FOCAL LENGTH SELF-CALIBRATION BASED ON DEGENERATED KRUPPA'S EQUATIONS: METHOD AND EVALUATION

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ABSTRACT

In this paper, a linear self-calibration approach for camera focal length estimation is proposed. No priori information of camera motion is needed for the method. The unknown focal length of camera is conveniently obtained in closed form from degenerated Kruppa's equations. These degenerated equations (one quadratic and two linear) provide the essential constraints on calibration of camera intrinsic parameters. Real images of indoor and outdoor generic scenes and 3D metric reconstruction are used to analyze the accuracy of the estimated focal length. Experiments results show that the method is both robust and efficient.

1. INTRODUCTION

Camera self-calibration aims to automatically establish the camera's intrinsic parameters without the use of any 3D information. As a key step for constructing 3D models, it involves the computation of metric properties of the cameras and/or the scene directly from several uncalibrated images. In general, camera calibration can be divided into two categories: Photogrammetric calibration [1] and self-calibration [2].

Photogrammetric calibration uses a precise pattern with known metric information to calibrate a camera. This pattern is usually composed of two or three orthogonal grid planes. Since metric information is provided with high precision, calibration can be done with good accuracy.

However, the use of precise patterns is expensive and inconvenient for practical applications. Besides, the photogrammetric approach cannot detect metric information changes over time. Self-calibration can overcome such problems. The approach just needs two or more images as input and produces the camera's intrinsic parameters as the output.

Traditional approaches for camera self-calibration are nonlinear since the constraint for the camera intrinsic parameter matrix is quadratic [3]. They often fail as the solution for such nonlinear optimization may easily fall into local minima.

Because of the above difficulty, many researches on self-calibration incorporated controlled motion information [4] [5]. Some linear techniques are also proposed [6]. Other recent efforts have been taken to linearize Kruppa's equations. For example, Ma [7] found the constant scale factor for Kruppa's equations for two special motion cases.

When some assumption on camera intrinsic parameters is considered, closed form calibration equations could be obtained [8]. This paper assumes that only the focal length of camera is unknown and skew factor is zero. Kruppa's equations then degenerate to one quadratic and two linear equations. The algorithm is partly referred in [9] and [10], but more rigorous derivation and evaluation are conducted in this paper.

2. BACKGROUND

The so called Kruppa's equation can be represented as

$$FCF^T = s[e_2]_\times C[e_2]_\times = s[e_2]_\times AA^T[e_2]_\times \quad (1)$$

where F is the fundamental matrix, e_2 is the right epipole and A is the camera intrinsic parameter matrix. (1) gives the general constraint for camera intrinsic parameter matrix. Since C is the symmetric matrix, (1) provides six equations to compute C . However, at most only two equations are independent. These six equations are quadratic in terms of C and thus fourth order in terms of elements of A . The common way to compute A is to first obtain C by iterative estimation and to obtain A next by Cholesky decomposition.

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The problem for the above derivation is to decide which two equations are independent. Hartley gives a simple derivation for Kruppa's equations based on singular value decomposition [11]. This derivation decomposes the above six equations to only two equations thus such inconvenience is avoided.

Specifically, let the singular value decomposition (SVD) [12] of fundamental matrix be $F = U\Sigma V^T$. Here $U = [u_1 \quad u_2 \quad u_3]$ $V = [v_1 \quad v_2 \quad v_3]$, and $\Sigma = diag(a, b, 0)$. Since the right epipole $e_2 = null(F^T) = u_3$, then $[e_2]_\times = UMU^T$, M is the skew symmetric matrix for $[0 \quad 0 \quad 1]^T$. Hence after some mathematical manipulations, the Kruppa's equations (1) becomes

$$\Sigma V^T CV\Sigma^T = sMU^T CUM^T \tag{2}$$

Only four elements in each matrix are nonzero, thus (2) gives only two equations on constraining A. The final result is

$$\frac{a^2 v_1^T C v_1}{u_2^T C u_2} = \frac{b^2 v_2^T C v_2}{u_1^T C u_1} = \frac{ab v_1^T C v_2}{-u_2^T C u_1} \tag{3}$$

In this paper, equations in (3) are fundamental to calibrate the camera. Since (3) provides two independent equations, and the number of unknown parameters in C is five, at least three images are needed to obtain C.

3. LINEAR METHOD FOR FOCAL LENGTH CALIBRATION

3.1. Linearize Kruppa's equations

Equation (3) provides the simplified form of Kruppa's equations. The common method to obtain C is to initially estimate the parameters, then conduct non-linear optimization methods to obtain more accurate results. Although equation (3) provides constraints on the camera's intrinsic parameter matrix, it is not easy to solve for these parameters as multiple solutions exist for these nonlinear equations. Generally, three fundamental matrices or three images are needed to fully calibrate a camera. However, these three images represent six constraints. It is difficult to know whether these six constraints are independent. Even if they are independent, solutions from any five of the six constraints could be different. There are thus $2^5 = 32$ possible solutions. We have to eliminate spurious solutions one by one.

We assume that only the focal length is unknown (but constant) and the skew factor to be zero. Therefore, with simple coordinate transformation, Kruppa's equations in (3) can be further linearized. Specifically, we transform the image coordinates so that aspect ratio appears to be one and principal point is at the image center. The new fundamental matrix now is $F' = T^{-T} F T^{-1}$, where F' and F are the new and the original fundamental matrix respectively, and T is the transformation matrix. Consider the singular value decomposition of F' is $F' = USV^T$, then (3) in turn yields:

$$\frac{a^2 v_1^T diag(f^2, f^2, 1) v_1}{u_2^T diag(f^2, f^2, 1) u_2} = \frac{b^2 v_2^T diag(f^2, f^2, 1) v_2}{u_1^T diag(f^2, f^2, 1) u_1} = \frac{ab v_1^T diag(f^2, f^2, 1) v_2}{-u_2^T diag(f^2, f^2, 1) u_1} \tag{4}$$

Where a, b are two singular values of F' and f is the focal length. Expanding the above equations, we further obtain

$$\frac{a^2 (v_{11}^2 f^2 + v_{12}^2 f^2 + v_{13}^2)}{u_{21}^2 f^2 + u_{22}^2 f^2 + u_{23}^2} = \frac{b^2 (v_{21}^2 f^2 + v_{22}^2 f^2 + v_{23}^2)}{u_{11}^2 f^2 + u_{12}^2 f^2 + u_{13}^2} = \frac{ab (v_{11} v_{21} f^2 + v_{12} v_{22} f^2 + v_{13} v_{23})}{u_{11} u_{21} f^2 + u_{12} u_{22} f^2 + u_{13} u_{23}} \tag{5}$$

Due to the orthogonality of U and V, the three fractions are rewritten as

$$\frac{a^2 (1 - v_{13}^2) f^2 + a^2 v_{13}^2}{(1 - u_{23}^2) f^2 + u_{23}^2} = \frac{(1 - u_{13}^2)(1 - v_{23}^2) f^2 + b^2 v_{23}^2}{(1 - u_{13}^2) f^2 + u_{13}^2} = -\frac{ab v_{13} v_{23}}{u_{23} u_{13}} = s \tag{6}$$

Note that the right most fraction degenerates to a constant factor s. Rearranging equation (6), we obtain

$$f^2 [a u_{13} u_{23} (1 - v_{13}^2) + b v_{13} v_{23} (1 - u_{23}^2)] + u_{23} v_{13} (a u_{13} v_{13} + b u_{23} v_{23}) = 0 \tag{7}$$

$$f^2 [a v_{13} v_{23} (1 - u_{13}^2) + b u_{13} u_{23} (1 - v_{23}^2)] + u_{13} v_{23} (a u_{13} v_{13} + b u_{23} v_{23}) = 0 \tag{8}$$

and

$$f^4 [a^2 (1 - u_{13}^2)(1 - v_{13}^2) - b^2 (1 - u_{23}^2)(1 - v_{23}^2)] + f^2 [a^2 (u_{13}^2 + v_{13}^2 - 2u_{13}^2 v_{13}^2) - b^2 (u_{23}^2 + v_{23}^2 - 2u_{23}^2 v_{13}^2)] + (a^2 u_{13}^2 v_{13}^2 - b^2 u_{23}^2 v_{23}^2) = 0 \tag{9}$$

From the above derivation, it is clear that when only the focal length is unknown, the Kruppa's equations can be further decomposed into one quadratic and two linear equations. Base on equation (6), we know these three equations are dependent.

3.2 The Algorithm

The developed algorithm is shown in Figure 1. It basically comprises three steps of feature detection, robust matching estimation and self-calibration. Firstly, Harris corner detection [13] is applied on a pair of images to detect feature points. Next, a robust matching estimation technique is implemented with the aim to find sufficient corresponding points between the two images. The routine consists of cross-correlation matching, relaxation, RANSAC and Least Median Square (LMedS) [14] for epipolar geometry estimation. These correspondences can be also established manually especially when the base line of the two images is long. It is necessary to note that if radial distortion is significant, it should be corrected at the self-calibration stage [15]. The fundamental matrix for each pair of images is then obtained. Finally, we produce the estimation of the focal length based on its closed form in equations (7) (8) (9).

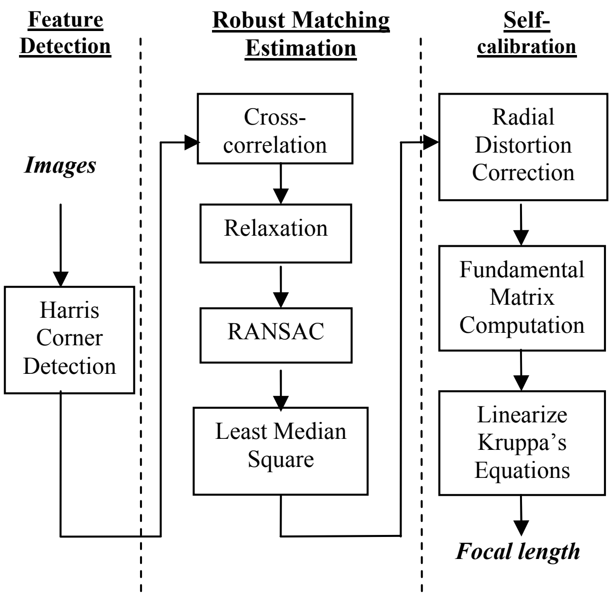

Figure 1 Algorithmic Block Diagram

4. EXPERIMENT

A Sony DSC-P31 digital camera is used to capture the images at a 640×480 resolution for all experiments. The camera is first calibrated utilizing a photogrammetric calibration algorithm [1] and six images of a calibration grid. The resulting focal length of 625 pixels is employed as the "ground truth" mentioned in the following experimental description.

4.1 Real images of indoor/outdoor generic scenes

We take four images of an outdoor building and three images of an indoor scene to compute the focal length using our proposed method. Results, as indicated in Table 1 are promising as the maximum relative error is only 5.7%. Figure 2 shows two sample images of the outdoor scene used for the experiment. Figure 3 shows the interface for our developed software RWImage depicting its use on experiments involving images of the indoor scene. The blue numbers highlighted in the images represent the feature matched points.

Table 1 Results for image pairs in different scenes

Focal length	Ground truth	Image pair (outdoors scene)		
		1&2	1&4	3&4
f	625.0	638.4	655.4	589.3
Focal length	Ground truth	Image pair (indoors scene)		
		1&2	1&3	2&3
f	625.0	617.7	607.8	624.6

Figure 2 Some images of the outdoor scene

4.2 3D reconstruction results using the calibrated focal length

Successful 3D reconstruction depends on the amount of a priori knowledge available on the parameters of the stereo system. If only the intrinsic parameters are available, we may implement the metric reconstruction from two views unique only up to an unknown scale factor. The scale factor can be determined if we know the distance between two points of the observed scene.

The metric structure preserves not only parallelism but also angles and ratios of lengths. We can thus compare the ratios of certain corresponding lengths on real objects and their images to test the precision of the camera intrinsic parameter estimates especially the focal length. For this experiment, four sets of corresponding lengths are both measured on the real object and calculated in its two images within an indoor setting. To illustrate, lengths of two edges of the paper box, represented in red lines in Figure 3, are measured using our software RWImage, to obtain their ratios. From the results showed in Table 2, we find that the ratios of different lengths are preserved well after the 3D reconstruction which indicates that the obtained intrinsic parameters, particularly the focal length, are accurate.

Table 2 Results for ratio preservation in 3D reconstruction

	Ratio of lengths			
	1	2	3	4
Real Object	0.731	1.057	1.295	0.566
3D Metric Reconstruction	0.716	1.108	1.275	0.580

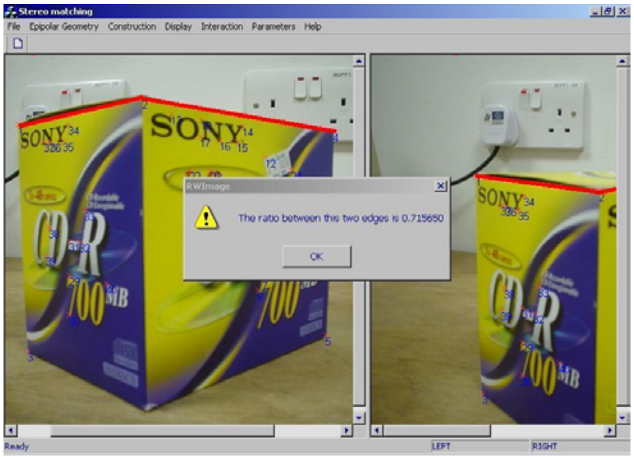

Figure 3 Interface of software RWImage

5. DISCUSSION

The proposed self-calibration technique would be an important step in automatic 3D modeling. However, it is still not sufficiently robust in handling 'critical motion'. A critical motion sequence is one that gives spurious results for self-calibration or structure recovery. To ensure stable results, singularities must be avoided, especially in the case of coplanar optical axes. One simple solution is this: before taking the second image, point the camera to the same point in the scene as in the first image. Next, tilt the camera slightly upwards or downwards before capturing the second image.

The number of matches obtained with the epipolar geometry estimation is limited. For future work, dense feature detection as well as other functions such as 3D model reconstruction will be added into the software. Although our self-calibration algorithm works well, it is better to embed bundle adjustment into the algorithm to further refine the estimation of camera's intrinsic parameters [15]. Automatic detection of cases where two images may generate generic singularities would also be nice to have.

6. CONCLUSION

This paper proposes and evaluates a linear self-calibration approach for camera focal length based on degenerated Kruppa's equations. The reliable assumption is that only the focal length is unknown and constant, and the skew factor is zero. The whole algorithmic process is also described in detail to enable duplication of results by other researchers. Compared with other linear techniques, the advantage of this method is that no priori information of motion is needed. Our developed software RWImage has been utilized to test the algorithm where experiments on both real images of indoor/outdoor generic scenes and 3D metric reconstruction show that the estimated focal length is of high accuracy.

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