CONTOUR SIMPLIFICATION USING NON-LINEAR DIFFUSION

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ABSTRACT

The non-linear diffusion of Perona and Malik method is applied to a contour. Unlike most of the contour diffusions techniques, the contour is described by the angle variation, and the non-linear diffusion procedure is applied to the contour turning angle. Perona and Malik model determines how strong diffusion will act on the original function, and depends of a factor K, estimated automatically. In areas with spatial concentration of strong changes of the angle this factor is also adjusted to reduce the contour small perturbations and noise effect.

Keywords: Non-linear Diffusion, Scale-Space, Contour, Polygonal Approximation

1. INTRODUCTION

The world has seen a rapid grow of visual information interchange through the generalized use of computers and internet. New handling techniques of visual data are required. Shapes are one of the most important semantic attributes of an image. Many applications for image recognition systems use shapes as one of the main tools. The new requirements and tendencies on image standards are also shape oriented. MPEG-4 and MPEG-7 are two good examples of the importance of shape applications. Considering this tendency, shape processing techniques become very important, and many potential applications can be find.

Introduced by Perona and Malik, non-linear diffusion [1, 2] provides good and precise solutions for automatic segmentation. This work is a specific solution of the scale space model defined by Witkins [3].

In this paper it is applied the same concept to contours aiming simplification and extraction of reference points. Non-linear diffusion of contours has been studied before. An example of an alternative solution for a non-linear diffusion method of contours was introduced in [4].

In this work, the contour is initially described in the tangent space. The nonlinear diffusion model is then applied to the angular component $\Theta(s)$ rather than to the plane coordinate components $[x(s), y(s)]$. Applying the Perona and Malik non-linear diffusion method to the contour angle, leads to a diffusion process that tends to a simplified path with sharp turning angles at locations of high tangent variations.

Very promising results were obtained using the SQUID data-base [5] with 1100 contours.

2. THE MODEL

The contour is considered as a trajectory of a point moving in a plane. This trajectory defines a path ℓ [6]:

$$\ell : S \subseteq \Re \rightarrow \Re^2,\ \ell(s) = [x(s), y(s)] \qquad (1)$$

where x and y are the plane coordinates, and s is the arclength of the curve.

Applying this model directly to the path of equation (1) can lead to instability, because in this case each component $[x(s), y(s)]$ of the path is treated separately. Since for any location s on the path the amount of change in the x-component is different from those in the y-component, the diffusion velocity is also different for each component. To avoid this problem we consider the original path in the tangent space. The nonlinear diffusion model is then applied to the angular component $\Theta(s)$ rather than to the components $[x(s), y(s)]$ as described below. The main idea behind the proposed strategy is to exploit the fact that the implementation of the model necessarily involves the

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discrete form of the digital contour as the only available information.

The solution of this problem results in a new non-linear scale-space of a contour, where the Perona and Malik model is applied to the function $\Theta(s)$. In this case, an evolution of the form:

$$\Theta_t = \nabla. \left(f \left(\|\nabla\Theta\| \right) \nabla\Theta \right), \tag{2}$$

is defined. The diffusion function $f(w)$ controls the degree of diffusion. Perona and Malik [1] used diffusion functions of the form:

$$f(w) = e^{-\left(\frac{w}{K}\right)^2} \qquad \text{or} \qquad f(w) = \frac{1}{1 + \left(\frac{w}{K}\right)^2} \tag{3}$$

As explained in [1], the parameter K in (3) determines how strong diffusion will act on the original function $\Theta(s)$. After some iterations only sharp changes in the contour course, i.e., strong changes in the contour direction where the difference $\|\nabla\Theta\|$ is larger than K, are not smoothed. The remaining contour variations are smoothed and merged to single polygon lines. The value of K is estimated automatically according to the procedure prescribed by Perona and Malik in [1]: the histogram of the absolute values of the differences $\|\nabla\Theta\|$ is calculated for the initial step function $\Theta_{\ell(s)}$. K is then chosen as a percentage F_k of the histogram.

3. THE NON-LINEAR DIFFUSION TECHNIQUE

The most significant drawback observed with this technique is that sharp contour changes due to noise or small geometrical features are treated in the same manner as the contour changes due to relevant shape variations. Indeed, a strong $\Theta(s)$ variation due to a local contour variation can be interpreted by the model as a significant contour variation. The corresponding vertex in the polygon will remain along the time evolution. To alleviate this shortcoming, the following strategy was implemented. Let e_{i-1}, e_i, e_{i+1} be three consecutive extremes of $\|\nabla\Theta\|$. Define the length between e_{i-1} and e_{i+1} as d_i. A local value of the parameter K, K_i, corresponding to the point e_i is estimated, if $\max\left(|\Theta_{i-1} - \Theta_i|, |\Theta_i - \Theta_{i+1}|\right) > K$ and $d_i < D$. Then, the local parameter K_i is given by:

$$K_i = K\frac{D}{d_i} \tag{4}$$

The global parameter K is still estimated in the same way. D is chosen to be the desired minimum distance value between two consecutive extremes of $\|\nabla\Theta\|$. In all cases that a local estimate K_i of the parameter K is used, D/d_i is greater than 1. This allows a growth of the K parameter every time that several extremes are closer than D points. Then, the non-linear diffusion process is more selective in contour regions where several extremes are close (typical noise situation) and the above mentioned shortcoming can be alleviated.

3.1. Implementation of non-linear diffusion of a contour

In a similar approach to the Perona and Malik [1, 7], equation (2) can be solved with an iterative method in scale-space. In one dimensional notation (see figure 1) the iterative solution will be:

$$\Theta_{t+1} = \Theta_t + \lambda \left(f(\Delta\Theta_{+1})\Delta\Theta_{+1} + f(\Delta\Theta_{-1})\Delta\Theta_{-1} \right) \tag{5}$$

where $\Delta\Theta_{+1} = \Theta_t(n+1) - \Theta_t(n)$ and $\Delta\Theta_{-1} = \Theta_t(n-1) - \Theta_t(n)$. λ is the time constant for each iteration and can assume a value between 0 and $1/4$.

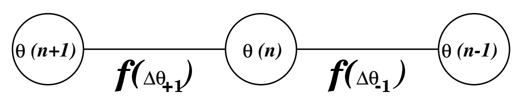
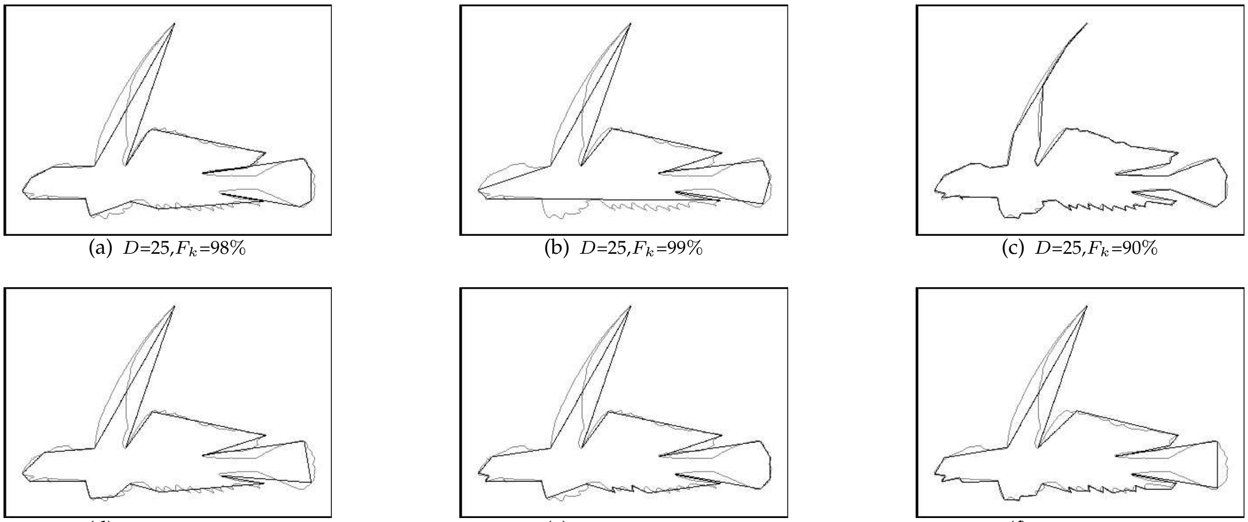


Fig. 1. The discrete computational scheme for the non-linear diffusion equation in the contour case.

4. RESULTS

Figures 2 and 3 show results of non-linear diffusion. The reference points (points where a large change in the turning angle value occur) that result from the non-linear diffusion process are used as vertices of the represented polygonal approximations. They correspond to the points where a sharp change of the contour direction occur. So, they are ideal vertices of a contour polygonal approximation. This approximation can be used as an initial approximation. Then, it can be improved with a scalable method like the proposed in [8].

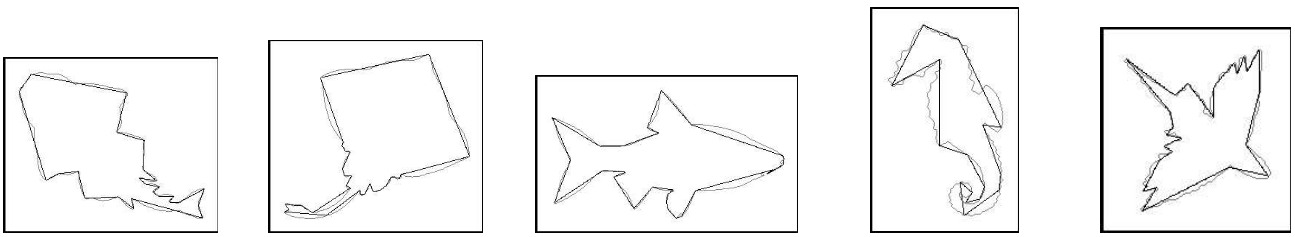
In figure 2 different examples of non-linear diffusion of a contour with different parameters are shown. By selecting the right value of D it is possible to overcome any noise or small geometric features effect. This happens, because parts of the contour with local concentration of $\|\nabla\Theta\|$ extremes will have a stronger diffusion acting on them. The selection of the parameter D becomes quite important, because this contour has several small geometric features. An example of substantial reduction of the artefacts due to noise can be seen in figure 2(d), where the diffusion was done



(a) D=25,F_k=98% (b) D=25,F_k=99% (c) D=25,F_k=90%

(d) D=50,F_k=98% (e) D=15,F_k=98% (f) D=0,F_k=96%

Fig. 2. Examples of non-linear diffusion with different parameter values.



(a) D=25,F_k=98% (b) D=25,F_k=99% (c) D=25,F_k=90% (d) D=50,F_k=98% (e) D=15,F_k=98%

Fig. 3. Examples of non-linear diffusion with different contours.

with $D = 50$ and $F_k = 98\%$. With $D = 0$ (figure 2(f)), the points that correspond to stronger Θ variations become vertices. In fact, the parameter D, can be considered as a measure of the detail that is intended for the final approximation. The parameter F_k is also a meaningful parameter. $1 - F_k$ defines the percentage of points that are intended to result as reference points after the diffusion process. So, it is an indication of the number of vertices of the resulting approximation. However, the two parameters interact between each other. The several examples of figure 2 show how these parameters can influence the diffusion process. Large values of D, result in approximations with a larger distance between vertices. Large values of F_k result in approximations with a lower number of vertices.

Figure 4 represents the turning angle evolution after different number of iterations. Although the diffusion process is only completed for a large number of iterations, it is possible to extract the reference points in lower scales. For instance, the third represented iteration, that results after 256 iterations, is conclusive in this case. This property allows the method to have an acceptable efficiency, and hence it can be used for interactive applications. The value of K, that is computed dynamically, can be a good indication of how much was the turning angle diffused. Hence can be used to control the end of the diffusion.

5. SUMMARY

A method of non-linear diffusion of the contour was specified. This method, based on the described applications for image non-linear diffusion [1], results in a contour polygonal approximation.

Applying the Perona and Malik model to the turning function of the contour we were able to extract geometric feature points. It uses the turning angle along the contour instead of the plane coordinates. This method makes use of a dynamic computation te-

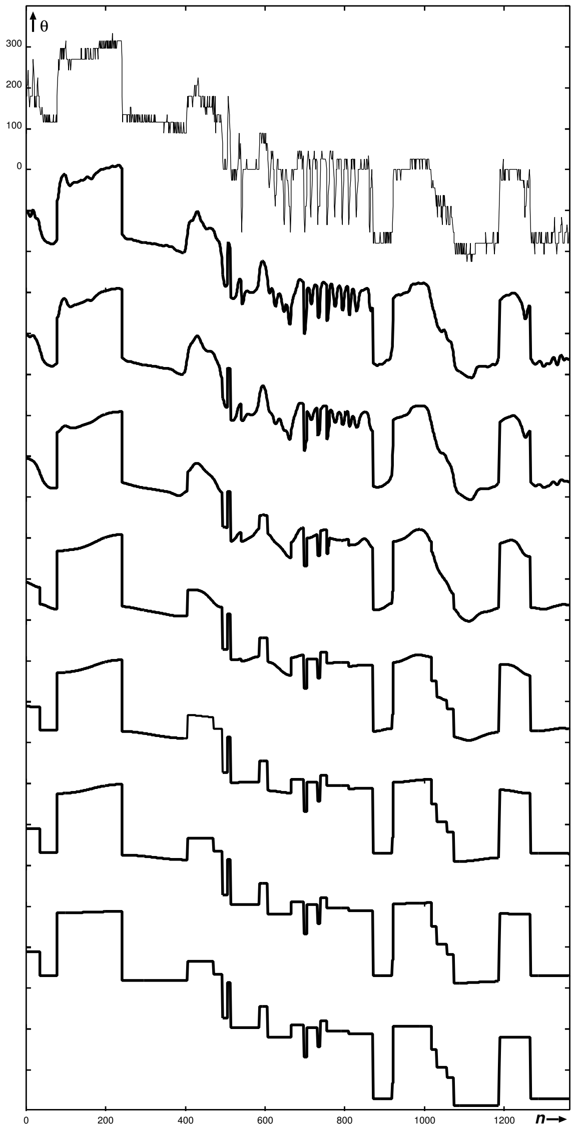


Fig. 4. Turning angle variation for different scales. From top to bottom: Original turning angle followed by the diffused turning angle after 32, 64, 256, 1024, 4096, 8192 and the final diffused turning angle.

chnique of the factor K of the diffusion function $f(w)$ to discriminate between local and non-local geometric features.

The computational requirements are more realistic than the corresponding application of non-linear diffusion to images, because much less information needs to be treated. Some applications of non-linear diffusion can be seen in [9]. In particular, very good shape retrieving results after shape simplification with non-linear diffusion were found.

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