EFFICIENT FUZZY-CONNECTEDNESS SEGMENTATION USING SYMMETRIC CONVOLUTION AND ADAPTIVE THRESHOLDING

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ABSTRACT

Fuzzy Connectedness segmentation emerged in recent years as an alternative to traditional "hard" image-segmentation approaches. It employs scale-based affinity, which incorporates both fuzziness and degree of hanging-togetherness of a region, to extract regions of interest from, especially, medical images. Computation complexity has been, however, one of its arguable issues that needs further theoretical investigation and improvement. Furthermore, the homogeneity parameter needs to be specified on per image fashion. In this paper, we propose an improved fuzzy connectedness segmentation method by utilizing a sequential grow-and-merge scheme that we called symmetric convolution, and an adaptive thresholding technique that incorporates an entropy-guided process to determine the homogeneity parameter. The proposed approach with symmetric convolution is proven valid and efficient. We employ a simulated on-line Brain database – BrainWeb to generate the testbed to evaluate the accuracy and robustness of the proposed algorithm.

1. INTRODUCTION

In recent years, medical imaging techniques have been extensively and significantly improved along with the development of rapid and relatively inexpensive computational resources. The acquired images are thus usually huge and the processing and analysis algorithms must be efficient enough to deal with a great amount of high-resolution images. In addition, images captured by medical scanners exhibit inherent inaccuracies. How the inaccuracies or ambiguities are preserved and represented as long as possible until they have to be solved is a also a challenging issue. In this paper, we pursue sound combination of effectiveness and fuzziness.

Image segmentation is a process of partitioning an image space into disjoint homogeneous regions. The success of an image analysis system heavily depends on segmentation performance. Many scanner factors, however complicate the image-segmentation issue. Fuzzy image segmentation techniques have been introduced to cope with the imprecise or ill-defined data [1]. Further, the Fuzzy Connectedness (FC) algorithm, apart from most of the other "crisp" segmentation techniques, captures the idea of "hanging togetherness" of image points [2]. We refer to the 2D image points as pixels and 3D image elements as voxels. Generally, an *N*-dimensional image point is called a spatial element or *spel*. Hanging togetherness is a grouping property that can depict the grouping state among image points. The algorithm analyzes the image and assigns a strength of connectedness to every pair of image points to represent the intensity features and patterns of intensity variations. It assumes the user has prior domain knowledge to select a suitable threshold value that corresponds to image-dependent homogeneity parameter [3]-[5].

Two problems exist: (1) how to design an efficient implementation that is theoretically valid and yields equivalent results to the existing methods; (2) how to automate the threshold specification process and make it intuitive to common users.

Our approach is first to incorporate previously proposed symmetric region growing into the fuzzy-connectedness framework and propose a symmetric convolution scheme that is theoretically proven valid [6]. With symmetric convolution, a medical image can be analyzed efficiently in terms of both time complexity and space usage. Secondly, we take into account the spatial characteristics and connectivity among *spels* within a region, to propose an adaptive threshold selection scheme that employs entropy fuzzy measures to enhance conventional fuzzy connectedness segmentation [7].

The remainder of this paper is organized as follows. In Section 2, we give a brief introduction to fuzzy connectedness. Detailed disposition can consult with Ref. [4] for a thorough deposition. In Section 3 we describe the symmetric convolution and propose fuzzy connectedness threshold-selection scheme. In Section 4, we present the experimental results and comparisons with some existing approaches. The images used were generated from BrainWeb (http://www.bic.mni.mcgill.ca/brainweb/) [8]. Conclusion remarks follows in Section 5.

2. FUZZY-CONNECTEDNESS: A BRIEF REVIEW

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Udupa *et al*. incorporated the concept of fuzzy connectedness with the image segmentation algorithm to deliver the notion of "hanging togetherness" [2]. The overall idea is to construct a fuzzy map of connectedness of every image points and their relations with the user-specified seed in the object of interests (OOIs). The fuzzy relations used to calculate membership functions are *fuzzy adjacency*, fuzzy *affinity*, and *fuzzy connectedness*.

A *fuzzy relation* $\alpha = \{((c,d), \mu_\alpha(c,d)\} \mid c,d \in Z^n\}$ in Z^n is said to be a fuzzy adjacency if it is relfexive and sysmetric, where c and d are spels in Z^n. A fuzzy relation $k = \{((c,d), \mu_k(c,d)\} \mid c,d \in C\}$ in certain scene C is said to be a fuzzy affinity in C if it is reflexive and symmetric. Fuzzy adjacency capture local phenomenon of "how close the spels are; the closer the points are, the more adjacency they are to each other. For all spels c, d, the adjacency function $\mu_\alpha(c,d)$ is assumed to be:

$$\mu_\alpha(c,d) = \begin{cases} 1, \text{ if } \sqrt{\sum_i (c_i - d_i)^2} \le 1, \\ 0, \text{ otherwise.} \end{cases}$$

Fuzzy affinity μ_k is a local parameter which takes the degree of adjacency of the spels as well as the similarity of their intensity value into account. That is

$$\mu_k(c,d) = \mu_\alpha(c,d)[\omega_1 h_1(f(c), f(d)) + \omega_2 h_2(f(c), f(d))], \text{ if } c \ne d,$$

$$\mu_k(c,d) = 1.$$

where ω_1 and ω_2 are weighted function satisfying

$$\omega_1 + \omega_2 = 1 .$$

h_1 and h_2 are Gaussian functions of $\frac{1}{2}(f(c) - f(d))$ and $|f(c) - f(d)|$ respectively. Therefore,

$$h_1(f(c), f(d)) = e^{-\frac{1}{2}[\frac{\frac{1}{2}(f(c)+f(d)-m_1)}{s_1}]^2},$$

$$h_2(f(c), f(d)) = e^{-\frac{1}{2}[\frac{(f(c)-f(d)-m_2)}{s_2}]^2}$$

where m_1, m_2 and s_1, s_2 represent the mean standard deviation of spel values and their differences for spels that are in the object of interest. The f function represents spel intensity. Affinity is used to assign the strength of connectedness to any pair of spels.

Finally, fuzzy connectedness captures the global phenomenon of hanging togetherness in a fuzzy relation between two spels. The strength of a path is the strength of the weakest link in the pair of spels, the strength of the connectedness between spel *c, d* is the strength of the strongest of all path.

3. SYMMETRIC CONVOLUTION AND ENTROPY-GUIDED ADAPTIVE THRESHOLDING

3.1. Symmetric Convolution

Symmetric region growing is an image segmentation paradigm that emphasizes its efficacy on both computational complexity and processing memory needed [6]. After segmentation, an image is partitioned into disjoint regions R_i: $S = \bigcup_{i=1}^{M} R_i$, where $R_i \cap R_j = \phi$ for $i \ne j$ and S represents the segmented image. Traditional (seeded) region-based segmentation methods usually involve recursive or iterate region growing and, say for 2D images, *xy*-inseparable computation. Given any *symmetric* criteria that account for local (or intra-region) variations in an image, the following theorems and corollary hold.

THEOREM 1: Consider a symmetric region growing algorithm $SymRG(\psi)$, such that $S(I, SymRG(\psi), A) = \bigcup_{i=1}^{M} R_i$.

The replacement of certain seed for a resulting region by another point in the same region gives the equivalent segmentation result.

THEOREM 2: Given $SymRG(\psi)$ and seed sets $A, B \subset I$, both sets containing corresponding pairs of seeds in the same regions. We then have $P_{AB}(I, SymRG(\psi)) \ne \phi \Leftrightarrow S(I, SymRG(\psi), A) \equiv S(I, SymRG(\psi), B)$.

COROLLARY 1: Consider $(SymRG(\psi), \boldsymbol{S})$, a complete segmentation algorithm based on symmetric region growing. Scan the digital image of interest, I, sequentially. Grow regions from each scanned point by applying criteria $\psi = \langle \boldsymbol{I}, \boldsymbol{X} \rangle$, until all image points have been visited. Examine the resulting regions using $\boldsymbol{S}$. If any point p of a region satisfies criteria $\boldsymbol{S}$ for region R_i, then assign the region to R_i; otherwise, relegate it to the background R_M. The resulting segmented image is $S(I, SymRG(\psi), \boldsymbol{S})$.

The above theorems and corollary suggest that the algorithm can grow-and-merge the regions *sequentially* and *uni-directionally* while producing the equivalent result to the traditional segmentation methods. Therefore, it is more efficient as well as capable of implementation in parallel fashion. As the fuzzy-connectedness emphasizes that the fuzzy adjacency, fuzzy affinity, and fuzzy connectedness are symmetric – complying with the requisite of the symmetric region-growing paradigm, the symmetric convolution is applicable to the modified fuzzy-connectedness segmentation algorithm.

3.2. Entropy-guided adaptive thresholding

Traditionally, Otsu's thresholding method was used to obtain optimal threshold value automatically [5]. It performs exhaustive search of potential thresholds that can maximize between-class variance of pixel intensity to achieve image segmentation. However, histogram information can be easily biased by the noises. Instead of choosing a threshold values, we apply fuzzy measurements, by minimizing the entropy measurement [7]. The entropy of a fuzzy image *X* as defined by De Luca et al. is [7]:

$$H(X) = \frac{1}{MN \ln 2} \sum_m \sum_n E(\mu_X(x_{mn})) \text{ and}$$

$$E(\mu_X(x_{mn})) = -\mu_X(x_i)\ln \mu_X(x_{mn}) - (1-\mu_X(x_{mn}))\ln(1-\mu_X(x_{mn}))$$

$$m = 1,2...,M \; , n = 1,2,...,N$$

E , reaching its maximum at 0.5, is increasing in the interval [0,0.5] and decreasing in the interval [0.5,1]. The following method is employed [7]:

$$\mu_X(x_{mn}) = \begin{cases} \dfrac{1}{1+|x_{mn}-\mu_b|/K}, & \text{if } x_{mn} \le t \\ \dfrac{1}{1+|x_{mn}-\mu_f|/K}, & \text{if } x_{mn} > t \end{cases}$$

where K is a constant value such that $1/2 \le \mu_X(x_{mn}) \le 1$ and

$$\mu_b = \frac{\sum_{l=1}^{T} l \times h(l)}{\sum_{l=1}^{T} h(l)} \; , \; \mu_f = \frac{\sum_{l=T+1}^{L} l \times h(l)}{\sum_{l=T+1}^{L} h(l)}.$$

The proposed entropy-guided approach utilizes the information of the mean of spel intensity values *m* which are in the object of interest as the weight of μ_X :

$$\ddot{\mu}_b = \varpi_1 \frac{\sum_{l=1}^{T} l \times h(l)}{\sum_{l=1}^{T} h(l)} \; , \; \ddot{\mu}_f = \varpi_2 \frac{\sum_{l=T+1}^{L} l \times h(l)}{\sum_{l=T+1}^{L} h(l)}.$$

where $\varpi_1 = [\left|l-m\right|/(L+1)]$ and $\varpi_2 = 1 - \varpi_1$.

Finally, substitute the entropy membership functions μ_f and μ_b for $\ddot{\mu}_f$ and $\ddot{\mu}_b$. The formula is rewritten as:

$$\ddot{\mu}_X(x_{mn}) = \begin{cases} \dfrac{1}{1+|x_{mn}-\ddot{\mu}_b|/K}, & \text{if } x_{mn} \le t \\ \dfrac{1}{1+|x_{mn}-\ddot{\mu}_f|/K}, & \text{if } x_{mn} > t \end{cases}$$

$$\ddot{H}(X) = \frac{1}{MN \ln 2} \sum_m \sum_n E(\ddot{\mu}_X(x_{mn})),$$

where

$$E(\ddot{\mu}_X(x_{mn})) = -\ddot{\mu}_X(x_i)\ln \ddot{\mu}_X(x_{mn}) - (1-\ddot{\mu}_X(x_{mn}))\ln(1-\ddot{\mu}_X(x_{mn}))$$

$$m = 1,2...,M \; , n = 1,2,...,N$$

4. RESULTS

The test images are simulated from BrainWeb [8]. Figure 1 illustrates an MR brain image obtained with T1-weighted and of size $181 \times 217 \times 181$.

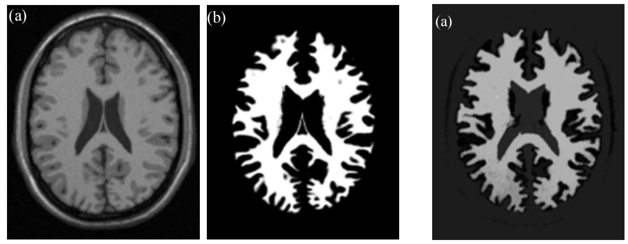

Figure 1: (a) The image slice from BrainWeb datasets with T1-weighted and the size of the volume image is $181 \times 217 \times 181$ with resolutions of $\Delta x = \Delta y = \Delta z = 1mm$; (b) The desired segmented result – "ground truth" of the white matter region provided by BrainWeb; (c) Result of fuzzy-connectedness segmentation.

Figure of merit (FOM) is employed to evaluate the performance of the proposed method. For any scene $\varsigma = (C, f)$, we denote by $\varsigma_t = (C, f_t)$ the binary scene which results from thresholding ς at *t*. That is for any $o \in C$,

$$f_t(o) = \begin{cases} 1, & \text{if } f_t \ge t \\ 0, & \text{otherwise.} \end{cases}$$

The FOM is defined as:

$$FOM = \left[\left(1 - \frac{|C_t \otimes C_G|}{|C|} \right) \times 100 \right],$$

Where C_G represents ground truth, $|C|$ the cardinality of *C*, $\otimes$ represents the exclusive-OR operation between two binary scene, and $|C_t \otimes C_G|$ denotes the number of 1-valued spel in $C_t \otimes C_G$. FOM represents the degree of match between the original object regions captured in C_G and object region in C_t resulting after thresholding.

Table 1 shows the comparison of segmentation results from various approaches. The optimal threshold is defined as the grey level which maximizes as possible the separation between of the two classes. That is, when the fuzziness is minimal. As expected, when the intensity non-uniformity and noise level increased, the Otsu's thresholding methods were not able to detect the optimal threshold because of the unimodal histogram, thus segmentation results are affected with pre-settings of the fuzzy connectedness method. In contrast to Otsu's method, the proposed modified entropy method can deal with the unimodal histogram effectively, as shown in Figure 2.

5. CONCLUSIONS

This work incorporates previously proposed symmetric region growing to allow the fuzzy-connectedness segmentation to perform more efficiently. By utilizing the fuzzy measures, intensity means of regions of interest, and entropy of the image, the proposed method yields a more robust empirical result.

6. ACKNOWLEDGMENTS

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Table 1

Each of the entry represents FOM measures which are resulted from Otsu, Entropy (Ent), Modify Entropy (MEnt) thresholding method.

	INU: 0%			INU: 20%			INU: 40%		
	Otsu	Ent	MEnt	Otsu	Ent	MEnt	Otsu	Ent	MEnt
N0: σ=0	98.27	95.88	97.53	96.40	92.67	98.29	92.76	92.12	96.06
Thr Lev	0.4392	0.3164	0.6172	0.4275	0.3359	0.5703	0.4118	0.3945	0.5195
N1: σ=0.001	98.87	96.44	97.69	95.15	90.73	97.80	93.51	92.52	96.01
Thr Lev	0.4196	0.3477	0.6055	0.4118	0.3281	0.5430	0.4078	0.3750	0.4883
N2: σ=0.003	96.65	89.01	98.02	93.55	88.94	97.67	91.04	89.56	95.03
Thr Lev	0.3922	0.2405	0.5859	0.3922	0.2852	0.5313	0.3726	0.3281	0.4883
N3: σ=0.005	97.54	93.88	97.31	88.77	87.00	94.20	94.81	90.79	97.14
Thr Lev	0.3765	0.2617	0.5547	0.3608	0.2813	0.5078	0.3843	0.2735	0.5469
N4: σ=0.007	93.59	87.01	97.76	95.24	88.57	97.30	88.39	86.56	90.57
Thr Lev	0.3804	0.1914	0.5703	0.3608	0.1836	0.5625	0.3569	0.2070	0.4570

Note: Intensity non-uniformity (INU) increases along rows and the noises (N) increases with the σ values along columns.

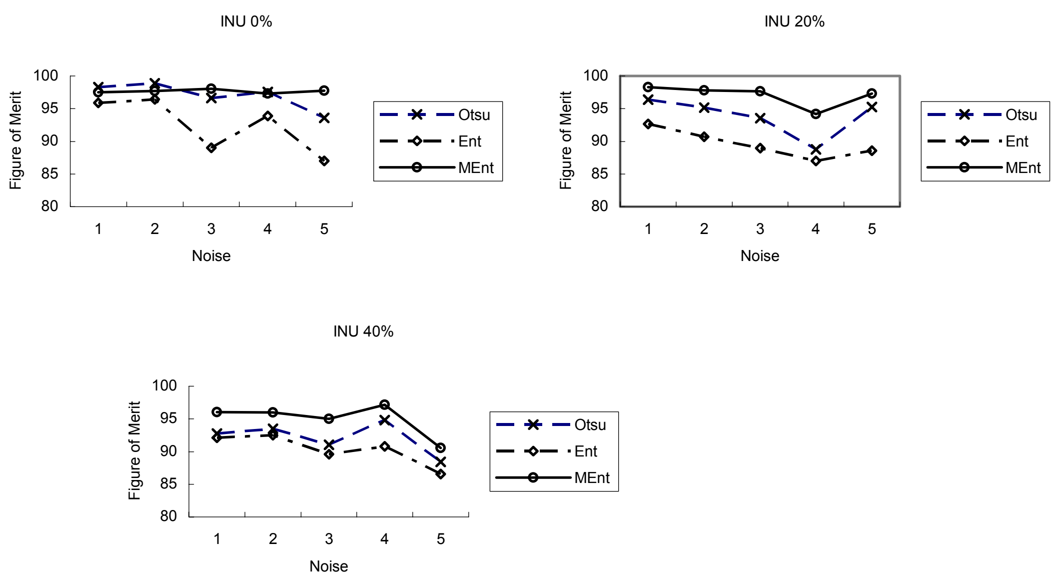

Figure 2: Line charts of Table 1.