FUZZY NON-RIGID MOTION ESTIMATION ROBUST TO ROTATION

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ABSTRACT

The increasing needs of register images, usually picked up from diverse moving sources of deformable nature, have arisen in diverse fields and have focused the attention of researchers in the last years. The critical step involved within image registration use to be the image matching. This paper proposes a robust method that faces the ill-posed nature of this image matching process, taking into account the image rotation effect. The matching ambiguity is described by a fuzzy parametric model that includes estimated relative rotation, and, finally, a spatially non-uniform fuzzy interpolation is used to translate the parametric info into a set of matching field vectors. The method obtains the spatial matching between the two images in a global spatial extent and with sub-pixel accuracy. Results of the method on real images and high non-rigid artificial deformation and rotation prove the validity of the approach.

1. INTRODUCTION

The transformations involved in deformable registration generally make the problem of image matching ill-posed, so that additional constraints are required to achieve a solution close to the actual. A common way to do this is to incorporate in the method a priori knowledge about the deformation characteristics to restrict the space of possible solutions.

In previous works, a method to estimate the motion field vectors that connect two images for non-rigid motion was presented [1] [2]. In this paper, an enhanced method that addresses the problem of significant relative rotation between certain image regions is proposed.

In order to skip the ambiguity in the image matching process, a block-matching approach of very reduced size is used and the field of matching is described with fuzzy parametric models. In a second processing phase a spatially non-uniform fuzzy interpolation translates the fuzzy parametric motion information into a set of matching field vector that relates spatially the pair of images.

The goal of this method is to parametrize for each pixel or node of a reticular mesh its field of matching. This field of matching, computed by a local similarity cost function, usually encloses a large area of several pixels, which is described with a fuzzy parametric model. A spatially non-uniform fuzzy interpolation translates the parametric data into a set of matching field vectors, which relates both images with sub-pixel accuracy. The method is capable of tracking high non-linear deformations. These ideas have an extension for volumetric problems [2].

The paper is organized as follows. Section 2 presents the basic ideas of the method, which is composed of two processing phases. Section 3 details the first phase, the estimation of fuzzy spatial correspondence between the images. Section 4 contains the method to regularize the previous fuzzy solution. In Section 5 some simulation results prove the validity of this approach. Finally, Section 6 contains the conclusions.

2. NON-RIGID MOTION ESTIMATION: PROPOSED METHOD

The proposed method is composed of two sequential stages:

1. Estimation of the fuzzy matching. This method combines a special similarity cost function with a very reduced matching-block. At difference with classical block-matching algorithms, in which hard decisions are taken, the goal pursued in this process is to spread the matching ambiguity and thereafter to parametrize the matching field with the highest similarity. In most of the cases this field has the shape of a cluster, a curve line or amorphous, each case leading to a different fuzzy parametric model.

2. Fuzzy parametric regularization. This phase converts the fuzzy information at each pixel into a set of numerical matching field vectors. This is accomplished by a spatially non-uniform fuzzy interpolation, an iterative process that consists of a discrete linear filtering of

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minimum entropy constrained to the parametric models at each pixel.

3. ESTIMATING THE FUZZY SPATIAL CORRESPONDENCE

The goal of this phase is to compute and parametrize for each pixel or node in a reticular mesh of the first image the field in the second image that is the most similar. There are three main keys in this process:

1. The similarity cost function, which is a trade-off between block correlation and difference, and the computing block is very small (3-5 pixel wide).

2. The cost operation results in a similarity map that informs about the likeness between both images in a vicinity of a pixel.

3. The similarity map may contain many pixels of high similarity degree (all being likely candidates); this ambiguity is resolved by fitting a parametric model in the map, resulting in a fuzzy motion estimation.

Finally, the block operations involved in this process have a clear difference respect to classical block-matching algorithms: here the block is very small, this fact leading to an increase in the ambiguity of the spatial matching and, on the other hand, an increase in the ability to track non-rigid motion.

3.1. Computing the Similarity Map

For each pixel (x,y) in image 1 a similarity map of (2N+1)-pixel width is computed, which contains the similarity between a block of image 1 centered at the pixel (x,y) and a block of image 2 centered at the pixel (x',y') in a (2N+1) search length over the image 2.

In this matching process, a light rotation in one of the blocks in relation to the other might cause severe errors in block assignments because of the well-known lack of effectiveness of correlation in this case. To solve this problem, rotation must be detected and compensated before correlation operation.

The relative orientation of blocks is computed by means of a log-polar mapping and Fourier-Mellin transform [4][5]. Log-polar mapping yields to the next coordinate change:

$$\sigma = \tfrac{1}{2}\log\left(x^2 + y^2\right) \qquad \varphi = \tan^{-1}\left(\tfrac{y}{x}\right) \tag{1}$$

This transformation map every point (x,y) of an image i_{xy} in another pair (σ,φ), where $0<\varphi\leq 2\pi$, in order to generate a warped image $i_{\sigma\varphi}$. An image rotation of an angle δ over i_{xy} will be reflected in $i_{\sigma\varphi}$ like an image translation of δ in the φ axis. And owing to the translation property of the Fourier transform, this image spatial shift affects only the phase representation, while magnitude keeps unchanged.

$$i_{\sigma\varphi}(\sigma,\varphi+\delta) \xleftrightarrow{F} I_{\sigma\varphi}(\omega_\sigma,\omega_\varphi)e^{-j\delta\omega_\varphi}$$

So, if an established magnitude correlation threshold is exceeded, certain degree of likeness found between both blocks is found, phase difference shows the relative rotation angle between blocks. This linear phase difference can be clearly detected in the shift of the inverse Fourier Transform of $e^{-j(\delta_1-\delta_2)\omega_\varphi}$, where δ_1 and δ_2 are the phase of the Fourier transform of both block.

Once the relative rotation between blocks is corrected, the similarity coefficient, ρ, is related directly to the correlation, r, and inversely to the difference between both image blocks, d. See [1] for details.

$$\rho(x,y) = \begin{cases} \gamma(x,y) & \text{if } \gamma(x,y) > \lambda \\ 0 & \text{if } \gamma(x,y) \leq \lambda \end{cases} \tag{2}$$

$$\gamma(x,y) = \frac{r(x,y)}{1+\alpha\, d(x,y)} \tag{3}$$

Parameter α is a positive constant that sets the importance of correlation versus difference. Parameter λ is a positive constant within [0,1], that works as a threshold of the similarity coefficient.

3.2. Translating Similarity Map in a fuzzy model

Depending on the location of the searching block, the similarity map shows three main types of shape:

1. If the block location corresponds to a small isolated detail in the image, the similarity map usually shows a small isolated cluster of high similarity degree.

2. If the block location corresponds to a border of the image, the similarity map usually shows a curve of high similarity degree.

3. If the block location corresponds to a low contrast or nearly flat intensity region, the similarity map usually shows an irregular shape of low similarity degree.

Based on the previous reasoning, the similarity map is parameterized with one of the following models, which correspond to each one of the previous cases:

1. Model 0, a point (px, py), equal to the center of mass of the cluster.

2. Model 1, a quadratic curve rotated by angle $\theta+\delta$, θ being the intrinsic angle to the parametric model, and δ being the angle that correspond to the relative rotation angle between blocks computed in the previous phase, and that must be accumulated to θ.

3. Model 2, undefined, no parameters.

The points (x,y) of the quadratic curve in model 1 are described by the following equations:

$$\hat{y} = f(\mathbf{a}, \hat{x}) = a_2\hat{x}^2 + a_1\hat{x} + a_0 \quad (4)$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\theta+\delta) & \sin(\theta+\delta) \\ -\sin(\theta+\delta) & \cos(\theta+\delta) \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} \quad (5)$$

Computation of the quadratic parameters a and angle θ are detailed in [1].

4. REGULARIZATION OF THE FUZZY SOLUTION

The previous phase estimates the motion or deformation between the two images as fuzzy parametric information, which informs about the location of the matching field, this area being more or less constrained depending on the associated model. Since the objective of the problem is to obtain the spatial information that relates both images in a direct and explicit way, it is necessary to convert the fuzzy solution into a matching field vector set or any other type of support.

The proposed solution is to use a regularization method based on an iterative procedure that combines 2D linear discrete filtering and constraint forcing by projecting onto the parametric models. This process is equivalent to a spatially non-uniform fuzzy interpolation. The solution achieved, supported in a dense set of 2D vectors, has minimum dispersion. Let vx(x,y) and vy(x,y) be the two coordinates of the matching field vector at pixel (x,y).

4.1. Low curvature energy interpolation

Some regularization methods rely on an iterative filtering of the discrete vector field with different types of kernels, such as Gaussian. The use of a kernel that minimizes the energy of curvature of the motion field vectors, or equivalently their second difference, is proposed in this work. The second difference correlation in terms of the 1-dimensional Z-transform is defined by

$$C(z) = \left(z - 2 + z^{-1} \right)\left(z - 2 + z^{-1} \right) \quad (6)$$

In the 2D case, the filtering kernel is defined as the pseudo-inverse of the energy of curvature

$$H(z_x, z_y) = \frac{M}{M + C(z_x) + C(z_y)} \quad (7)$$

where M is a constant that sets the cut-off frequency of the low-pass filter. The larger the constant M is the larger the cut-off frequency, the larger the ability to track high deformable motion, and the slower the speed of the process. This iterative filtering, implemented with the FFT algorithm, can be speed up by embedding the process into a second order discrete loop.

4.2. Parametric constraint-forcing

Besides the global curvature minimization constraint result of the linear filtering, at each iteration and for every pixel (x,y) the corresponding parametric model constraint is imposed to the values of the matching field vector (vx(x,y), vy(x,y)). The parametric model constraint is accomplished by projecting the matching field vector onto the model. Depending on the model,

1. If model is 0, the matching field vector is replaced by point (px, py).
2. If model is 1, the matching field vector is replaced by its projection onto the quadratic curve,
3. If model is 2, the matching field vector (vx(x,y), vy(x,y)) remains the same.

Projecting a point (xi, yi) onto a θ-rotated quadratic curve is done as follows:

$$\begin{bmatrix} x_o \\ y_o \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} xi \\ yi \end{bmatrix} \quad (8)$$

$$(x_o, y_o) \rightarrow \left\{ y = a_2x^2 + a_1x + a_0 \right\} \rightarrow (x_p, y_p) \quad (9)$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x_p \\ y_p \end{bmatrix} \quad (10)$$

The problem of projecting a point onto a quadratic parabola has an analytic solution and it can be also obtained numerically by means of a gradient algorithm.

5. RESULTS - DISCUSSION

The method proposed in this paper for estimating the spatial matching between a pair of images was tested with a great rotation and deformation applied over the 256x256 Lenna's image.

The upper left hand side of Fig. 1 shows the original test image, and the upper right hand side the effect of warping original image with the rotation pattern showed in Fig. 2.

By choosing the original Lenna as image 1 and the warped version as image 2, the automatic method proposed in this paper was applied over this pair of images. The parameters of the experiment were: N=13, λ=0.4, α=3, size of the block 5x5, size of the images 256×256 and M=10. The lower left hand side of Fig. 1 shows the result of "de-warping" the image 2 with the motion field vectors achieved by the proposed method; the lower right hand side of Fig. 1 shows the pixels that have an associated model. Remaining points were obtained by interpolation. In [2] similar results of the method on medical images are shown.

From the graphical results we observe that the method gives an accurate estimation of the spatial correspondence between the two images. However, the problem of de-warping correctly the deformed image in its full extent is not perfectly accomplished: in low-contrast or flat intensity areas, such as the chick or background, the method does not have a proper reference to estimate the deformation owing to scarce number of pixels with a model assignment. Obviously this limitation cannot be perceived over Lenna's image.

An interesting point of discussion about this method is its computational load. The first processing phase includes a block-matching computation: at difference with the classical approach, the block size in the proposed method is very small, which, in spite of the additional parametric model translation, does not imply a larger computing cost. The filtering process in the second phase is implemented with an FFT algorithm and the projections onto quadratic curves are scarce since most of the pixels in the image correspond to low-contrast areas. Finally, in comparison to optical flow algorithms, these have a clear advantage in computing load but they show clear limitations in the accuracy of the solution.

Fig. 1.. Upper-left: original image; upper-right: artificially warped image; lower-left: reconstructed image; lower-right: original image showing in white color the points where a parametric model is detected.

6. CONCLUSIONS

Estimation of deformable motion in pairs of images is an ill-conditioned problem. In this paper a method that addresses this problem is proposed. In order to skip the ambiguity in the matching, a block-matching approach of very reduced size is used and the field of matching is described with fuzzy parametric models. In a second processing phase a spatially non-uniform fuzzy interpolation translates the fuzzy parametric motion information into a set of matching field vector that relates spatially the pair of images with sub-pixel accuracy. The method has been successfully tested on real images with high non-linear deformation and rotation effect.

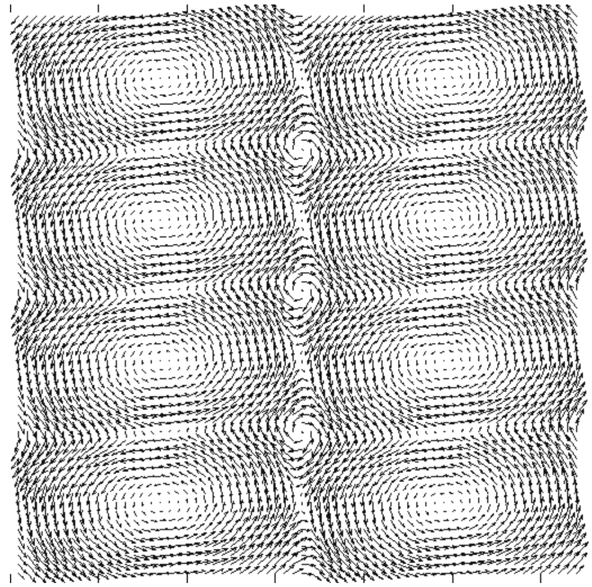

Fig. 2. Deformation pattern.

7. REFERENCES

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