

MULTIPLICATIVE MULTIREOLUTION DECOMPOSITION FOR 2D SIGNALS: APPLICATION TO SPECKLE REDUCTION IN SAR IMAGES

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ABSTRACT

In this paper, the new concept of Multiplicative Multiresolution Decomposition (MMD) with perfect reconstruction is presented. This kind of analysis/synthesis representation is suitable for images corrupted by multiplicative noise (e.g. Synthetic Aperture Radar (SAR) images). The design and implementation of a nonlinear multiplicative filter bank are addressed. An application to speckle reduction in SAR images is also described. The performances of MMD speckle reduction method are studied and compared with those of the most often used adaptive filters.

1. INTRODUCTION

Wavelet filtering have been successfully exploited for signal denoising in additive noise environment through thresholding techniques developed by Donoho [2, 3]. Since the wavelet transform is a linear operator, wavelet signal denoising in the above environment appears to be a natural scheme [2, 1]. In multiplicative noise environment such as Synthetic Aperture Radar (SAR) imaging, the use of wavelet decomposition for noise reduction is usually performed in the image logarithm domain [4]. The purpose of the logarithm processing consists of transforming the multiplicative noise to an additive one. An important drawback of such transformation is that it leads to biased estimation of the information contained in the original image [5]. To deal directly with multiplicative noise model, attempts to speckle reduction in SAR images using multiplicative decomposition has been investigated in [8, 10].

In this paper, we develop a new denoising method based on nonlinear low level representation, which is suitable for multiplicative noise model. This representation is based on a nonlinear multiresolution-multistage subband decomposition [9]. Application of the new proposed technique to speckle reduction in SAR images is reported.

2. MULTIPLICATIVE REPRESENTATION FOR IMAGES

In this section, we present a bi-dimensional extension of a new nonlinear multiplicative decomposition using filter banks with critical sub-sampling and perfect reconstruction [9].

Consider a description of the analysis and synthesis filter banks in terms of four input-four output systems with equal symbol rates at both input and output. We obtain the wanted structure by performing a polyphase decomposition of the 2D signal (image). The four polyphase components x_{11} , x_{12} , x_{21} and x_{22} of the input image $I(n, m)$ are defined by

$$x_{ij}(n, m) = I(2(n-1) + i, 2(m-1) + j) \quad \forall i, j \in \{1, 2\}$$

In this case, the polyphase linear filters h_{ij} and f_{ij} are defined by

$$h_{ij}(k, l) = h(2(k+1) - i, 2(l+1) - j) \quad \forall i, j \in \{1, 2\}$$

$$f_{ij}(k, l) = f(2(k+1) - i, 2(l+1) - j) \quad \forall i, j \in \{1, 2\}$$

where h and f are bi-dimensional linear filters. Consider D and R_{ij} , $i, j \in \{1, 2\}$ the analysis and synthesis nonlinear filters, respectively. Figure 1 shows the analysis/synthesis bi-dimensional multiplicative nonlinear filters banks with polyphase scheme.

According to figure 1 and for a perfect reconstruction, we need to have $\hat{x}_{ij} = x_{ij}$.

Let us set $f_{11} = h_{11}^{-1}$, $f_{12} = h_{12}^{-1}$, $f_{21} = h_{21}^{-1}$, $f_{22} = h_{22}^{-1}$, $h_{12} = \alpha h_{11}$, $h_{21} = \nu h_{11}$ and $h_{22} = \gamma h_{11}$, where α , ν and γ are positive scalars. The nonlinear analysis filter is then given by

$$y_{2V} = \begin{cases} \beta \frac{x_{12}}{x_{11}}, & \text{for } x_{11} \geq x_{12} \\ \beta \left(2 - \frac{x_{11}}{x_{12}}\right), & \text{for } x_{12} > x_{11} \\ \alpha, & \text{for } x_{11} = x_{12} = 0. \end{cases} \quad (1)$$

$$y_{2H} = \begin{cases} \beta \frac{x_{21}}{x_{11}}, & \text{for } x_{11} \geq x_{21} \\ \beta \left(2 - \frac{x_{11}}{x_{21}}\right), & \text{for } x_{21} > x_{11} \\ \nu, & \text{for } x_{11} = x_{21} = 0. \end{cases} \quad (2)$$

$$y_{2D} = \begin{cases} \beta \frac{x_{22}}{x_{11}}, & \text{for } x_{11} \geq x_{22} \\ \beta \left(2 - \frac{x_{11}}{x_{22}}\right), & \text{for } x_{22} > x_{11} \\ \gamma, & \text{for } x_{11} = x_{22} = 0. \end{cases} \quad (3)$$

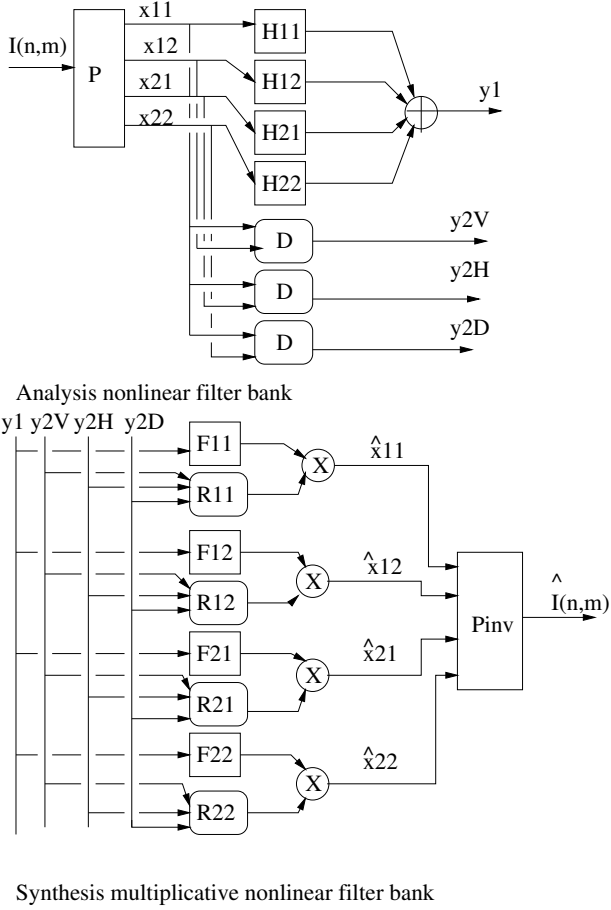


Figure 1: Analysis/Synthesis Multiplicative nonlinear filter banks with polyphase scheme.

for $x_{11} \neq 0$, we can deduce the following nonlinear filters r_{ij}

$$r_{11}(y_{2H}, y_{2V}, y_{2D}) = \frac{1}{1 + \alpha \frac{x_{12}}{x_{11}} + \nu \frac{x_{21}}{x_{11}} + \gamma \frac{x_{22}}{x_{11}}} \quad (4)$$

$$r_{12}(y_{2H}, y_{2V}, y_{2D}) = \alpha \frac{x_{12}}{x_{11}} r_{11}(y_{2H}, y_{2V}, y_{2D}) \quad (5)$$

$$r_{21}(y_{2H}, y_{2V}, y_{2D}) = \nu \frac{x_{21}}{x_{11}} r_{11}(y_{2H}, y_{2V}, y_{2D}) \quad (6)$$

$$r_{22}(y_{2H}, y_{2V}, y_{2D}) = \gamma \frac{x_{22}}{x_{11}} r_{11}(y_{2H}, y_{2V}, y_{2D}) \quad (7)$$

where β is a positive scalar. Hence, for $x_{11} \neq 0$ (what is equivalent to have $y_{2V} \neq \alpha$, $y_{2H} \neq \nu$ and $y_{2D} \neq \gamma$), according to equations (1),(2) and (3), the nonlinear filters r_{ij} are expressed as function of the nonlinear outputs y_{2H}, y_{2V}, y_{2D} .

3. MULTIREOLUTION-MULTISTAGE REPRESENTATION

Several sub-band decomposition are cascaded by applying analysis filter banks to one or more coefficient outputs of the preceding stage. Herein, we consider the frequent case where the approximation sub-band is re-decomposed (we assume that the sub-band y_1 (see figure 1) corresponds to the approximation sub-band). The sub-band y_1 is split to its polyphase components $y_{11}, y_{12}, y_{21}, y_{22}$ and then filtered. Consider, at the resolution $j = 1$, that $y_{11}^{(j)} = x_{11}, y_{12}^{(j)} = x_{12}, y_{21}^{(j)} = x_{21}, y_{22}^{(j)} = x_{22}$, and the polyphase operator P given by:

$$y_1^{(j)} \rightarrow^P y_{11}^{(j)}, y_{12}^{(j)}, y_{21}^{(j)}, y_{22}^{(j)}$$

with $y_{11}^{(j)}(n, m) = y_1^{(j)}(2n - 1, 2m - 1)$,

$$y_{12}^{(j)}(n, m) = y_1^{(j)}(2n - 1, 2m),$$

$$y_{21}^{(j)}(n, m) = y_1^{(j)}(2n, 2m - 1), y_{22}^{(j)}(n, m) = y_1^{(j)}(2n, 2m),$$

and let P^{-1} be the inverse polyphase operator. Figure 2 shows an example of a multi-stage image decomposition.

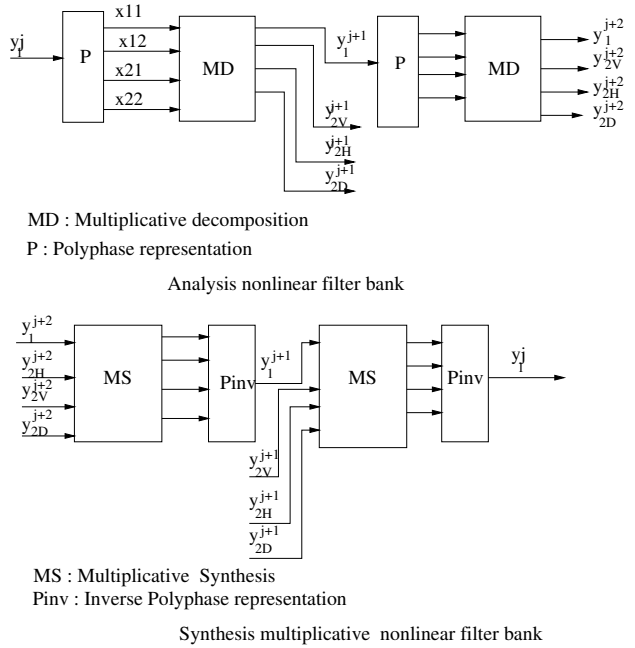


Figure 2: Multi-stage two band decomposition at resolution j .

Thus, for any $J > 0$, the original discrete signal $y_1^{(1)} = I$ measured at the resolution 1 is represented by the set S

$$S = \left(y_1^{(J)}, (y_{2H}^{(j)}, y_{2V}^{(j)}, y_{2D}^{(j)})_{2 \leq j \leq J} \right) \quad (8)$$

Inversely, an approximation of the reconstructed signal at resolution $j = 1$ is obtained by using multiresolution synthesis sub-band and the representation by the set of signals S . We have tested this analysis/synthesis method on 3-look

SAR images of figure 3-a. Figure 3-b shows the multiresolution analysis through three resolutions, it shows at the top the approximated signal at resolution $J = 3$ and below the details of the original image from resolution $j = 3, 2, 1$. The reconstructed image is illustrated by Figure 4. By comparing figures 3-a and 4, one can notice the high quality of the reconstruction. The smooth as well as the most irregular parts of the signal are well rebuilt. This illustrates the numerical stability of the decomposition and reconstruction process.

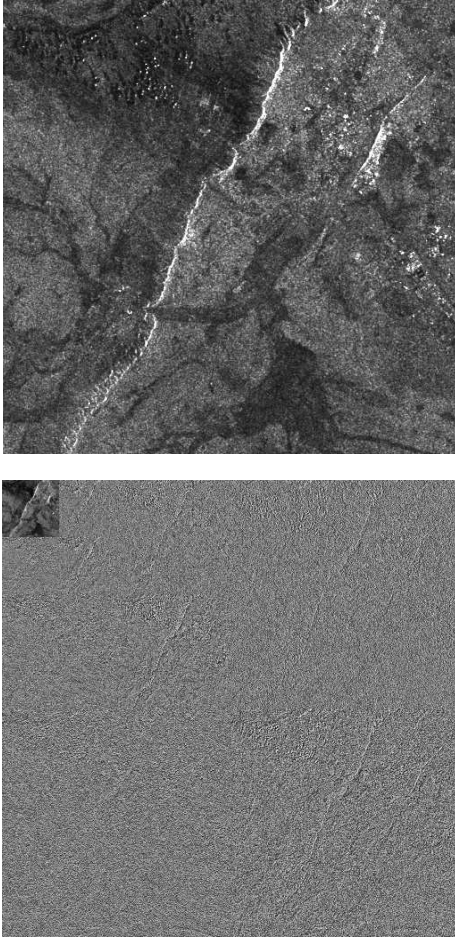


Figure 3: Original SAR ERS-1 3-look image of Laghouat region (south of Algeria) 512 x 512 and analysis image using multiplicative multiresolution decomposition at the resolution $j = 3$.

4. IMAGE DENOISING

In this section, a noisy image is considered, the latter is transformed using the above multiplicative multiresolution filter bank. The coefficient y_2^j in transform domain (except at the coarsest level) are subjected to thresholding. Let us

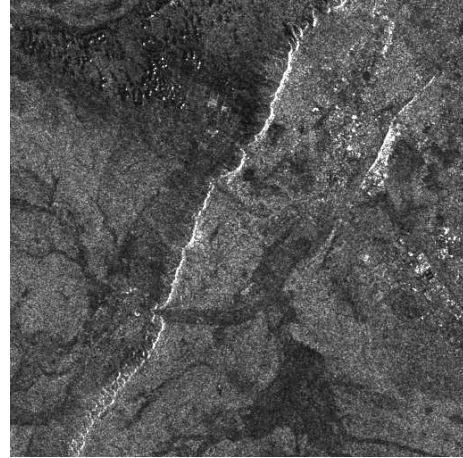


Figure 4: The reconstructed image by synthesis multiplicative and multiresolution nonlinear filter bank at resolution $j = 3$.

consider a threshold $0 < t_j < 1$, the soft thresholding, at the resolution j , in the multiplicative case, consists of :

$$\begin{cases} w_s^j = \min \{ \beta, w^j + t_j \} & \text{if } w^j \leq \beta \\ w_s^j = \max \{ \beta, w^j - t_j \} & \text{otherwise} \end{cases}$$

where w^j are the multiresolution details y_{2V} , y_{2H} and y_{2D} at resolution j . The threshold t_j is determined by the analysis of the noise under the multiresolution decomposition. However, to design an efficient filter within the proposed multiplicative decomposition, we must consider noise statistics in thresholding process. Thus, consider C_n the normalized standard deviation (ratio of standard deviation and mean) of the noise and C_n^j the normalized standard deviation of the noise at resolution j . The reconstructed image at resolution $j - 1$ is function of the coefficients $r_{k,l}^{(j)} = r_{kl}(y_{2H}^{(j)}, y_{2V}^{(j)}, y_{2D}^{(j)})$. We suppose that when the coefficients $\frac{1-C_n^j}{4} \leq r_{kl}^{(j)} \leq \frac{1+C_n^j}{4}$, the coefficient $r_{kl}^{(j)}$ represents a smooth area and the thresholded coefficient $r_{s,kl}^{(j)}$ are set to a value β . When the area is heterogeneous, we propose to threshold the coefficient $r_{s,kl}^{(j)}$ as follows

$$r_{s,kl}^{(j)} = \begin{cases} \nu_{kl} r_{kl}^{(j)} + \gamma_{kl} & \text{for } r_{kl}^{(j)} \geq \frac{1+C_n^j}{4} \\ \frac{1}{\nu_{kl}} r_{kl}^{(j)} - \gamma_{kl} & \text{for } r_{kl}^{(j)} \leq \frac{1-C_n^j}{4} \end{cases} \quad (9)$$

where $\nu_{k,l} = \frac{1}{\sqrt{1+C_n^{(j)2}}}$, $\gamma_{kl} = \frac{1}{4}(1 - \frac{1}{\sqrt{1+C_n^{(j)2}}})$.

The deduced multiresolution multiplicative filter performances have been compared with those of the most often used filters [6, 7] (refined Lee filter and MAP filter). The result of the MMD filtering using noise statistics is illustrated by figure 5. All the filters smooth correctly in homogeneous areas. However, the MMD filter is based on multiresolution window, which leads to a better smoothing. The refined Lee and MAP filters have equivalent results. The sharpness of edges and the linear feature or subtle details (point target response) are enhanced in the proposed algorithm, whereas speckle is well smoothed within the homogeneous areas.

However, such a visual comparison is not sufficient to accurately evaluate the denoising techniques. Hence some quantitative criteria must be used to evaluate speckle reduction and, at the same time, radiometric and texture integrity.

We propose to evaluate the following points:

1. Reduction of speckle, which could be estimated by the Equivalent Number of looks (ENL),
2. Preservation of the mean backscattering coefficient value, evaluated by the estimation bias ($Bias$) in dB ,
3. Preservation of target point response and edges, evaluated by the edge contrast loss after smoothing by ΔC ,
4. Texture and edge preservation, which is measured by $BiasT$.

Table 1 shows that all filters reduce correctly speckle in an homogeneous area, there are all based on the same test of homogeneity. The MMD filter presents a lower bias and ΔC values, then it preserves better the mean backscattering coefficient value and the edge contrast. In addition, it preserves better the texture information.

	mean	Bias	ENL	ΔC	Bias T.
Image 1	52.96	0	3.77	0	0
Image 2	50.45	-0.484	16.09	0.708	0.1986
Image 3	50.45	-0.485	16.10	0.6966	0.2000
Image 4	52.59	-0.091	17.52	0.432	0.1144

Table 1 : Filter Performances with image 1: Original image, image 2: Refined Lee, image 3: Refined MAP and image 4 : MMD filter.

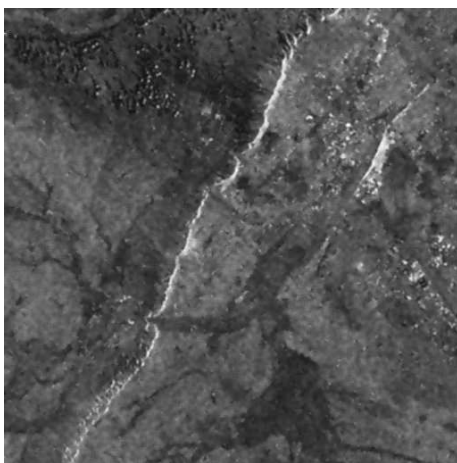


Figure 5: Results of multiplicative multiresolution decomposition filter applied on SAR ERS-1 3-look image using noise statistics.

5. CONCLUSION

In this paper, we have introduced a new multiplicative multiresolution multistage representation for 2D signals exploiting a nonlinear decomposition. The proposed representation is well suitable for a multiplicative noise model. The decomposition design has been performed by nonlinear polyphase filter bank. The multiresolution decomposition is obtained by iterating the decomposition on the linear component. A denoising technique based on our new nonlinear representation was proposed and applied to the speckle reduction in SAR images. In a further work, we suggest to study the statistics of the noise MMD's coefficients in order to improve the MMD filter performances.

6. REFERENCES

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