MEDICAL IMAGE SEGMENTATION WITH MINIMAL PATH DEFORMABLE MODELS

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ABSTRACT

This paper presents an algorithm that segments medical images by extracting object contours. It delineates object boundaries by detecting a path with the minimum energy on the image. A worm algorithm based on deformable models is proposed to find the minimal path by using the dynamic programming technique. The proposed algorithm overcomes the shortcomings of traditional deformable models such as fastidious initialization and inefficiency on segmenting objects with complex shapes or topologies. After presenting the algorithm, its performance on various synthetic and medical images is shown. Experimental results indicate that the proposed algorithm is robust to noise and edge discontinuities.

1. INTRODUCTION

Image segmentation is a fundamental operation in medical image analysis. In recent years, segmentation techniques that combine *deformable models* with local edge extraction have achieved considerable success in medical image segmentation [1][2].

With given initial contours, deformable models are able to evolve to obtain the object contours. Normally, they are manually initialized as polygons near the actual object contours. Deformable models can be broadly divided into three categories according to their representation and implementation. *Parametric deformable models* [3][4] are represented explicitly as parameterized contours in a Lagrangian framework. They have been used extensively [1][2] but their main drawback is the inability to adapt to topology. *Geometric deformable models* [5][6], on the other hand, are represented implicitly as level sets of higher-dimensional, scalar level set functions and evolve in an Eulerian fashion [6][7]. These deformable models require the initial curves to be properly placed and may be fastidious to use when dealing with several adjacent objects [7]. *Minimal path deformable models* [7][9] minimize the energy of deformable models by finding the path with minimal energy between specific points. From this point, the first two types of deformable models can be thought as traditional deformable models. In [9], the user only needs to point out the two end points and the final contour will be between these two points. However, little attention is paid on how to design an "ideal potential" to determine the precise contour. Han *et al*. [10] developed a minimal path finding algorithm based on [9] by using a graph search method. In order to reduce the "metrication errors" between two end points, a potential window is defined. Nevertheless, it may be captured by the local minimum in the "wriggling" process. Udupa *et al.* proposed a "live wire" algorithm which also detects object boundaries by looking for piecewise shortest paths [11][12]. However, this method requires continuous human interaction in the segmentation process to obtain the piecewise "live wire" segments.

In this paper, we present a new approach for extracting the object contour by searching for a minimal path from a single starting point. In order to make the path robust to noise and avoid local minima, the proposed algorithm looks for the path with minimal integrated energy from the given starting point. The algorithm behaves like an intelligent worm that moves along the direction with minimal cost.

The paper is organized as follows. Section 2 provides a background on previous related works. In Section 3, details of our proposed algorithm are presented and implementation issues are discussed. Section 4 demonstrates the performance of our method through experimental results, followed by the conclusions in Section 5.

2. MINIMAL PATH DEFORMABLE MODELS

In the original parametric deformable models [3], the energy $E(\mathbf{C})$ is defined as

$$E(\mathbf{C}) = \int_0^1 \left(\frac{1}{2} \left[\alpha \left| \frac{\partial \mathbf{C}}{\partial s} \right|^2 + \beta \left| \frac{\partial^2 \mathbf{C}}{\partial s^2} \right|^2 \right] - \gamma f\left(\mathbf{C}(s)\right) \right) ds \quad (1)$$

where $\mathbf{C}(s)$ is a curve and parameterized by $s \in [0, 1]$. Here,

α, β and γ are real positive constants, and f is an edge map of the image. The object contour is approached by $\mathbf{C}(s)$ when $E(\mathbf{C})$ is minimized. In practice, the minimization is performed iteratively by considering the curve as a function $\mathbf{C}(s, t)$ of time t.

In the above model, the external image force (last term of (1)) is significant only in the immediate vicinity of the object boundaries. Elsewhere, the model's evolution is driven dominantly by the internal forces (the first two terms of (1)), which lead to shrinking and smoothing of the contour. As a result, the original model requires an initial guess close to the actual object boundaries, or at least located outside of the object, so that the shrinking forces move the contours close to the boundaries where the external force becomes important.

Another possible problem of the original deformable models is the need to select the parameters that control the trade-off between smoothness and proximity to the object. Caselles *et al.* [9] set $\beta = 0$ and replaced f with $g(|\nabla I|)^2$, so that the energy function (1) becomes:

$$E(\mathbf{C}) = \alpha \int_{\Omega} \left|\mathbf{C}_s\right|^2 ds + \gamma \int_{\Omega} g(|\nabla I(\mathbf{C})|)^2 ds \qquad (2)$$

where ∇I is gradient of the image and $g(\cdot)$ is a potential which is strictly decreasing, so that $g(r) \rightarrow 0$ as $r \rightarrow \infty$. They deduced a new formula for minimizing (2) in a Riemannian space:

$$\min \int_{\Omega} g(|\nabla I(\mathbf{C})|) \cdot |\mathbf{C}_s| \, ds \qquad (3)$$

where the Euclidean length of the contour $\mathbf{C}$ is: $L(\mathbf{C}) = \int_{\Omega}|\mathbf{C}_s|ds$. Hence, the classical energy minimization can be transformed to search of the global minimal path weighted by $g(|\nabla I(\mathbf{C}(s))|)$. In other words, it shows that deformable models can be realized by the minimal path approach, whose main contribution to the deformable model method is the complexity reduction in computing high order gradients and avoiding the minimization of the corresponding Euler equation.

Our proposed method is inspired by the above works. Minimal path deformable models are used to look for object boundaries. A potential energy map is designed to evaluate the minimal path. In general, minimal path finding algorithms need two or more initialization points, whereas our algorithm requires a *single starting point*. Also, to avoid being captured by local minima, our algorithm uses an *intelligent "worm"* to look for the minimal path. Details are given in the following section.

3. FINDING THE MINIMAL PATH

Given the potential energy map of the image that has lower values near the edges or features, our goal is to find a single contour that best fits the boundary of a given object of interest. The minimal path is found from a single initial point using a worm algorithm.

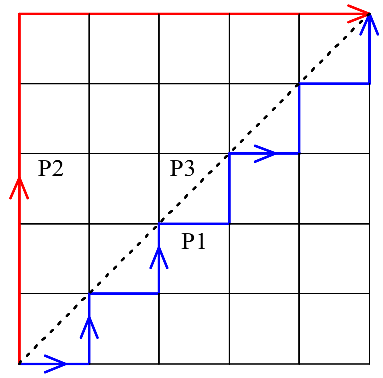


Figure 1. Illustration of metrication errors.

3.1. Worm algorithm

The worm body is composed of a sequence of points and has a length L_w and energy E. The actual edges along the object contour may not be continuous and smooth. Hence, the worm needs to be long enough to skip disjointed parts. In addition, since it is designed to avoid local minima, its energy consists of not only external energy coming from the potential energy map but also internal energy generated by its own topology. At each step, the worm compares all the possible ways ahead and moves itself along the one with minimal energy. Since raster images can be deemed as graphs with rectangular grids, the number of possible paths is not infinite but subject to the length of the worm and definition of the point neighbourhood. Then the problem is transformed to be graph searching and *dynamic programming* [11] is employed to select the minimal path.

Dynamic programming is introduced to iteratively optimize deformable models by Amini *et al.* [14]. The shortcomings of dynamic programming include its large memory consumption and high computational complexity. The complexity is $O(n \cdot m^2)$, where n is the number of tasks and m is the number of stages. In Amini's model, $n = 1$ and $m = L$, where L is the length of the longest possible contour and so its complexity is $O(L^2)$. In our algorithm, $n = L'$, where L' is the actual contour length and $L' \leq L$, and $m = L_w$. Thus, the complexity of our algorithm is $O(L' \cdot L_w^2)$. Although, the worm length L_w is about 10 in our experiments, the length L of the contour of a small object often exceeds 100. Hence, our algorithm has a lower complexity, especially for large objects.

Dynamic programming is used as a graph search method in our approach to get the minimal path. It is efficient but suffers from "metrication errors" [8]. Since pixels can be thought as nodes, raster image can then be assumed as a regular graph with unit weight on every link. Also, the distance between two points is restricted to be "city block" distance in graph search algorithms.

(a) (b) (c)

Figure 2. CT cardiac image (a) original image (b) edge image (c) distance transform map.

Under such an assumption, both P1 and P2 are shortest paths in Figure 1. However, by refining the graph grids, only P1 can approach the ideal shortest path P3 under Euclidean distance measure. To solve this problem, the Euclidean distance transform [15] is applied on the edge map of the image to get the potential energy map (see Figure 2). Then the weight on each link is replaced by the corresponding value in potential energy map. With this measure, paths like P2 in Figure 1 can be removed by the dynamic programming algorithm while paths like P1 are kept. Therefore, with refinement of grids, the shortest paths under Euclidean distance measure will be guaranteed to be the result.

3.2. Energy function for the worm

We use F_{ext} and F_{int} to represent the external and internal forces on the worm respectively. F_{ext} describes the force that comes from the image's potential energy map and F_{int} characterizes the stiffness of the worm body. The energy of the worm is then defined as:

$$E = \int_{\Omega} \left(\alpha(s) F_{ext}(s) + F_{int}(s) \right) ds \qquad (4)$$

where s is a small part of the worm body, and Ω is the whole one.

In our algorithm, F_{ext} is the force that attracts the worm to the edges of the image. F_{ext} is stronger when the worm is further away from the edges and vice versa. The edges are detected by Canny edge detector [16]. As mentioned earlier, the distance transform [14] is applied on the edge map and used to represent F_{ext} as shown in Figure 2.

The internal force F_{int} is used to constrain the bending of the worm. This makes the worm robust to noise and prevents it from twisting. It is defined as follows:

$$F_{int}(s) = \beta(s) \left| \frac{\partial^2 \mathbf{C}(s)}{\partial s^2} \right|^2 \qquad (5)$$

where $\mathbf{C}(s)$ is the body of the worm. In addition,

$$\beta(s) = \begin{cases} \beta_1 & F_{ext}(s) = 0 \\ \beta_2 & others \end{cases} \quad where\ \beta_1 << \beta_2 \qquad (6)$$

In above equation, $F_{ext} = 0$ means the worm is at an edge according to the definition of external force. To help the worm fit itself into complex object shape, F_{int} should be insignificant at edges because object contour is composed by these edges.

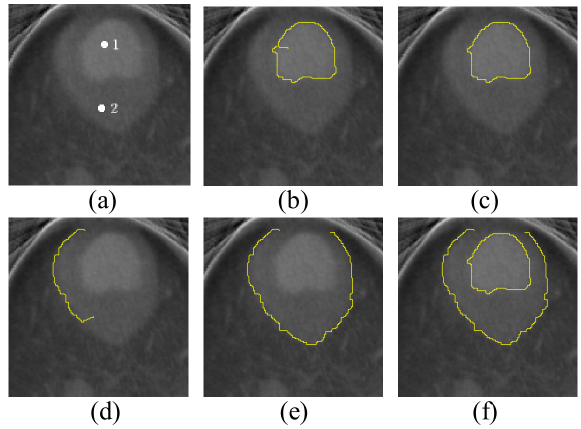


Figure 3. CT cardiac image (a) with two initial points, (b)(c) segmentation of endocardium wall, (d)(e) segmentation of epicardium wall, (f) the final segmentation result.

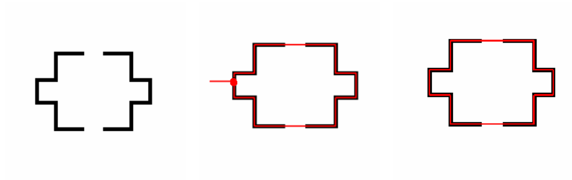
3.3. Implementation

The stopping condition and the contour detection are two important implementation issues. In our algorithm, a worm starts from an initial point, moves along the minimal path and stops when the contour is closed or an end of an open contour is achieved. If the minimal path is found to be closed, it will contain the object contour. This path is not naturally the desired object contour as the starting point may not be initialized on the contour as shown in Figure 3(b). We need to detect the point from which the path begins to be the actual object contour and the path before that point is discarded to obtain a closed contour as shown in Figure 3(c). This is also called *actual contour detection*. When the worm stops at an image boundary as shown in Figure 3(d), the obtained path may contain only part of the object contour. In this case, second search needs to be conducted to get another part of the contour.

3.4. Summary of algorithm

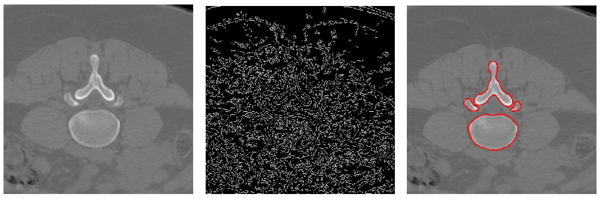
In summary, the new minimal path search algorithm is as follows:

i) Mark an initial starting point on or near the boundary of the desired object.
ii) Move the worm along the minimal path.
iii) Check if any stopping condition has been met. If yes, proceed to (iv). Otherwise, go back to (ii).
iv) If the contour is closed, cut off the trail before entry point and stop. For open contour, go to step (ii) to do the second search.



(a) (b) (c)

Figure 4. (a) A synthetic shape with sharp corners and two breaks (b) the path stopping at entry point (c) contour detection result.



(a) (b) (c)

Figure 5. MRI spine image (a) original image (b) edge image (c) segmentation result with four parts.

4. RESULTS & DISCUSSIONS

In this section, we present several experiments on synthetic and real images to demonstrate the performance of the proposed algorithm. For each object contour in the image, an initial point should be specified by the user. In addition, values of parameters used in the algorithm are decided based on empirical observations, which seems to be the most widely used strategy [3][4][8][10].

Figure 3 shows how our algorithm segments a relatively noisy CT cardiac image. Given two initial points as shown in Figure 3(a), the endocardium wall and the epicardium wall are delineated in Figure 3(f). Clearly, our algorithm is robust since the epicardium wall is identified even in the presence of noise and disturbance from background tissues.

Figure 4 clearly shows that our algorithm is able to delineate sharp corners and to connect disjointed parts to obtain a closed contour.

Figure 5 shows the performance of the proposed algorithm on segmenting an MRI image of the spine. Although the edge information is complicated (Figure 5(b)), four contours are easily obtained with four given starting points as shown in Figure 5(c).

5. CONCLUSIONS

In this paper, we propose a segmentation scheme which uses a worm algorithm to extract object boundaries in medical images from a single initial point. Compared with other deformable models, our approach requires a simple initialization task and has a low computational complexity.

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