H-THINNING FOR GRAY-SCALE IMAGES

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ABSTRACT

Thinnings are very important operators for image analysis developed by mathematical morphology. Their most popular applications are the discrete homotopic skeletons. They are de£ned for the sets as for the functions but their use in the gray-scale case remains delicate and marginal. This article explores one variation of the thinning's original de£nition in which the nonsigni£cant local variations of luminance are ignored. The simpli£ed skeletons obtained are presented on various examples.

1. INTRODUCTION

The fundamental idea leading to the skeleton is that shapes may be entirely described using a minimal set of points: a disk by its center, an ellipse by its median axis... The skeleton of a set X is formed by the points of X at equal distance of the borders of X: they correspond to the meeting points of £re in a *prairie £re* scenario where £re hearths correspond to set borders (see £gure 1). More formally, the skeleton of a set X is de£ned as the set of all centers of maximal balls included in X.

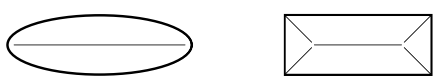

Fig. 1. Shapes and skeletons.

Mathematical de£nitions of the skeleton are due to L. Blum [2] and L. Calabi [3] while essential interactions with image analyzing has been developed by mathematical morphology [7, 4, 1, 11].

One major contribution of mathematical morphology in this £eld concerns the question of the skeleton computation in the discrete case. Several solutions have been suggested. The £rst one is due to C. Lantuéjoul [4] who expresses the skeleton in terms of residues of openings. A second family of solutions, available only in the discrete case, consider the local maxima of the distance function ; different algorithms have been proposed by F. Meyer [8] and L. Vincent [13]. Blum's and Lantuéjoul's de£nitions are equivalent in the continuous case. In the discrete case, Vincent's and Lantuéjoul's solutions are equivalent but the so-de£ned discrete skeleton is only a subset of the real skeleton.

In all cases, an important property of the continuous skeleton has been left when passing in the discrete case: the homotopic behavior. In the continuous case, the original set and its skeleton present the same number of grains and pores. In the discrete case, it is not guaranteed since maximal balls are approximated and since certain smaller details are eliminated.

The de£nition of homotopic skeletons in the discrete case has been the object of several works. In mathematical morphology framework, we can mention the solution proposed by L. Vincent for connecting the crest lines of the distance function [14, 15] and the contributions of C. Lantuéjoul, S. Beucher and J. Serra for the de£nition of a thinning-based skeleton [4, 11, 1]. This last solution is certainly the most famous and the most powerful one. It stems from the interpretation of the skeleton as a *prairie £re*. The £re propagation is simulated while preserving the homotopic properties of the sets via the hit or miss transform and the associated thinning operation.

Basically, skeletons are de£ned for sets. Two mainly numerical versions exist: the £rst one is an extension of Lantuéjoul's formula [9, 6, 12], the second one is an adaptation of the thinning-based algorithm [1, 5]. The numerical versions inherit all the properties of the binary versions, in particular it is noise sensitive. In addition, local variations of luminance inside the shapes can completely modify the characteristics of the skeleton.

In this article, numerical skeletons based on thinnings are studied and tested on various examples. An adaptation of the original de£nition is then proposed: h-thinnings resulting provide simpli£ed skeletons which give better results in many cases.

2. THINNING-BASED SKELETONS

Homotopic skeletons stem from Blum's interpretation of the skeleton as a *prairie £re*. Precisely, the goal is to simulate the propagation of a £re while preserving the homotopy of

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sets. Such operation is carried out in mathematical morphology via the Hit or Miss Transform (HMT).

Considering a bi-phase structuring element $(T_1, T2)$, where T_1 and T_2 are supposed to be disjointed ($T_1 \cap T_2 = \emptyset$), the HMT of a set X extracts the points p of the space such that T_1^p is entirely included in the set X and T_2^p is entirely included in the complementary set X^c, what gives :

$$\text{hmt}(X) = \epsilon_{T_1}(X) \cap \epsilon_{T_2}(X^c) = \epsilon_{T_1}(X) \cap [\delta_{\tilde{T}_2}(X)]^c$$

In this formula, T_1^p and T_2^p denote the structuring elements centered at point p. ϵ and δ denote the standard erosion and dilation :

$$\epsilon_T(X) = \{p \in E,\, T^p \subset X\} \quad , \quad \delta_{\tilde{T}_2}(X) = [\epsilon_{T_2}(X^c)]^c$$

Note that $hmt(X) \subset X$ if and only if the center of the bi-phase structuring element belongs to T_1.

Depending on the choice of the bi-phase structuring element, the HMT allows to extract end points, triple points or border points of sets...

The operation which eliminates from a set X the points of the HMT is called thinning:

$$Thin(X) = X \backslash hmt(X)$$

and in that case of course , the center of the structuring element belongs to T_1.

The £gure 2 illustrates the effect of a thinning on a set X. Note that the procedure is isotropic only if rotated structuring elements are sequentially considered.

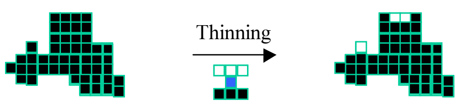

Fig. 2. Thinning of a set X. The two phases of the S.E. are represented in black (T_1) and weight (T_2).

The HMT produces homotopic transform if the modi£cation of the center in the two-phase structuring element preserves the homotopy. There is essentially two adequate con£gurations in the case of the squared grid [1].

3. GRAY-SCALE CASE

The adaptation of the precedent de£nitions to the numerical case has been proposed by S. Beucher [1]. Note that HMT and thinning do not correspond to increasing operators, so the thinning of a function may not be de£ned as the thinning of its level sets.

First, the HMT of a function f acting on E is de£ned as the operation which extracts the points $p \in E$ such that:

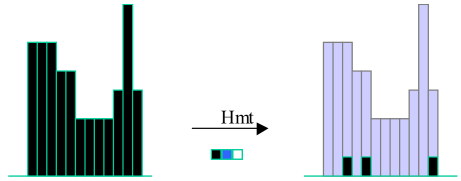

Fig. 3. Points extracted by the gray-scale Hit or Miss Transform.

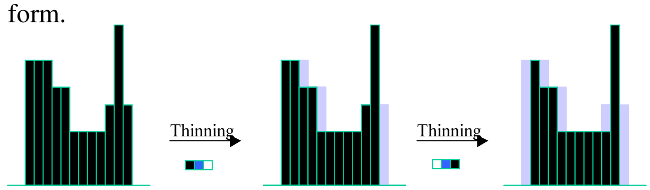

Fig. 4. Gray-scale image pro£le and effect of a sequence of thinnings.

- T_1^p is entirely included in a light structure of the image: no point of T_1^p is of level lower than p:

$$\forall y \in T_1^p,\, f(y) \geq f(p) \Leftrightarrow \epsilon_{T_1}[f(p)] \geq f(p)$$

- T_2^p is entirely included in the background of the image: all point of T_2^p is of level smaller than p:

$$\forall y \in T_2^p,\, f(y) < f(p) \Leftrightarrow \delta_{\tilde{T}_2} f(p) < f(p)$$

If now 0 belongs to T_1, then the erosion ϵ_{T_1} is anti-extensive, $\epsilon_{T_1}[f] \leq f$, and £nally:

$$0 \in T_1,\;\; p \in hmt(f) \Leftrightarrow \delta_{\tilde{T}_2}[f(p)] < f(p) = \epsilon_{T_1}[f(p)]$$

Note that the HMT of a function is a set of points. The £gure 3 illustrates the points selected in different con£gurations.

Then, the purpose is to thin the function at the border points extracted by the hit or miss transform:

$$\forall p \in E,\, Thin(f)(p) = \begin{cases} \delta_{\tilde{T}_2}[f(p)] & \text{if } p \in hmt(f) \\ f(p) & else \end{cases}$$

The function level is lowered at each point p of the hmt. The output level of p is then the highest level of the background of the image near p.

The £gure 4 illustrates how the thinning allows to simplify a function while preserving its homotopy (i.e. the repartition of its regional minima and maxima).

Finally, homotopic skeletons of sets or functions are build by applying sequential thinnings (associated with rotated structuring elements). The iteration is pursued until the result is stable. Examples of skeletons are presented on £gure 8: the result obtained in the gray-scale case is not satisfactory. The skeleton presents some large ¤at zones bordered by £ne walls ; it does not correspond to median crest line of the objects... Our goal is now to propose a new de£nition of gray-scale skeleton resolving this problem.

4. H-THINNING AND SIMPLIFIED SKELETONS

As said before, the homotopic transforms preserve the function extrema. By thinning, the input and output functions have the same number of extrema. The maxima are thinned while the minima are thickened. This property may become a drawback in real applications since noise and local luminance variations cause the appearance of many spurious regional extrema...

Our goal here is to build an "almost homotopic" thinning where only the significant extrema are preserved while the others are destroyed.

We take as starting point the notion of h-extrema originally introduced proposed by M. Schmidt and F. Prêteux [10]). The high of an extremum being defined as the difference between its altitude and the altitude of its higher neighbor, this extremum is either neglected if this difference is less than h, either preserved if the difference is higher than h (and in that case, it is called an h-extremum).

We want to build an "almost homotopic" thinning where only the h-extrema are preserved. We start by introducing the h-Hit or Miss Transform (h-HMT) of a function f which extracts the points p so that:

- all the points of T_1^p belongs to the same light structure (low luminance variations ($\leq h$) are neglected):

$$\forall y \in T_1^p,\ f(p) - f(y) \leq h \Leftrightarrow f(p) \leq \epsilon_{T_1} f(p) + h$$

- all the points of T_2^p belong to the background:

$$\forall y \in T_2^p,\ f(y) < f(p) \Leftrightarrow \delta_{\tilde{T}_2} f(p) < f(p)$$

It comes the following definition:

$$p \in \text{h-hmt}(f) \Leftrightarrow \delta_{\tilde{T}_2}[f(p)] < f(p) \leq \epsilon_{T_1}[f(p)] + h$$

Then, the h-thinning of a function f is defined as the standard thinning except that the function is thinned at the place of the points extracted by the h-HMT:

$$\forall p \in E,\ \text{h-thin}(f)(p) = \begin{cases} \delta_{\tilde{T}_2}[f(p)] & \text{if } p \in \text{h-hmt}(f) \\ f(p) & else \end{cases}$$

The figures 5 and 6 illustrate the effect of h-thinnings and their non-homotopic behavior. Only the regional maxima of contrast higher than h are preserved. The others are eliminated.

Skeletons based on h-thinning have been experimented on several examples. First, it can be noted that the skeleton obtained in the example "tools" with h-thinnings is very near from the result computed on the binary mask: see figures 7, 8 and 9. Of course, the skeleton computed by h-thinnings being not homotopic, some important connections may be broken: see the figures 10 or 11. This remark points out the limits of our algorithm.

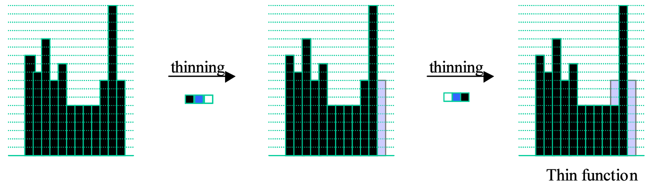

Fig. 5. Standard thinning-based gray-scale skeleton.

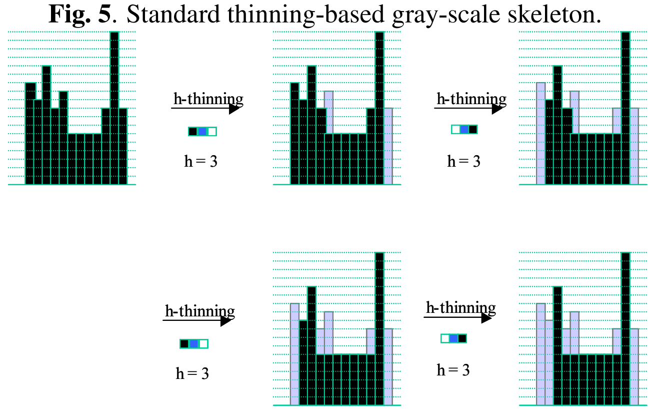

Fig. 6. Gray-scale skeleton based on h-thinnings.

5. CONCLUSION

Numerical skeleton remains a important question in image analysis. Algorithms based on thinnings are very promising even if their use in real application is delicate. In this article, we have proposed to modify the thinning's original definition in order to produce a simplified skeleton and to improve the result when the shapes are not uniformly enlightened.

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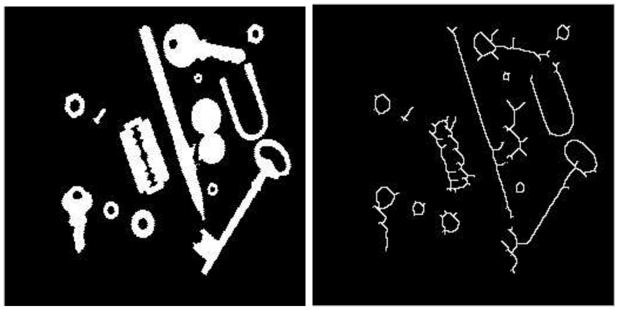

Fig. 7. Binary image and its skeleton by thinning

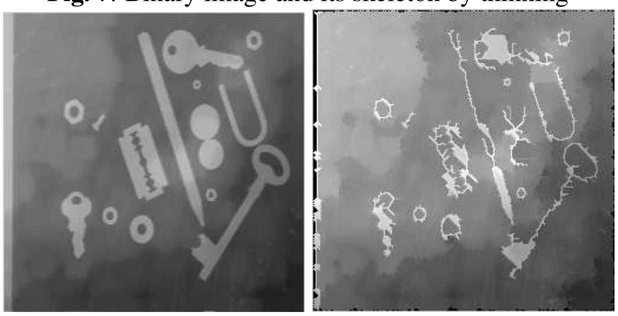

Fig. 8. Gray-scale image and its skeleton by thinning

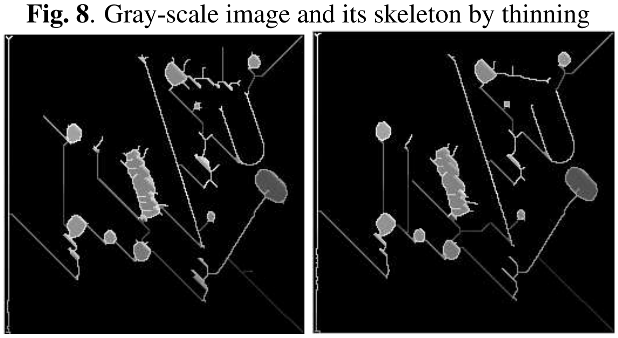

Fig. 9. Gray-scale image and its skeleton by h-thinning with $h = 10$ and $h = 20$.

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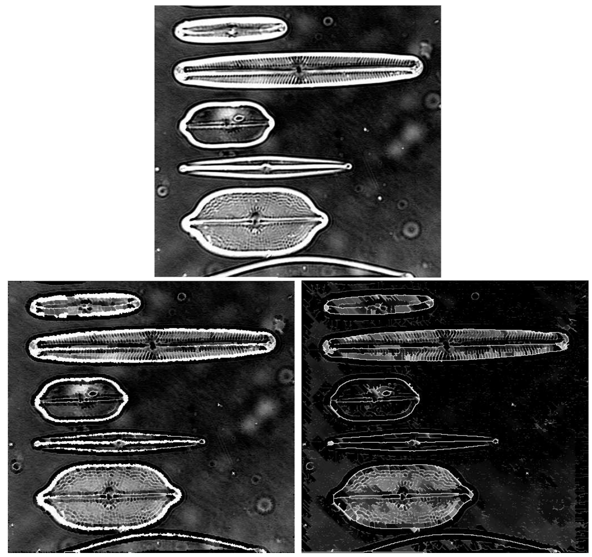

Fig. 10. Original gray-scale image, skeletons by thinning and skeleton by h-thinning ($h = 20$.)

Fig. 11. Original gray-scale image (£ngerprint), skeletons by thinning and skeleton by h-thinning.