WAVELET APPROXIMATION-BASED AFFINE INVARIANT 2-D SHAPE MATCHING AND CLASSIFICATION

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ABSTRACT

In this paper, an algorithm for matching and classifying 2-D shapes that undergo affine transformation is developed. The algorithm uses the 1-D Dyadic Wavelet Transform (DWT) to decompose a shape's boundary into multiscale levels. The curve moment invariants of the approximation coefficients are used as the shape features. Two different dissimilarities are calculated from the Euclidean distances between the decomposed scale levels of the shapes. These dissimilarities are used in shape matching and clustering by using hierarchical clustering algorithm with ward's linkage rules. The presented algorithm is invariant to the affine transformation and to the boundary starting point variation. The algorithm is also capable of finding and clustering similar shapes even if there are small deformations between their boundaries.

1. INTRODUCTION

Matching and classifying shapes that undergo geometrical transformations had attracted a large attention in shape analysis and recognition systems in the last decade. A typical shape analysis system consists of preprocessing, shape representation and description, feature extraction, dissimilarity/similarity measure, and matching/classification stages.

In general, there are two main types of unsupervised classification (clustering) algorithms, namely partitional and hierarchical [1]. Hierarchical clustering procedures are among the best known of unsupervised classification methods, because of their conceptual simplicity. Hierarchical clustering can be represented by a tree, called a dendrogram, using linkage rules. The linkage determines how clusters are connected to form a new one, and what the new cluster distance will be.

In this paper, hierarchical clustering is used with wavelet transform which provides multiscale decomposition of the shape boundary. Multiscale decomposition methods of shapes are considered as one of the most promising techniques in shape analysis. This is because of the ability to such methods to take into account the evolution of a shape as it is subjected to more and more smoothing. The proposed algorithm is invariant to affine transformation and to boundary starting point variation. The algorithm is also capable of finding and clustering similar shapes even if there are small deformations between their boundaries.

The Wavelet Transform (WT) has been used by many researchers for shape matching and classification. In [2], [3], and [4], invariant functions computed from the dyadic wavelet transform of the shape boundary were derived. These functions were sensitive to noise, in the first decomposed scale levels, due to the dependant of such functions on the detail coefficients. Two dimensional geometric moments have been used with the WT by [5], and [6]. Other techniques used a combination of the WT and Fourier Transform (FT) [7], the combination of the wavelet multiscale features and the Hopfield neural networks [8], and the combination of the line moment, WT, and FT. [9]. In [10], 1-D continues wavelet transform (CWT) was used to represent the shape boundary (w-representation). For classification, the Fourier descriptors (as global features) and the w-representation as a local features were combined. Wavelet moment invariants were derived in [5] to distinguish between seemingly similar shapes. For classification application, these invariants were used with Min-distance classifier.

2. APPROXIMATION-BASED SHAPE MATCHING AND CLASSIFICATION ALGORITHM

The proposed wavelet-based algorithm (in Figure 1) starts extracting the outer boundary of the shape using the bug following technique [11]. The extracted 2-D boundary is then broken into two 1-D sequences ($x(k)$ and $y(k)$). If the shape is subjected to a geometric transformation, such as the affine transformation, then the boundary will be distorted by the same transformation. The original and the distorted boundaries are related by:

$$\begin{bmatrix} \hat{x}(k) \\ \hat{y}(k) \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} x(k) \\ y(k) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \tag{1}$$

where $x(k)$ and $y(k)$ are the original boundary sequences, $\hat{x}(k)$ and $\hat{y}(k)$ are the distorted boundary sequences, c_{11}, c_{12}, c_{21}, and c_{22} are the coefficients of an affine transformation, and b_1, and b_2 represent the translation. The translation parameters can be removed by subtracting the centroid of the shape from the boundary sequences $x(k)$ and $y(k)$.

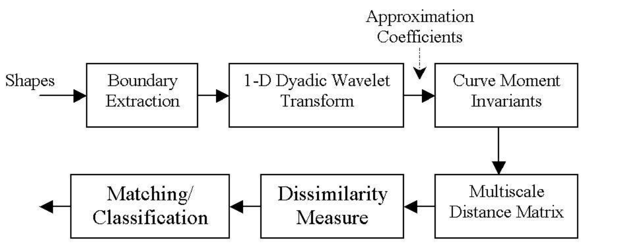

Fig. 1. The multi-scale matching and classification system

2.1. Wavelet Decomposition and Feature extraction

After extracting and parameterizing the shape boundary, a 1-D Dyadic Wavelet Transform (DWT) is applied to the boundary sequences $x(k)$ and $y(k)$. A 1-D DWT is applied to $x(k)$ and $y(k)$ to obtain the different approximations and details coefficients. The boundary sequences ($x(k)$ and $y(k)$) are decomposed to a certain wavelet scale level L,

$$\begin{bmatrix} x_{l_0}(k) \\ y_{l_0}(k) \end{bmatrix} = \begin{bmatrix} \sum_n a_{x,L,n}\tilde{\phi}_{L,n}(k) \\ \sum_n a_{y,L,n}\tilde{\phi}_{L,n}(k) \end{bmatrix} + \sum_{l=l_o}^{L} \begin{bmatrix} \sum_n d_{x,l,n}\tilde{\psi}_{l,n}(k) \\ \sum_n d_{y,l,n}\tilde{\psi}_{l,n}(k) \end{bmatrix} \quad (2)$$

where $a_{x,L,n}$ are the approximation coefficients for $x(k)$ at scale L , $a_{y,L,n}$ are the approximation coefficients for $y(k)$ at scale L, $d_{x,l,n}$ are the detail coefficients for $x(k)$ at scale l, $d_{y,l,n}$ are the detail coefficients for $y(k)$ at scale l, $\tilde{\phi}_{L,n}(k)$ are the dual scaling functions at scale L, and $\tilde{\psi}_{l,n}(k)$ are the dual wavelet functions at scale l. In this paper, only the approximation coefficients are considered for shape clustering. Figure 2 illustrates the approximations coefficients plotted as 2-D contours.

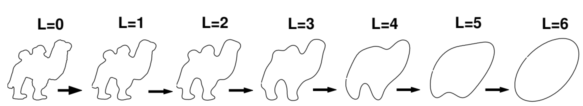

Fig. 2. Multiscale representations of a shape using wavelet decomposition.

Since the multiscale decomposition represents the shape into different smoothed boundaries, in this paper, the approximation coefficients at each scale level are assumed to be a boundary of a different stand-alone shape. Because the wavelet coefficients are not invariant to the affine distortion, the moment invariants found in [12] are calculated for these coefficients. The moments are calculated at each scale level using the curve moments defined in [13]. The advantages of applying these moment invariants are their invariance to the affine transformation and to the staring point variation of the shape boundary. Also the moments reduce the size of the extracted feature vectors from the shapes.

2.2. Shape Matching and Clustering

Shape matching and clustering are obtained based on the computation of the dissimilarity measures calculated from the distance matrix. The distance matrix between two shapes is computed from the Euclidian distances between the curve moment invariants of the wavelet approximation coefficients at all scale levels. Two dissimilarities are introduced in this paper: A direct dissimilarity measure, DS1, is calculated by taking the diagonal values of the distance matrix which measure the distance between the corresponding scale levels of each shape. If U_{AB} is denoted as the distance matrix between shapes A and B, then

$$DS1_{AB} = \frac{\sum_{j=j_o}^{J} U_{AB}(j,j)}{J - j_o + 1}, \quad (3)$$

where J is the highest and j_o is the lowest used scale levels.

Another dissimilarity measure (Hausdorff-like distance measure) is computed by tracking and capturing the minimum values across each row in the distance matrix.

$$V_{AB}(j) = min_{i \in B}\{U_{AB}(j,i)\}. \quad (4)$$

This measure is asymmetric which means that two shapes could have two different dissimilarity values. Symmetry is achieved by taking the maximum values of both directions.

$$DS2_{AB} = \frac{\sum_{j=j_o}^{J} max_{j \in A}\{V_{AB}(j), V_{BA}(j)\}}{J - j_o + 1}. \quad (5)$$

The advantage of using the asymmetric dissimilarity DS2 over the traditional symmetric one DS1 is that dissimilar shapes are easily detected by DS2. Both DS1 and DS2 are used in this paper in shape matching and clustering.

For shape clustering, hierarchical clustering algorithm is applied using ward's linkage rules. Ward's method is considered one of the best hierarchical methods [1]. The hierarchical clustering is represented by a tree called dendrogram. The classes are obtained by horizontally thresholding the dendrogram at a specific distance value.

3. EXPERIMENTAL RESULTS

The robustness of the algorithm to small boundary deformation and the invariance to the affine transformation are tested in this experiment. Sample of the shapes used in the experiment is shown in Figure 3. The shapes are categorized into 20 different groups and each group contains four similar shapes with small boundary deformation at different starting point positions. All shapes in our experiment are resampled to have 256 points, so the DWT decomposes the sequences $x(k)$ and $y(k)$ into 8 different scale levels.

In order to test the performance of the proposed dissimilarities in grouping similar shapes, DS1 and DS2 are computed (excluding the first and last levels) and enforced to obtain a small number of clusters. Figure 4 shows the clustering results by using DS1 and thresholding the dendrogram to obtain ten classes. Figure 5 shows the clustering results using DS2 and ten classes. The asterisks in this figure indicate the differences of DS2 results from DS1 ones. In spite of the differences in the results between the two dissimilarities using few clusters, it can be shown that identical clustering results for DS1 and DS2 dissimilarities can be obtained using large number of clusters.

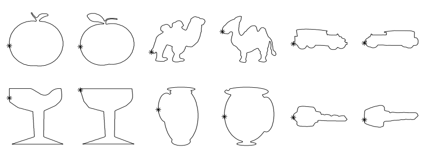

Fig. 3. Sample of the shapes used in the experiment

To test the affine invariance of the algorithm, 20 shapes each representing a different group are selected and affine distorted. Sample of the affine distortion for one of these shapes is shown in Figure 6. These distorted shapes are obtained by applying the affine transformation equation from [14]. The affine transformation parameters used in this experiment are shown in Table 1.

Shape	a	b	c	d	e	f
scale	1	1	1	1	1	0.7
rotation	0	60^o	120^o	0	0	0
skew	0	0	0	0.4	0.9	0

Table 1. Affine transformation parameters used in the experiment.

Figure 7 shows the clustering results for the affine distorted shapes using DS1 and thresholding the dendrogram to obtain 20 clusters. It can be seen that the algorithm is invariant to the affine transformation.

A comparison between the proposed algorithm and one of the recent matching techniques that uses the wavelet detail coefficients of the shape boundary is conducted. Khalil and bayoumi in [3] and [4] showed that their invariant functions outperform the geometrical moments, Fourier descriptors, and Alferez and Wang's [2] invariant functions. Figure 8 shows the matching results of a cup shape obtained from

Fig. 4. Clustering using DS1

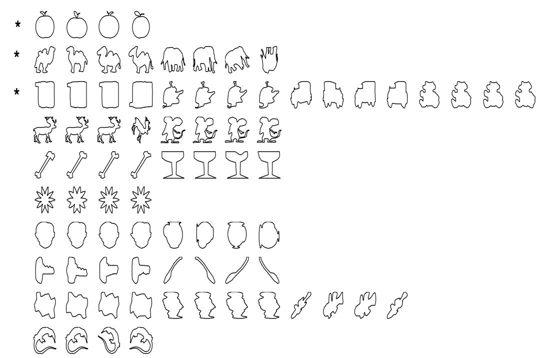

Fig. 5. Clustering using DS2

both the proposed algorithm and the invariant functions derived in [3] and [4]. Figure 9 illustrates the clustering result using Khalil's detail-based functions at scale levels 3 and 4. It can be noticed from these results that the approximation coefficients are more appropriate in matching and clustering shapes than the detail coefficients. Although the detail coefficients may have higher discrimination than the approximation coefficients, the later are less sensitive to noise and more convenient for finding globally related shapes.

4. CONCLUSIONS AND FUTURE WORK

In this paper, a new and simple multiscale algorithm for matching and clustering shapes according to their wavelet approximation coefficients is presented. Two dissimilarities are calculated from the moment invariants of the approximation coefficients. The algorithm is invariant to the affine transformation and to the starting point variation of the shape's boundary. A comparison with a recently published wavelet detail-based invariant functions is conducted. Although the detail-based functions could have higher shape discrimination at some scale levels, the results show that the approximation-based algorithm is more efficient in matching and grouping shapes into meaningful clusters.

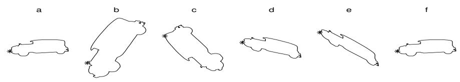

Fig. 6. Sample of the affine distorted shapes.

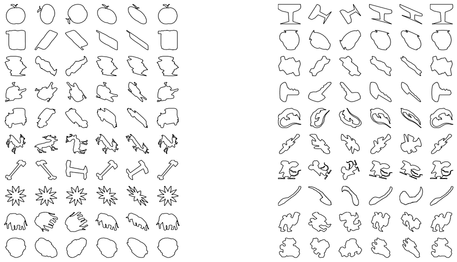

Fig. 7. The affine distorted shapes clustered into 20 different classes.

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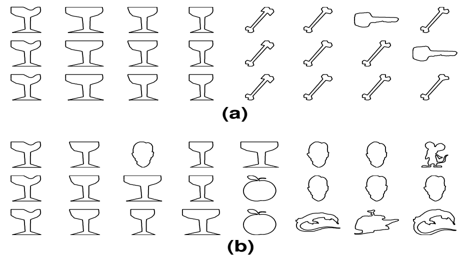

Fig. 8. Matching results using a) the proposed algorithm at L=1,2 and 3, b) the detail-based invariant functions at L=1-2,2-3, and 3-4.

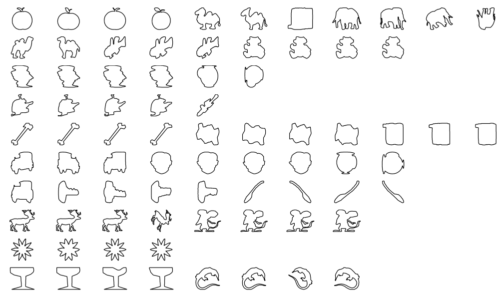

Fig. 9. Clustering using the detail-based invariant functions computed from scale levels 3 and 4.

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