Blurred Image Recognition based on complex moment invariants

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[Abstract] An important class of radiometric degradations we are faced with often in practice is image blurring. Special attention is paid to the recognition of the blurred image by moment invariant approach. Some important rules of complex moments for the blurred image are presented. Based on these rules, a useful subset of moment invariants is introduced, that are not affected by the blur, rotation, scale, and translation of the images. The experiments have shown that these invariants can be successfully used in recognition of the blurred image.

[Key Words] Image Recognition Moment Invariant Gaussian PSF function Complex moment

1. Introduction

Moment invariants are widely used in automatic target recognition[1][2], character recognition[3][4][2][5], computer vision[6] and medical analysis[7]. But the real imaging systems and the imaging conditions are usually imperfect, the observed image presents only a degraded version of the original scene. An important class of degradations we are faced with often in practice is image blurring, which can be caused by camera defocus, atmospheric and by sensor or/and scene motion, etc. Finding a set of invariants, which are also not affected by blurring, is a key problem in pattern recognition. The complex moment is a simple and straightforward way to derive invariants[8][3][9][10][11]. Unfortunately, How to construct the complex invariants which is also invariant to convolution with PSF, has not been reported.

From viewpoint of complex moment, we propose a new way to deal with the problem of recognizing blurred images. Firstly, the transfer equation of complex moment after image blurring is obtained. Then, the complex moment invariant subset that is also invariant with respect to blur is introduced. As an application, we apply our complex moment invariant subset to the task of recognizing blurred, rotational, scaled and translated objects successfully.

2. Definition of Image blurring

Blurring can be usually described by a convolution

$$g(x,y)=f(x,y)\otimes h(x,y) \qquad (1)$$

where f is an original image, g is an acquired image and h is a point spread function (PSF) of image system. For a Gaussian PSF $h(x,y)=e^{\frac{x^2+y^2}{2\sigma^2}}/2\pi\sigma^2$

3. Central moment of the image

Central moments of (p+q)th order is defined as

$$u_{pq}^{(f)}=\iint_D (x-x_0^{(f)})^p(y-y_0^{(f)})^q f(x,y)dxdy \qquad (2)$$

Where $x^{(f)}_0, y^{(f)}_0$ is the centroid of the image f(x,y).

4. Complex moment of the blurred image

The complex moment $C^{(f)}_{pq}$ of p+q order of the image function f(x,y) is defined as

$$C_{pq}^{(f)}=\frac{1}{[u_{00}^{(f)}]^{1+\frac{p+q}{2}}}[\iint_{R^2}(x+jy)^p(x-jy)^q f(x,y)dxdy] \qquad (3)$$

The expression $C_{pq}C_{rs}...C_{tu}$ is a rotational invariant if only p+r+...+t=q+s+...+u.

Theorem 1 The complex moment $C^{(g)}_{pq}$ of the blurred image g(x,y)= f(x,y)⊗h(x,y) is

$$C_{pq}^{(g)} = \sum_{u=0}^{p}\sum_{v=0}^{q}[C_p^u C_q^v (u_{00}^{(h)})^{\frac{u+v-(p+q)}{2}} C_{uv}^{(h)} (u_{00}^{(f)})^{-\frac{u+v}{2}}]C_{p-u,q-v}^{(f)} \quad (4)$$

where $C^{(g)}_{pq}$, $C^{(h)}_{pq}$ and $C^{(f)}_{pq}$ are complex moments of the blurred image, PSF and original image, respectively. $u_{00}^{(f)}$ is the zeroth-order central moment of f(x,y). C_p^u=p!/(p-u)!u!

As Gaussian PSF can be seen as the approximation of many centrally symmetric PSF, we will discuss an important rule of the complex moment for Gaussian PSF.

Theorem 2 In a complex moment C_{pq}, as long as at least one of the subscripts (p or q) is zero, it will not be affected by Gaussian blurring.

This theorem gives an explanation why some moment invariants undergo changes after blurring whereas others remain unchanged after Gaussian blurring. The first of Hu's seven moment invariants $C_1=C_{11}$ have $C_{11}^{(h)}C_{00}^{(f)}[u_{00}^{(f)}]^{-1} = 2\sigma^2 \cdot 1/u_{00}^{(f)} \neq 0$ added to it and so have changed after Gaussian blurring. As zero is found in all the subscripts of the factors in $C_2=C_{20}C_{02}$ and $C_3=C_{30}C_{03}$, there is no change. Because $C_{10}=u_{10}+ju_{01}=0$ and $C_{01}=u_{10}-ju_{01}=0$, it can be proved that $C_{12}^{(g)} = C_{12}^{(f)}$ and $C_{21}^{(g)} = C_{21}^{(f)}$. As either there is zero in the subscripts or it is C_{12} (or C_{21}) in all the factors of the invariants $C_4=\mathrm{Re}(C_{21}C_{12})$, $C_5=\mathrm{Re}(C_{30}C_{12}^3)$, $C_6=\mathrm{Re}(C_{20}C_{12}^2)$, $C_7=\mathrm{Im}(C_{30}C_{12}^3)$, these invariants remain unchanged. It can be said that the union of the above two cases constitutes the subset of moment invariants that will not be affected by Gaussian blur, rotation, scale and translation of the images (See (5)).

$$\{C_{2,1}^s C_{1,2}^r \prod_{i=1}^{n} C_{pi,0} \prod_{i=1}^{m} C_{0qi} \mid \forall s,r,p_i,q_i \geq 0, s+\sum_{i=1}^{n} p_i = r + \sum_{i=1}^{m} q_i\} \quad (5)$$

By set (5), we have got the invariants $\Phi_1,\ldots,\Phi_7$ with their explicit constructions as follows

$$\begin{cases} \phi_1 = C_{02}C_{20} = [(u_{20}-u_{02})^2 + 4u_{11}^2]/u_{00}^4 \\ \phi_2 = C_{03}C_{30} = [(u_{30}-3u_{12})^2 + (3u_{21}-u_{03})^2]/u_{00}^5 \\ \phi_3 = C_{21}C_{12} = [(u_{30}+u_{12})^2 + (u_{21}+u_{03})^2]/u_{00}^5 \\ \phi_4 = \mathrm{Re}(C_{30}C_{12}^3) \\ \quad = \{(u_{30}-3u_{12})(u_{30}+u_{12})[(u_{30}+u_{12})^2 - 3(u_{03}+u_{21})^2] \\ \quad + (3u_{21}-u_{03})(u_{03}+u_{21})[3(u_{30}+u_{12})^2 - (u_{03}+u_{21})^2]\}/u_{00}^{10} \\ \phi_5 = \mathrm{Re}(C_{20}C_{12}^2) = \{(u_{20}-u_{02})[(u_{30}+u_{12})^2 - (u_{21}+u_{03})^2] \\ \quad + 4I_{11}(u_{30}+u_{12})(u_{03}+u_{21})\}/u_{00}^7 \\ \phi_6 = C_{40}C_{04} = [(u_{04}+u_{40}-6u_{22})^2 + 16(u_{31}-u_{13})^2]/u_{00}^6 \\ \phi_7 = \mathrm{Re}(C_{40}C_{02}^2) = \{(u_{04}+u_{40}-6u_{22})[(u_{20}-u_{02})^2 - 4u_{11}^2] \\ \quad + 16u_{11}(u_{31}-u_{13})(u_{20}-u_{02})\}/u_{00}^7 \end{cases} \quad (6)$$

In section 5.1, experiments have shown that these invariants are approximatively invariant even when the image is convolved by other centrally symmetric PSF.

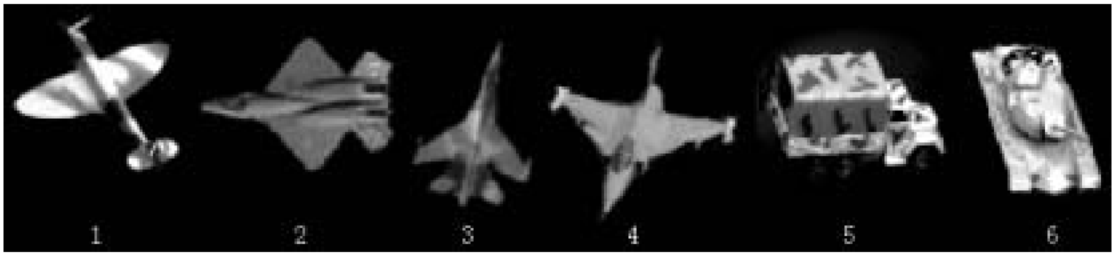
5. Experiments

5.1 Recognition of the blurred objects

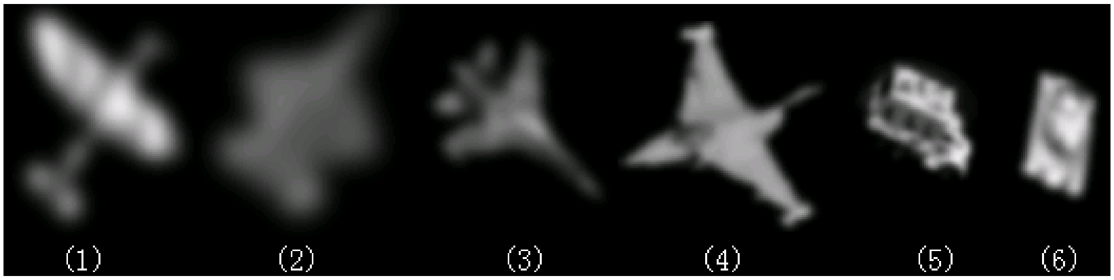
This experiment deals with recognition of some aircrafts. Original aircrafts are shown in Fig.1(a). Fig.1(b) shows the same set of aircrafts rotated and blurred by an uncertain symmetrical PSF. The task is to recognize these blurred aircrafts. The invariants $\Phi_1,\ldots,\Phi_7$ in (6) are normalized as follows.

$\Phi'_1=\Phi_1^{1/2}$; $\Phi'_2=\Phi_2^{1/3}$;
$\Phi'_3=\Phi_3^{1/3}$; $\Phi'_4=[\mathrm{sgn}(\Phi_4)]|\Phi_4|^{1/6}$;
$\Phi'_5=[\mathrm{sgn}(\Phi_5)]|\Phi_5|^{1/4}$; $\Phi'_6=\Phi_6^{1/4}$,
$\Phi'_7=[\mathrm{sgn}(\Phi_7)]|\Phi_7|^{1/4}$

(where $\mathrm{sgn}(x) = \begin{cases} 1 & x>0 \\ 0 & x=0 \\ -1 & x<0 \end{cases}$)



(a)



(b)

(a) Templates (b) Objects to be recognized

Fig.1 Objects to be recognized and templates

Then all invariant values are placed in the same magnitude which will be easy to be processed in classification. The normalized invariants $\{\Phi'_1,\ldots,\Phi'_7\}$ of the aircrafts are used as feature set for recognition, their values are given in Table 1. It can be seen that $\{\Phi'_1,\ldots,\Phi'_7\}$ are not affected by blurring, rotation and scale of the image. Minimum distance classification is carried out and the distance matrix is given in Table 2. It can be seen that the distances at diagonal of the matrix are minimum and all of the aircrafts can be classified correctly.

Table 1 Moment invariants of the blurred rotational objects and the templates

$*10^{-4}$		Φ'_1	Φ'_2	Φ'_3	Φ'_4	Φ'_5	Φ'_6	Φ'_7
1	Fig.1(a)	47.8474	141.1097	73.4281	83.3506	-57.9352	163.6817	-79.3472
	Fig.1 (b)	50.3668	143.0995	74.9502	84.9036	-60.9531	166.1236	-78.7547
2	Fig.1.(a)	77.9306	100.1901	63.3092	69.2839	64.0336	173.7815	115.9888
	Fig.1.(b)	77.3934	96.2911	59.7012	65.1108	60.4290	169.1724	114.0160
3	Fig.1 (a)	69.1369	202.3008	104.1412	122.8092	93.5254	279.8661	138.3047
	Fig.1 (b)	66.4622	199.8464	99.4797	118.2163	89.2360	278.3136	134.8050
4	Fig.1 (a)	78.1634	140.3792	31.7251	44.9366	35.0881	210.8760	121.7225
	Fig.1 (b)	79.8016	142.2358	31.7490	45.2324	35.3869	211.1912	123.5178
5	Fig.1 (a)	59.4569	62.4597	29.7197	-35.7155	-33.1490	88.8306	70.0342
	Fig.1 (b)	60.9398	63.3953	30.4147	-36.5181	-33.4431	89.6381	71.5451
6	Fig.1 (a)	61.3502	23.0256	10.9850	11.0857	14.8430	81.8102	62.4638
	Fig.1 (b)	60.9120	23.0111	10.9846	9.9497	14.1056	81.0471	61.9137

Table 2 Distance matrix

		Fig.1 (a)					
		1	2	3	4	5	6
Fig.1(b)	(1)	**5.5187**	237.8777	300.1030	235.9313	228.1788	238.4857
	(2)	234.1631	**9.1680**	174.7817	74.6643	172.2705	152.5944
	(3)	293.8568	160.2553	**9.4371**	145.8933	320.2542	316.1980
	(4)	237.3455	74.9844	152.6280	**3.1033**	189.3794	192.4519
	(5)	226.4518	180.2102	327.0279	186.4743	**2.6875**	82.1511
	(6)	235.3250	162.8534	324.4995	191.3114	79.7129	**1.7064**

6. Conclusions

From viewpoint of complex moment, we present a new approach to deal with the recognition and restoration of the blurred image. Based on Theorem 1 and Theorem 2, an important subset (in(5)) of complex moments are introduced which are invariant simultaneously to image blurring and to various geographic transform. Experiment results show that our method performs well in the tasks of recognizing blurred images.

References

[1] S.A. Dudani, K.Jbreeding, and R.B. McGhee, "Aircraft identification by moment invariants" IEEE Trans. Comput. vol. C-26, No.1. pp.39-45, Jan.1983.

[2] Belkasi M. S O, Shridher M,Ahmadi M.Pattern recognition with moment invariants:a comparative study and new results, *Pattern Recognition,* 1991, 24(12):1117-1138

[3] Wai-Hong Wong, Wan-Chi Siu, Kin-Man Lam "Generation of moment invariants and their uses for character recognition," *Pattern Recognition Letters* 16 (1995)115-123

[4] J. Flusser and T. Suk, "Affine moment invariants: A new tool for character recognition," *Pattern Recognition Letters*, Vol 15, 433-436, 1994

[5] Jan Flusser and Tomas Suk, "Pattern recognition by affine moment invariants," *Pattern Recognition*, Vol.26, No.1, pp167-174,1993

[6] F.MinDru, T.Moons and L.Van Gool, "recognizing color patterns Irrespective of viewpoint and illumination" *Computer Vision and Pattern Recognition,* 1999. IEEE Computer Society Conference on, Vol.1, pp.23-25 June 1999-373 Vol.1

[7] Ock-Kyung Yoon, Dong-Min Kwak, Dong-Whee Kim, Kil-Houm Park, "MR brain image segmentation using fuzzy clustering" 1999 *IEEE International Fuzzy Systems Conference proceedings August 22-25 1999, Seoul Korea.*

[8] Abu-mostafa Y S ,Psaltis D. Recognitive aspects of moment invariants. *IEEE Transactions on Pattern Analysis and MachineIntelligence*,1984,6(6) pp.698-706.

[9] J. Flusser "On independence of rotation moment invariants," *Pattern Recognition* 33 (2000), pp.1405-1410

[10] Irene Rothe, Herbert Susse, and Klaus Voss "The method of Normalization to determine Invariants" *IEEE Trans. on Pattern Analysis and Machine Intelligence,* 1996, Vol.18(No.4) pp.374-375

[11] Chong-huah Lo, and Hon-son don "3-D Moment Forms: Their Construction and Application to Object Identification and Positioning" *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 1989, Vol.11(No.10) pp1053-1063