STATISTICAL SOLUTION TO WATERSHED OVER-SEGMENTATION

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ABSTRACT

A region based algorithm for segmentation motivated by a parallel implementation is introduced. It is obtained by combining the watershed transform with further merging based on a statistical approach which is first independently introduced. This algorithm leads to a statistically reliable segmentation.

1. INTRODUCTION

One of the most important tasks of an image analysis system is image segmentation, the identification of homogeneous regions in an image. This operation aims at splitting an image into regions in order to simplify its analysis by higher level tools.

Our applicative framework is that of low power vision microsystems based on massively parallel asynchronous computing resources [1]. In this paper, we head for a segmentation algorithm that complies with this framework.

In the literature, several methods for segmentation are distinguished. Common segmentation methods are *edge detection, split and merge, region growing, clustering* and *active contours* techniques, which can be sorted in 3 different approaches :

1. The bottom-up approach leads to merging algorithms, consisting in aggregating small regions into larger ones.
2. The top-down approach which leads to splitting algorithms, consisting in recursively dividing an image into smaller and smaller regions.
3. The mixed approach which leads to split and merge algorithms, combining splitting and merging.

In region based segmentation algorithms, the operations applied to sets of pixels are selected according to measures on regions of increasing size (during algorithm's execution) in the case of *merge* algorithms, or on regions of decreasing size in the case of *split* algorithms. Many algorithms use these measures on predetermined regions, whose size change during the different steps of the algorithm in order to allow multi-resolution, without adapting shape to the actual image data [2]. In *merge* type algorithms, this kind of multi-resolution analysis methods with predetermined regions wrongly aggregate pixels because the merging operations are done on fixed regions not corresponding to image true regions. This raises a need of *splitting* operations. The same problem occurs in the case of *split* algorithms : separating neighbor points forces to split a region into several sub-regions and then to merge these multiple sub-regions.

Geodesic active contours and energy minimization methods [3] [4][5][6] suppress the use of predetermined patterns for multi-resolution image analysis by adapting to the arbitrary region shape. However, these methods can lead to over-segmentation because of the conservation or the creation of insignificant regions.

Our asynchronous parallel applicative framework strongly favors the use of the watershed transform, which is the favorite segmentation tool from the mathematical morphology approach to image processing. However, the watershed transform usually suffers from over-segmentation. In this paper we attempt to compensate for the over-segmentation tendency by a further merging operation based on a statistical approach. Before getting to the watershed, our statistical approach is first independently introduced as a general merging mechanism relying on reliability criteria.

First, the paper introduces a statistically grounded *merging criterion* allowing meaningful region merging. With regions of an unspecified size, the merging criterion must follow statistical rules accounting for the region size. If we want to merge two large regions the criterion as to be sharper than if we want to merge two small regions. For this, we consider regions as the outcome of stochastic processes. The reliability of statistical mean or standard deviation estimators improves with the number of samples used. The larger the region, the more reliable the statistical measures, and the more acute the merging criterion can be. For these reasons, we propose a merging criterion based on the knowledge of region area and statistical regional measures. This criterion determines the statistical reliability of the merging.

Second, we propose an algorithm to integrate this merging criteria into a segmentation method working on massively parallel and asynchronous computing models.

2. HEADING FOR A STATISTICALLY RELIABLE REGION MERGING

A discrete region of the image is considered as an outcome of a stationary, energy bounded, ergodic stochastic process on the considered region. The sample set of the region will be noted $(X_1, ..., X_n)$. Two regions are said as similar if both of them are the realization of the same stochastic process. Similarity will be measured by a probability. The arithmetic mean of the underlying stochastic process on the region will be noted m et its standard deviation σ.
We define on the sample ordered set $(X_1, ..., X_n)$, the sample arithmetic mean $\bar{X}$ and the sample median $\tilde{X}$. We will talk about this measures because the mean is a standard operator, and the median is an operator used in geodesic processes which is more thrifty in terms of needed parallel computer ressources. It is well known that :

$$E(\bar{X}) = m$$

$$\sigma^2(\bar{X}) = \frac{\sigma^2}{n}$$

According to central limit theorem :

$$\frac{\bar{X} - m}{\frac{\sigma}{\sqrt{n}}} \rightarrow \mathcal{N}(0, 1)$$

$$\frac{\tilde{X} - m}{\frac{\sigma}{\sqrt{n}}} \rightarrow \mathcal{N}(0, \sqrt{\frac{\pi}{2}})$$

where $\mathcal{N}$ is the normal gaussian probability distribution.
Arithmetic mean or median distribution of sampled sets from a region have a dispersion inversely proportional to $\sqrt(n)$. The value of the arithmetic mean or median of the distribution is probably located in an interval of the type $I = [\ \bar{X} - \frac{K}{\sqrt{n}}\ ;\ \bar{X} + \frac{K}{\sqrt{n}}\ ]$, where K is the level of reliability needed. We will call this interval I, the reliability interval.

What about merging neighbor regions only when their reliability interval intersection is not void ? If this intersection is empty, the probability of the two region to be similar is low, and the merging doesn't seem to be relevant. If the intersection is not empty, the probability for the two regions to be similar is rather high, and the merging is more likely to be relevant. K value allows to set up the merging criterion. If K value is high, there will be more merging reducing over-segmentation, but each merging will not be as reliable as for a lower K because the intersection of reliability intervals is a function growing with K. For a small value of K, the size of the reliability intervals is small, and there is only a few merging operations. That leads to over-segmentation. While reliability intervals take into account surface and pixel values of the sample set from the considered region, K can be defined only once for all regions of the image. This value is chosen by the user depending on the desired level of segmentation.
When it is possible to merge a region with more than one is its neighbors, the two regions with the highest probability to be similar are merged. This two region have the largest intersection of reliability intervals. For example, if we want to merge a region with a small one and a big one, both having the same arithmetic mean, the merging will be more reliable with the smaller one, because its reliability interval includes the reliability interval of the larger region. In this way, merging is applied between most similar regions first.

Let R_1 and R_2 two regions to merge. M_1 and M_2 the sample arithmetic mean, n_1 and n_2 the respective number of points in each sample set. The reliability intervals for each estimation of the sample arithmetic mean are the following ones :

$$I_1 = [\ M_1 - \frac{K}{\sqrt{n_1}}\ ;\ M_1 + \frac{K}{\sqrt{n_1}}\ ]$$

$$I_2 = [\ M_2 - \frac{K}{\sqrt{n_2}}\ ;\ M_2 + \frac{K}{\sqrt{n_2}}\ ]$$

Let's assume that $M_1 < M_2$. Reliability interval intersection $I_1 \cap I_2$ n is not empty if :

$$M_1 + \frac{K}{\sqrt{n_1}} > M_2 - \frac{K}{\sqrt{n_2}}$$

We have :

$$M_2 - M_1 > \frac{K}{\sqrt{n_1}} + \frac{K}{\sqrt{n_2}}$$

If region R_1 size is largely greater than region R_2 size, we can make the approximation :

$$(M_2 - M_1)\sqrt{n_2} > K$$

Thus, the decision to merge a small and a big regions depends on the difference between their arithmetic means, and is inversely proportional to the square root of the smallest region size. Using median instead of mean is just a matter of a $\frac{pi}{2}$ factor, which is negligible from a practical viewpoint.
The merging criterion presented so far uses comparisons between measures on adjacent regions. An implementation of this kind of comparisons on a parallel asynchronous model is not performing because of the necessary sequentialization introduced by this method. In order to overcome this problem, we propose a second criterion for merging regions. As already mentioned, we will use the watershed as our main segmentation tool because of the possibility to implement it on a parallel asynchronous computing model. The strength of the watershed in our framework comes from the parallelization of asynchronous operation between the different regions. Orientated by this property, we introduce another reliability criterion based on regional measures

which can be performed simultaneously on each regions. Following the watershed transform, each region is considered as a catchment basin with its area and depth. The area is the number of pixels in the region, and the depth the difference between the minimal pixel values found in the region and on its border. With these definitions, a small area basin will have to be deep enough to be significant, and a shallow basin will have to be large enough. With this approach, reliability of the region seems to depend on depth and area, two local measures that can be performed simultaneously on all the regions of the image.

To decide whether a region is reliable or not, we will use the reliability interval of the depth of the basin. First, we evaluate the reliability interval for the minimum pixel value of the basin and of the border line.
Assuming that the probability law of the stochastic process on the region is uniform, and after renormalization, the study of minimum convergency gives a reliability interval of the following type for the minimum value X_1 of the region :

$$I_1 \ = \ [\ X_1 - \frac{K\ (X_n - X_1)}{n-K}\ ;\ X_1\ ]$$

We'll note X_{border}, the minimum pixel value on the border of the region. We get a reliability interval for X_{border} :

$$I_{border} \ = \ [\ X_{border} - \frac{K\ (X_n - X_{border})}{n-K}\ ;\ X_{border}\ ]$$

Consequently, the reliability interval I of the depth is :

$$I \ = \ [\ X_1\ ;\ X_{border} - \frac{K\ (X_n - X_{border})}{n-K}\ ]$$

The region is considered reliable only if I is large enough *i.e.* :

$$(n-K)\ (X_{border} - X_1) - K\ (X_n - X_{border}) > \Delta$$

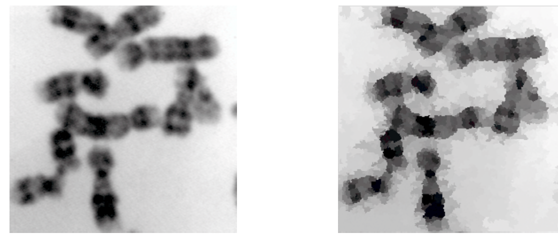
$$n\ (X_{border} - X1)\ -\ K\ (X_n - X1) > \Delta$$

K is much smaller than n (a value of $K = 3$ yields a reliability of 95% on X_{border} measure), and the value of X_n and X_{border} is of the same order. We have the following approximation :

$$n\ (X_{border} - X1) > \Delta$$

The merging criterion of a region is a function proportional to both depth and area. Only the area dependency differs from our first criterion : linear instead of square root. This result is consistent, because maximum convergence is faster than mean convergence when n increases.
Our criterion allows to determine whether a region is statistically significant or not in the image. The reliability estimation is computed using only region parameters, and without using neighbor data. This property allows to implement it on each region of the image with a thirsty parallel algorithm, thanks to the use of simple statistical measures such as range and area.

3. PARALLEL ALGORITHM FOR REGION MERGING



(a) Original image 3 (b) Segmentation 4

Fig. 1. Segmentation example of a medical image.

We introduce a *merge* segmentation algorithm using the second reliability merging criterion presented before. It is based on iterated convergent watershed transforms.

3.1. Initialization

The first step of the algorithm is a watershed segmentation on a gradient image obtained by application of vertical and horizontal Sobel filters. This initialization returns catchment basins separated by lines of locally maximal gradient. It usually leads to over-segmentation, in particular due to noise, but at least only merging is further necessary to achieve high quality segmentation.

3.2. Region Merging

The merging approach is a regional and statistical one, based on the results presented in section 2. At the end of the initialization, the image is composed of regions of different size and depth.
The first step of the merging algorithm is to detect the statistically unreliable regions, and to process the other ones. A region with area noted A, and depth noted H is considered unreliable if for the fixed constant Δ :

$$A * H < \Delta$$

In this case, the region is leveled and removed, otherwise the region is kept. The levelling operation replace the gradient values by the minimal value of the gradient on the pixels composing the border line of the region :

$$\nabla I_{Region} \ = \ Min_{Bords}(\nabla I)$$

If removed, the catchment basin is flattened. It will be later aggregated to a reliable region or merge with other unreliable regions. Then, we have the remaining reliable regions and a set of flattened unreliable regions.

The second step of the algorithm is the growing of the existing reliable regions by aggregating flattened unreliable regions satisfying the condition :

$$\nabla I_{border of flattened region} \geq \nabla I_{border of reliable region}$$

The latter is already used to add points to a region during the watershed segmentation. When the reliable regions have grown, there are some remaining unreliable regions not aggregated. These regions are then merged by a supplementary watershed transform. At the end of this second step, we get a set of regions constituted by the reliable regions which have grown, and by the created regions. Compared with the initialization state, the number of regions has necessarily decreased.

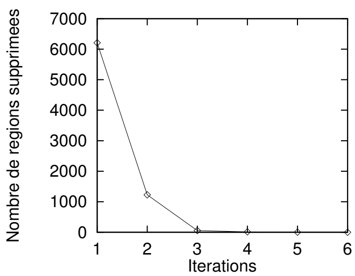


Fig. 2. Number of unreliable regions at each iteration.

The number of regions decrease as steps 1 and 2 are iterated. When all the regions of the image are satisfying the reliability criterion, the algorithm is terminated. We can see experimentally that convergence to final image is very fast. Fig. 2 shows a typical convergence of the algorithm. It is exponential, therefore very fast.

3.3. End of the algorithm

When the number of regions removed during step 1 becomes null, each of the region satisfy to the reliability criterion. The last step of the algorithm is to return the segmented image. In order to do this, the sample arithmetic mean of the intensity is calculated on each region. The value of the intensity in each pixel of the region is then replaced by the arithmetic mean, which provide a segmented image.

3.4. Results

First results of our algorithm match high quality standard ones. Compared to the standard segmentation algorithms, ours provide improved segmentation quality on images featuring tonally complex areas. However, it suffers from the following weakness : region segmentation is straight at the border line of regions separated by a high value gradient, but on rather homogeneous regions, the shape of the border line come from the first watershed segmentation during the initialization, and this segmentation is mostly due to the noise in the region. This leads to a fractal aspect shape for this kind of regions.
Algorithm is currently compared with other ones, and extensive results will be provided later.

4. CONCLUSION

Subject to particular constraints of parallel implementation, the proposed segmentation method takes advantage of the regional measures to provide interesting properties, for example the ability to separate fuzzy regions(cf. fig 1). More than this, it ensure the statistical reliability of each region present in the segmented image. Most segmentation methods do not satisfy this property. Future development is expected to show a link with energy based methods described in [4] [5] [6].

5. REFERENCES

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