A WAVELET-BASED TWO-STAGE NEAR-LOSSLESS CODER

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ABSTRACT

In this paper,we investigate a two-stage near-lossless compression scheme. It is in the spirit of "lossy plus residual coding" and consists of a wavelet-based lossy layer followed by an arithmetic coding of the quantized residual to guarantee a given L^∞ error bound in the pixel domain. Our focus is on the selection of the optimum bit rate for the lossy layer to achieve the minimum total bit rate. Unlike other similar lossy plus lossless approaches using a wavelet-based lossy layer, the proposed method does not require iteration of decoding and the IWT(Inverse Wavelet Transform) to locate the optimum bit rate. We propose a simple method to estimate the optimal bit rate and provide a theoretical justification for it. It is based on the 'critical rate' argument from the Rate-Distortion theory and 'whiteness' of the residual.

1. INTRODUCTION

We consider a near-lossless coding problem with a two-stage coder [1], where the amount of distortion, as measured by the maximum allowable deviation of pixel values, is guaranteed quantitatively. The first stage consists of a wavelet-based lossy coder such as SPIHT. For the second stage, we use an arithmetic coder to encode the residual. Since it is difficult to derive a meaningful relation between distortions in the pixel domain and the wavelet domain in the L^∞ error sense, pixel domain techniques such as predictive coding have been the most popular choice in practice. Furthermore, since prediction-based coders are very competitive in lossless and high bit-rate coding, it is expected to be so in many near-lossless coding scenarios, where the maximum pixel-domain error specified by L^∞ norm (hereafter denoted as δ)is small. Unfortunately, this is no longer true when δ gets larger as the quality of prediction deteriorates. Also, predictive coders do not offer the advantage of progressive transmission as with wavelet-based coders. Thus it is highly desirable to combine the advantages of both worlds so that we can provide the convenience of embedded bit-stream along with the guaranteed maximum distortion in the L^∞ sense in pixel domain. Indeed, there have been various attempts at achieving such a goal in the near-lossless/lossless coding problem. In [1], the authors compare various near-lossless schemes with a focus on the wavelet-based two-stage scheme. A similar approach is taken in [2]. Marpe et al. [3] also propose to use a two-stage wavelet-packet based approach to lossy/lossless coding. However, none of the aforementioned works addresses the problem of bit rate assignment except through exhaustive search by repeatedly encoding the residuals at every possible lossy layer coding rate.

We elaborate on the choice of efficient bit rate assignment for the wavelet-based first-stage lossy-layer coder in the two-stage near-lossless setting. It does not directly rely on any analytical model for the source, yet works reasonably well for most real sources. In Section 2, we explain the structure of the near-lossless coding scheme we used. In Section 3, we discuss the modelling of the reconstruction error from the first stage lossy coder. In Section 4,we propose a simple bit rate assignment scheme when an arithmetic coder is used for residual coding. Section 5 provides comparisons with other near-lossless coders , and Section 6 concludes the paper.

2. A TWO-STAGE NEAR-LOSSLESS CODER

Figure 1 shows a block diagram of the two-stage near-lossless coder proposed in this paper. Let I and I_{lossy} represent the original image and the lossy reconstruction, respectively. Then the error $e = I - I_{lossy}$ is quantized as follows:

$$
\begin{aligned}
e_{q-idx} &= \lfloor \frac{e+\delta}{2\delta+1} \rfloor, \text{if e} > 0 \\
e_{q-idx} &= \lfloor \frac{e-\delta}{2\delta+1} \rfloor, \text{if e} < 0
\end{aligned}
$$

where the maximum amount of deviation from the original pixel, i.e. bound on the error magnitude is indicated by δ and the quantization index is represented by e_{q-idx}. In the two-stage scheme,I_{lossy} is transmitted as the lossy layer and losslessly coded qunatization index, $e_{q-idx,coded}$, is sent as the residual layer.

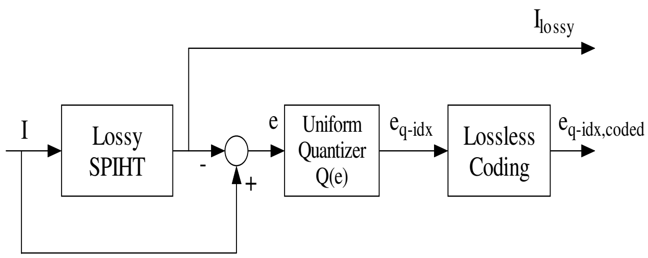

Fig. 1. Block diagram of the two-stage near-lossless coder

3. MODELLING THE RESIDUALS

3.1. Modelling the residuals with GGD's

It is well-known that wavelet coefficients are well-modelled by a Generalized Gaussian Distribution (GGD). Since large coefficients are encoded first in typical wavelet coders, it is expected that the distribution of the encoding residual in the wavelet domain will still fit well into a GGD (usually with shorter tail than the wavelet coefficient itself). Indeed, it was observed through numerous experiments that the residuals both in the pixel and the wavelet domains could be reasonably well-modelled by GGD's.

A GGD $f(x)$ is defined as follows:

$$f(x) = ae^{-|bx|^c} \tag{1}$$

,where $a \equiv \frac{bc}{2\Gamma(1/c)}$, $b \equiv \frac{1}{\sigma}\sqrt{\frac{\Gamma(3/c)}{\Gamma(1/c)}}$, $\Gamma()$ is the Gamma function , c is the shape parameter and σ is the standard deviation of the source. Thus a GGD is completely specified by σ and c. Here the shape parameter c for a random variable X can be estimated using the following equation.

$$c = G^{-1}(\eta) \tag{2}$$

where $G(x) \equiv \frac{\Gamma(2/x)}{\sqrt{\Gamma(1/x)\Gamma(3/x)}}$, $\eta \equiv \frac{E(|X|)}{\sigma}$ is a normalized MAV(Mean Absolute Value).

3.2. Critical rate and residual distributions

Figure 2 shows the standard deviation(STD) and the mean absolute value(MAV) of the wavelet and the pixel domain residuals as functions of the lossy layer coding rate. We can see the mean absolute value of the encoding residual distribution in both domains converge as the lossy bit rate increases. Since we used a 9/7 biorthogonal filter, it did not exactly preserve the encoding residual variance between the two domains, but they were very close. Correspondingly, the shape parameters also converged as is evident from (2). Since a GGD is completely specified by its variance and shape parameter, this means the encoding residuals in both domains converge in distribution. (See Figures 2 and 3.) In fact, the lossy layer coding rate where the residual distributions converge varies depending on a particular source to be coded and is thought to be related with the 'critical rate' phenomenon. For most sources, it can be argued that above a certain 'critical' rate, the encoding residuals become 'white'[4, 5]. Also note that the wavelet domain residuals can be regarded as the wavelet transform of the pixel domain counterpart. Since the pixel domain encoding residual is white when the lossy rate is above the 'critical' rate,wavelet transforming it does not change its zero order entropy[6]. Now, it can be shown that, given a variance,the shape parameter of a GGD is related to its zero order entropy in a one-to-one manner. Thus the shape parameters in both domains will actually converge when the lossy rate is above the critical rate as can be seen in Figure 2.

Next, we want to show the 'quantized' versions of the encoding residuals in both domains also converge. This will provide a theoretical justification for the bit rate assignment scheme proposed in the next section. Observe that once quantization is performed on the residuals in both domains, it is no longer true that one is obtained by DWT(or IDWT) of the other. Furthermore, since the quantized distributions often converge *before* the original distributions do, it is not true that we are quantizing the same distributions. Thus, it is not obvious if the 'quantized' residuals will also converge. However, experiments suggest that is indeed true and the following arguments provide one way to explain it.

Definition 1. Refinement/processing of distribution Given a distribution F_X on χ,divide χ into k mutually disjoint sets $\hat{X}_1, \hat{X}_2, ..., \hat{X}_k$ satisfying

$$\chi = \bigcup_{i=1}^{k} \hat{X}_i.$$

Define a new distribution $F_{\hat{X}}$ as

$$F_{\hat{X}}(\hat{X}_i)=\sum_{x \in \hat{X}_i} F_X(x)$$

Then F_X is called a *refinement* of $F_{\hat{X}}$ and $F_{\hat{X}}$ is called a *processing* of F_X. Then there is a well-known fact regarding divergence of random variables repeated below for convenience.

Lemma 3.1 *Let $F_{\hat{X}}$ and $F_{\hat{Y}}$ be the processing of F_X and F_Y respectively. Then*

$$D(F_X||F_Y) \geq D(F_{\hat{X}}||F_{\hat{Y}})$$

Proof: By log-sum inequality,

$$\begin{aligned} & \sum_{x \in \hat{X}_i} F_X(x) log \frac{F_X(x)}{F_Y(x)} \\ \geq \quad & (\sum_{x \in \hat{X}_i} F_X(x)) log \frac{\sum_{x \in \hat{X}_i} F_X(x)}{\sum_{x \in \hat{X}_i} F_Y(x)} \\ = \quad & F_{\hat{X}}(\hat{X}_i) log \frac{F_{\hat{X}}(\hat{X}_i)}{F_{\hat{Y}}(\hat{X}_i)} \end{aligned}$$

Hence,

$$
\begin{aligned}
D(F_X||F_Y) &= \sum_{i=1}^{k}\sum_{x\in\hat{X}_i} F_X(x)log\frac{F_X(x)}{F_Y(x)} \\
&\geq \sum_{i=1}^{k} F_{\hat{X}}(\hat{X}_i)log\frac{F_{\hat{X}}(\hat{X}_i)}{F_{\hat{Y}}(\hat{X}_i)} \\
&= D(F_{\hat{X}}||F_{\hat{Y}})
\end{aligned}
$$

Theorem 3.2 *Let $\hat{X}$ and $\hat{Y}$ be the quantization indices obtained from uniform-quantizing the random variables X and Y, respectively. (X: wavelet-domain error residual, Y: pixel-domain error residual) Then $F_{\hat{X}}$ converges to $F_{\hat{Y}}$ at least as fast as F_X does to F_Y.*

Proof: Observe that quantization partitions the event space of the original random variable. Thus the distribution of the quantization indices is nothing but a *processing* of the original distribution. Since $D(F_X||F_Y) \geq 0$ with equality iff $F_X(x) = F_Y(y)$ and we have $F_X(x) \rightarrow F_Y(y)$, it follows from lemma 3.1 that $F_{\hat{X}}(\hat{x}) \rightarrow F_{\hat{Y}}(\hat{y})$ at least as fast as its unqunatized counterpart.

4. OPTIMAL LOSSY LAYER CODING RATE

Beyond the 'critical rate', the wavelet-based lossy coder(e.g. SPIHT) cannot have any advantage over the zero-order entropy coding in the pixel domain because the wavelet/pixel domain encoding residuals are (ideally) white. In fact, the wavelet-based lossy coders usually become even worse than the zero-order entropy coders because the residuals no longer have correlation structures that the wavelet coders attempt to take advantage of. Thus a typical plot of lossy coding rate vs. total bit rate has a 'critical rate' where the total bit rate starts to increase due to the inefficiency of the first stage lossy coder as we can see from the Figure 4.

As mentioned in Section 3.2, the zero-order entropy of the pixel domain encoding residual is the same as that of the wavelet domain counterpart beyond the 'critical rate'. Since we have the wavelet domain residual in the memory while progressively encoding the source, we can calculate its zero-order entropy 'on the fly'. Observe that we can iteratively update the zero-order entropy of the wavelet-domain (quantized) residual by updating only one symbol frequency corresponding to the residual being coded at the moment, implying virtually no effect on the speed or complexity of the lossy coder. Also notice that although the residuals in both domains do not have the same zero-order entropy below the 'critical rate', we can still locate the point where the total bit rate(=lossy layer coding rate + residual layer coding rate) starts to increase. Note we do not need any iteration of decoding/inverse wavelet transform.

5. COMPARISON WITH OTHER CODERS

Table 1 compares the performance of the proposed coder with JPEG-LS and the 'SPIHT+Context-AC'(a wavelet-based two-stage near-lossless coder with a context-based entropy coding for residuals [1]). In terms of PSNR, the proposed scheme always performed best for Lena. As for total bit rates, wavelet-based coders were better than the JPEG-LS even at the high bit rates, where predictive coders are known to be competitive. Note the extra savings in total bit rates achieved by the 'Context-AC' come at the expense of exhaustive iteration. Or, the estimated optimal rate could be used as a starting point to reduce the number of iterative search in case the 'Context-AC' is used to code the residual for better compression.

6. CONCLUSION

In this paper, we proposed a simple method of choosing an optimal lossy layer coding rate in a wavelet-based two-stage near-lossless coding. It is based upon the 'critical rate' phenomenon and works reasonably well for most real sources with an almost unnoticeable effect on coding speed/complexity. Its PSNR and total bit rate performances were comparable to those of other near-lossless coders.

7. REFERENCES

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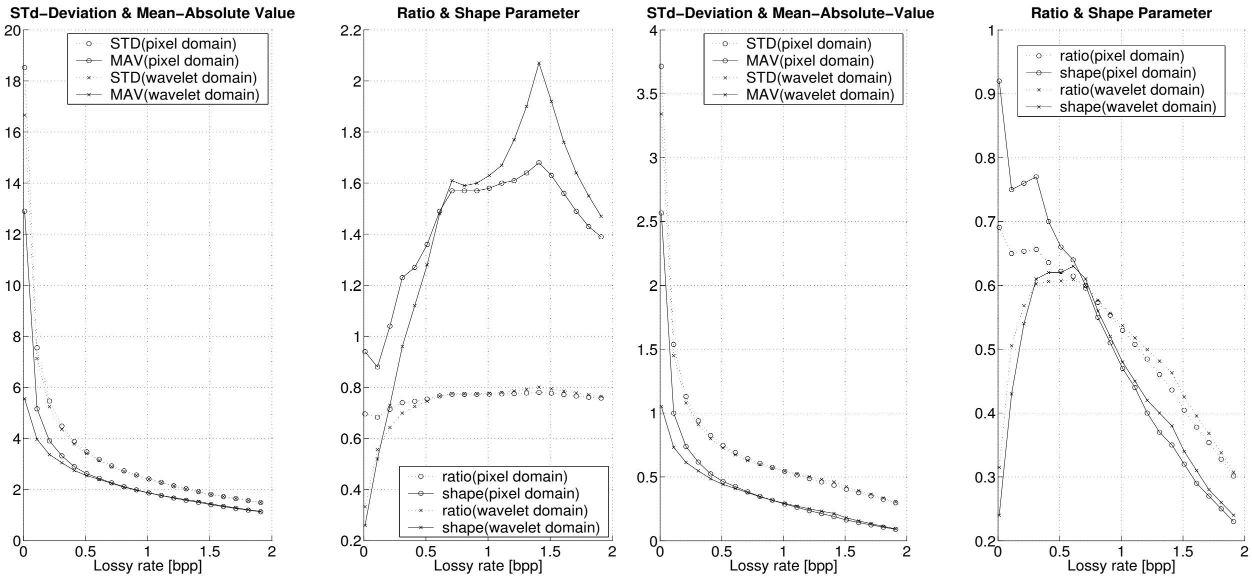
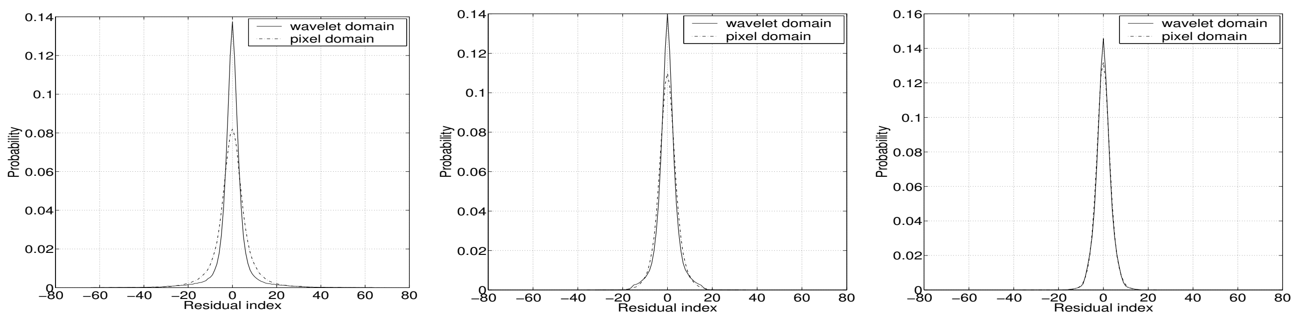


(a) Unquantized residual, i.e. δ=0

(b) Quantized residual, quantized with δ=2

Fig. 2. Convergence of residual distributions for Lena, 'ratio'= η in (2)



(a) δ=0 , lossy layer rate=0.11

(b) δ=0 , lossy layer rate=0.31

(c) δ=0 , lossy layer rate=0.51

Fig. 3. Residual distributions for Lena

Tolerance	Method	bpp	PSNR
$\delta = 1$	JPEG-LS	2.72	49.90
	SPIHT+context-AC [1]	**2.69**	**49.89**
	SPIHT+AC (proposed)	2.77	**49.89**
$\delta = 2$	JPEG-LS	2.09	45.15
	SPIHT+context-AC	**2.02**	45.16
	SPIHT+AC	2.12	**45.17**
$\delta = 6$	JPEG-LS	1.24	37.17
	SPIHT+Context-AC	**0.86**	38.54
	SPIHT+AC	0.92	**38.76**
$\delta = 7$	JPEG-LS	1.14	35.99
	SPIHT+Context-AC	**0.73**	37.95
	SPIHT+AC	0.79	**38.27**

Table 1. Comparison with other coders for Lena

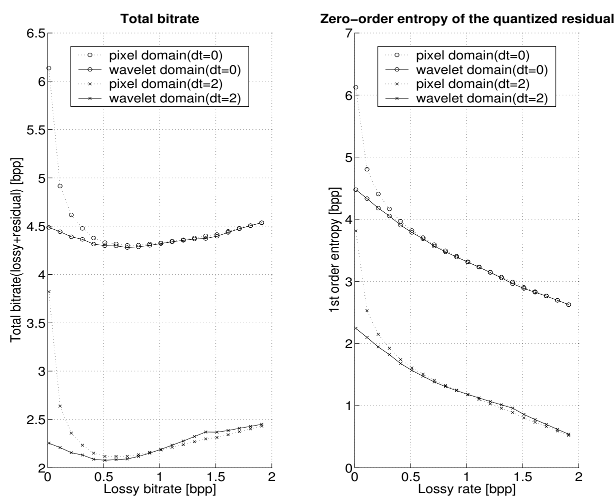


Fig. 4. Bit rates for Lena