This means that $S(c_1^n)$ and $S(c_2^n)$ are two different segments that are connected via a common sub-segment. Therefore, their common sub-segment has to be partitioned optimally between $S(c_1^n)$ and $S(c_2^n)$ dependent on the mean color represented in c_1^n and c_2^n . This is done in the splitting phase (for detailed information s. [RE98]). The splitting phase ensures that there are no overlapping segments.

3. PROPERTIES OF ISLAND HIERARCHIES

The hexagonal island hierarchy is a very special island hierarchy with some properties. Many other different island hierarchies are possible:

1. **Homogeneity:** All islands of level $n+1$ comprise the same number of *sub-islands*.
2. **k-Neighborhood:** Each island of level n overlaps with exactly k different islands of level n .
3. **Plainness:** Two islands of level $n+1$ overlap each other in at most one island of level n .
4. **Reducibility:** The number of islands are reduced from level to level by factor 2^d where d is the dimension of the under lying grid.
5. **Center Connectivity:** Each island of level $n+1$ consists of a center island of level n and its neighbor islands.
6. **Strong (resp. weak) Saturation:** All sub-islands (except the center island) of an island of level $n+1$ are

sub-islands of *exactly* (resp. *at least*) two different islands of level $n+1$. Each center island has *exactly* (resp. *at least*) one parent island.

7. **Coverability:** Each island of level n (except the top-most island) is a sub-island of at least one island of level $n+1$.
8. **Density:** Two neighboring islands are always sub-islands of a common island.

The *homogeneity*, *k-neighborhood*, *plainness*, *reducibility*, *center connectivity* and *strong saturation* properties are just important for a simple design of the CSC segmentation method. These properties does not affect the segmentation abilities of the CSC but they lead to a nice hierarchical topology. The *weak saturation* and *coverability* property play a more important role. These properties ensure that segments may grow in all directions without limitations which is essential for a good segmentation method. The *density* property ensures that each two color similar and overlapping segments can be linked to a new segment. Consider the example of fig. 1b. The figure shows two color similar segments S_1 and S_2 that share a common pixel. S_1 was detected in level 0 island I_1 and S_2 in level 0 island I_2 . Because I_1 and I_2 are both sub-islands of level 1 island I_H the segments S_1 and S_2 can only be linked to a new segment within I_H . If I_H would be missing (violation of the density property) then there is a barrier between the islands I_1 and I_2 . S_1 and S_2 cannot be linked directly together although they are overlapping. Due to the barriers it is not possible for segments to grow freely in all directions. A CSC segmentation based on a hierarchies that violates the density property is therefore definitely not optimal.

Thus an island hierarchy that should be used for segmentation with the CSC has to fulfill at least the *weak saturation*, *coverability* and *density* property. The other properties are optional. But it would be better if all of them could be fulfilled. Then the CSC can be used unmodified for the 3d-segmentation-task.

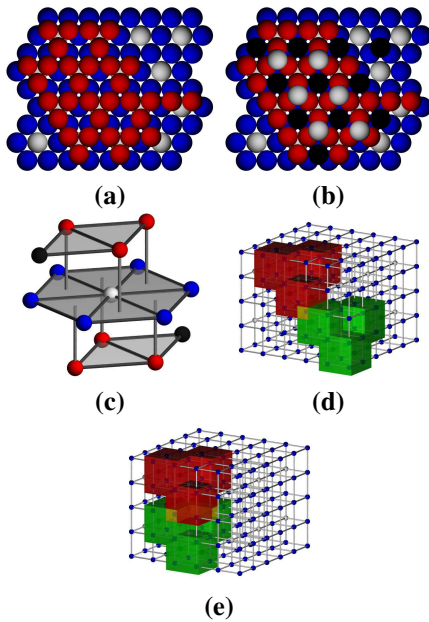


Fig. 2. **a** First two layers of the MDSP. White spheres: Island centers. Blue and red spheres: Neighbors of island centers. Black spheres: Holes. **c** Third layer of the MDSP. Two C_{19} islands of level 0 may overlaps in **d** two or **e** three common voxels.

4. THE SPHERE-ISLAND-HIERARCHY

In [ST03] we tried to find a generalization of the hexagonal island hierarchy in 3d. We used the most dense sphere packing (abbr.: MDSP see fig. 2) as the underlying grid of the 3d island hierarchy. The MDSP [CO98] is very similar to the 2-dimensional hexagonal grid: The distance between two adjacent spheres (neighbors) is always the same. Each sphere touches exactly 12 other spheres – 6 neighbors in the same layer and 3 neighbors each in the layer above and below. We define in a first step a 3d island of level 0 as a set of 13 spheres – one center sphere and its 12 neighbors. Each second sphere of each second row of each second layer has to be a center sphere to ensure that two 3d islands of level 0 overlaps in at most one common sphere (Plainness) and that each sphere except the center sphere is covered by exactly two different islands (Strong Saturation). As one may see in fig. 2 there are some (black) spheres that are not a neighbor of any center sphere. Thus, such spheres (holes) are not covered by any 3d island of level 0 (violation of the coverability property). To avoid this problem we assign to each island of level 0 two additional holes that lie close to its island center (s. fig. 2d). Now each 3d island of level 0 overlaps with exactly 14 other islands of level 0. Generally a 3d island of level $n+1$ consists of one center island of level n and its 14 neighbors. Also a 3d island of level $n+1$

overlaps with exactly 14 other islands of level $n+1$. The center connectivity is fulfilled for all islands of level $n > 0$ but for islands of level 0. The center of an island of level 0 is never adjacent to a hole. From level to level the number of islands decreases by factor 8 (Reducibility).

The 3d sphere-island-hierarchy fulfills all properties of the hexagonal island hierarchy but the very important density property. A lot of barriers between overlapping islands exists within the sphere-island-hierarchy. Therefore segments are not able to grow freely in all directions. Moreover overlapping segments may occur in the segmentation result. To this problem we introduce in [ST03] an enhancement of the CSC. In a special linking-and-splitting phase overlapping segments will be handled: If two overlapping segments are color similar they will be merged to a new (meta) segment. Otherwise the two segments will be separated. This linking-and-splitting phase is repeated until there are no more overlapping segments. This additional phase did not solve the problems caused by the violation of the density-property satisfactory since the effects of the barriers on the segmentation could not be made undone. Without the barriers in the island hierarchy segments may grow very different.

5. NON-PLAINNESS ISLAND HIERARCHIES

To avoid barriers in the 3d-dimensional island hierarchy we are looking for a new hierarchy that fulfills at least the *coverability*, the *weak saturation* and the *density* properties which are the most important properties. This new hierarchy may violate other properties which are not so important for segmentation. The C_{27} island hierarchy is based on the orthogonal grid. Each second voxel of each second row of each second layer is center voxel. An island of level 0 is a cube consisting of a center voxel and its 26 neighbors. Islands of level 0 can be considered as macro voxels that form an orthogonal grid by themselves. Islands of level $n+1$ consists now of a (center) island of level n and its 26 overlapping neighbors. The C_{27} island hierarchy fulfills all three demanded properties. But the overlapping structure of the islands is much too complex. Two islands may overlap in 1, 3 or 9 common voxels. Islands may have up to 8 parents. Now we used a computer program to find an island hierarchy that fulfills all 3 demanded properties but does not have such a complex overlapping structure like C_{27} . The program starts with the C_{27} island hierarchy and replaces its islands by smaller islands $C_x \subset C_{27}$. Further we consider just islands with more than 15 and less than 23 sub-islands. We prove in [ST04] that if the islands consists of less than 15 sub-islands the coverability property could not be fulfilled. The upper limit was just a guess. We found solutions for $x = 19$ and for $x = 23$. As we are looking just for island hierarchies with small islands we reject the solutions

for $x = 23$. It turns out that all solutions for $x = 19$ are just rotated or mirrored versions of a common prototype island hierarchy (denoted by C_{19}). Islands of type C_{19} can be imagined as three overlapping cubes with side length 2. There are only two different overlapping types between two islands of some level n (see figure 2d-e): Two islands may overlap in 2 or 3 common sub-islands. An island may have only 2 or 3 parent islands. This is not too complex compared to the sphere-island-hierarchy it is but much more simple than C_{27} .

The homogeneity, coverability, weak saturation and density are fulfilled for C_{19} . This was tested by the computer. The strong saturation and the plainness are obviously violated. But what's about the center connectivity? Each island of type C_{19} overlaps with only 12 other islands of the same level and not with 18. But an island consists of exactly 19 sub-islands. This means that each island of level $n+1$ does not consist of just a center island and its 12 neighbors. Thus the *center connectivity property* cannot be valid. But this property is not important for a working CSC algorithm.

6. RESULTS

The advantages of the 2d-CSC are its good performance and its accuracy in finding color homogeneous regions without any chaining mismatches. We introduced in this paper a method how to generalize the hexagonal island hierarchy to a three-dimensional non-plainness island hierarchy C_{19} . C_{19} does not fulfill all properties of the hexagonal island hierarchy but the most important.

Figure 3a) shows two CSC-segments of an CT head image (256^3 voxels). The segments are visualized in different mean gray values although they have a similar mean gray value in the original image. The first segment is a combination of the skull and the spinal column. Both have a similar gray value and are connected in the image. Thus they form one CSC-segment. The second segment is the lower jaw. Although it has the same gray value it is not spatial connected with the skull. Figure 3b) shows two CSC-segments extracted from a MRT brain image.

To check whether C_{19} is an improvement in comparison to the sphere-island-hierarchy and the additional linking-and-splitting phase we did a segmentation of 21 MRT brain images using both methods. As gray similarity measurement we used a simple thresholding method, i.e. two gray values are similar if their distance is less than a certain threshold. For segmentation of the brain images we used 10, 12 and 14 as thresholds. After that we counted the number of segments in each image and computed the mean standard deviation of the segments in each image. In each of the 63 test the segmentation with C_{19} leads to less segments and a lower mean standard deviation at the same time than the segmentation with the sphere-island-hierarchy (and

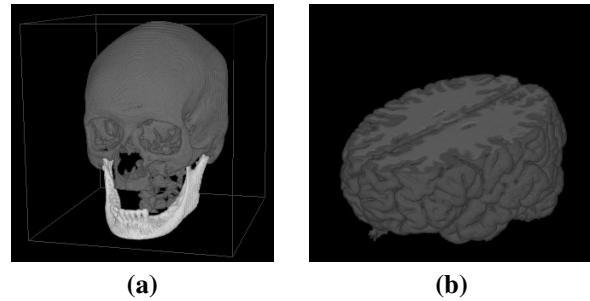


Fig. 3. a Segmentation result of a CT-image (head). b Segmentation result of a MRT-image (head).

the additional splitting-and-linking phase). Therefore using C_{19} for segmentation leads to a better segmentation result than using the sphere-island-hierarchy and the additional splitting-and-linking phase.

7. REFERENCES

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