

# PERFORMANCE EVALUATION OF BACK-PROJECTION AND RANGE MIGRATION ALGORITHMS IN FOLIAGE PENETRATION RADAR IMAGING

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## ABSTRACT

In this paper, two relatively novel Synthetic Aperture Radar (SAR) imaging techniques, namely back-projection algorithm and range migration algorithm, are discussed. The back-projection algorithm originates from the medical imaging reconstruction technique called computer-aided tomography whereas the range migration algorithm is derived from seismic migration techniques. In this paper, both the back-projection and range migration algorithms are applied to FOLIAGE PENETRATION (FOPEN) SAR imaging and the performance comparisons are made. The simulation and experimental data processing results show that both algorithms are suitable for FOPEN radar imaging and the theoretical performances can be achieved.

## 1. INTRODUCTION

FOLIAGE PENETRATION (FOPEN) imaging is the Synthetic Aperture Radar (SAR) imaging for foliage penetration application to image the targets concealed by foliages, which is of special interests for both military and civilian applications [1]. To reduce the radio energy attenuation induced by the leaves and branches and achieve foliage penetration, the VHF/UHF bands are often selected as the operation frequency of FOPEN radar systems. Therefore, when using large bandwidth waveform (up to several hundreds MHz) to get high range resolution ( $\leq 1\text{m}$ ), the fractional bandwidth of transmitting waveform may be as high as 0.5 or more, i.e., the Ultra-WideBand (UWB). On the other hand, to get high resolution in azimuth as well as range, the integration angle of FOPEN imaging may be more than 60 degrees, which will induce very large range migration. Briefly, using the UWB waveform and large integration angle bring new complexities and challenges for the traditional SAR imaging techniques. Some hypotheses are not valid

and some classical techniques, such as range-Doppler processing technique, cannot be applied at all.

In this paper, two relatively novel SAR imaging algorithms, namely back-projection algorithm and range migration algorithm, are discussed. Originally, the back-projection algorithm was applied in the medical image reconstruction known as computer-aided tomography technique [2]. Also, the range migration algorithm is derived from the seismic migration techniques [3]. In this paper, both of them are applied to FOPEN SAR imaging and their performances are evaluated by using simulation and experimental data. It will be shown that both of the two algorithms are suitable for FOPEN radar imaging where theoretical performances can be achieved.

## 2. SUMMARY OF BACK-PROJECTION ALGORITHM

The back-projection algorithm can be easily derived by starting with the two-dimensional Fourier transform pair. Consider the complex reflectivity function  $g(x, y)$  with its two-dimensional Fourier transform  $G(X, Y)$ , we have

$$g(x, y) = \frac{1}{4\pi^2} \int_L \int G(X, Y) e^{j(xX+yY)} dXdY \quad (1)$$

where  $L$  is the support of the function  $g(x, y)$ . Let  $(\omega, \theta)$  represent the polar coordinates in the  $(X, Y)$  plane. Therefore,  $G(\omega, \theta)$  denotes the values of  $G(X, Y)$  along a line at an angle  $\theta$  with the  $X$  axis. Therefore, expressing  $g(x, y)$  in polar coordinates with  $|\omega|$  being the Jacobian, the reconstructed image  $\tilde{g}(x, y)$  can be expressed as

$$\tilde{g}(x, y) = \int_{-(\theta_m/2)}^{\theta_m/2} \int_{\omega_1}^{\omega_2} G(\omega, \theta) e^{j\omega(x \cos \theta + y \sin \theta)} |\omega| d\omega d\theta \quad (2)$$

According to the projection-slice theorem, we have

$$G(\omega, \theta) = P_\theta(\omega) = \int p_\theta(t) e^{-j\omega t} dt \quad (3)$$

where  $p_\theta(t)$  is the projection of  $g(x, y)$ , i.e., the range profile, at a look angle  $\theta$ . Therefore, the SAR image reconstruction is reduced to solving the following integral

$$\tilde{g}(r, \phi) = \int_{-(\theta_m/2)}^{\theta_m/2} \int_{\omega_1}^{\omega_2} P_\theta(\omega) e^{j\omega r \cos(\theta - \phi)} |\omega| d\omega d\theta \quad (4)$$

where  $(r, \phi)$  represents the polar coordinates in the  $(x, y)$  plane. The processing in Equation (4) can be separated into two steps:

$$Q(t) = \int_{\omega_1}^{\omega_2} P_\theta(\omega) |\omega| e^{j\omega t} d\omega \quad (6)$$

$$g(r, \phi) = \int_{-(\theta_m/2)}^{\theta_m/2} Q(r \cos(\theta - \phi)) d\theta \quad (7)$$

where  $t = r \cos(\theta - \phi)$ . The first step, as shown in Equation (6), can be regarded as a convolution processing between  $p_\theta(t)$ , the projection of  $g(x, y)$  at a look angle  $\theta$ , and a filter whose frequency response is  $|\omega|$ . Therefore, the back-projection algorithm is also called the ‘‘convolution back-projection’’ or ‘‘filtered back-projection’’ algorithm. Subsequently, the final image  $g(r, \phi)$  can be obtained by integrating the function  $Q$  over the look angle  $\theta$ .

Note that in SAR, the radar signal is modulated by a carrier frequency  $\omega_c$ , the transformed term in Equation (6) need to be shifted to the baseband prior to taking the one dimensional inverse Fourier transform.

### 3. SUMMARY OF RANGE MIGRATION ALGORITHM

The range migration algorithm is also known as the wave-number domain processing technique. It works with motion compensation to a straight line, requiring a one dimensional interpolation known as Stolt interpolation and compensating completely the curvature of the wavefront. Therefore, the reconstructed image quality does not suffer from the space variant defocusing and geometric distortion induced by the wavefront curvature. These characteristics make the range migration algorithm particularly attractive for imaging situation including imaging large scenes at fine resolution, imaging at short range, and imaging with a low carrier frequency, such as FOPEN imaging.

The range migration algorithm mainly consists of four processing steps: (1) two dimensional Fourier transform of the range compressed data; (2) two dimensional phase compensation; (3) Stolt interpolation; and (4) two dimensional inverse Fourier transform.

The first step of range migration algorithm is to apply a two dimensional Fourier transform to the range compressed data. In this step, the data is converted to spatial frequency domain  $(R, Y)$ , where  $R, Y$  are the Fourier pairs of range and azimuth variable respectively.

After the two dimensional Fourier transform, the phase compensation is made to partially remove the residual range curvature of scatterers. Therefore, the phase term of received signal changes to

$$\exp[-jx_n \sqrt{R^2 - Y^2} - jy_n Y] \quad (8)$$

where  $(x_n, y_n)$  are the coordinates of pixel to be imaged. Next, the Stolt interpolation is conducted, that fully compensates the range curvature of targets by an appropriate warping of data. The warping is operated by a one dimensional variable changing, where the new variable  $X$  is given by

$$X = \sqrt{R^2 - Y^2} \quad (9)$$

The corresponding phase term becomes

$$\exp[-jx_n X - jy_n Y] \quad (10)$$

This is the kernel of two dimensional Fourier transform and the spatial frequency domain data are converted to the Cartesian coordinates  $(X, Y)$ . Finally, the image can be reconstructed by taking two dimensional inverse Fourier transform.

### 4. SIMULATION RESULTS ANALYSIS

To verify the performance of the above two algorithms, numerical simulations are applied to the following SAR imaging system geometry:

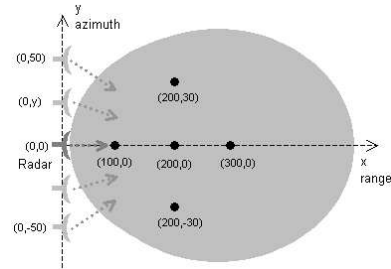


Fig. 1: SAR imaging system geometry

where five ideal scatterers located in the imaging area. Suppose the radar move along from  $-50$  to  $50$  meters in azimuth direction. The transmitting signal is chirp pulse with a center frequency of the  $225$  MHz and bandwidth of  $150$  MHz. Fig. 2 and Fig. 3 show the imaging results by using the back-projection and range migration algorithm respectively. We can see that both of them can successfully reconstruct the image of scatterers.

In order to depict the resolution performance, Fig. 4 shows the cut of image reconstructed by the two algorithms around the scatterer  $(200,0)$  along the range and azimuth direction. It can be found that both the back-projection and range migration algorithms can achieve the theoretical resolution performance in range and azimuth direction.

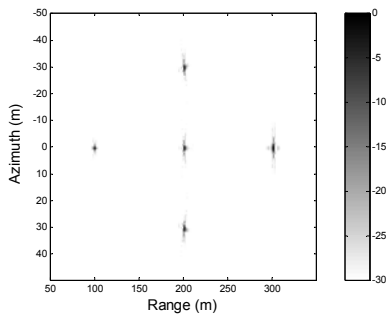


Fig. 2: Simulation result by using back-projection algorithm

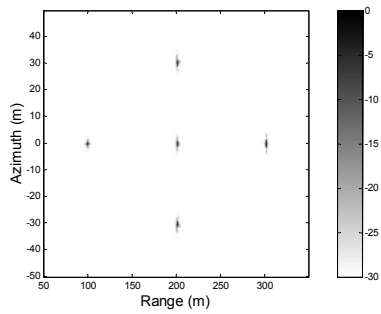


Fig. 3: Simulation result by using range migration algorithm

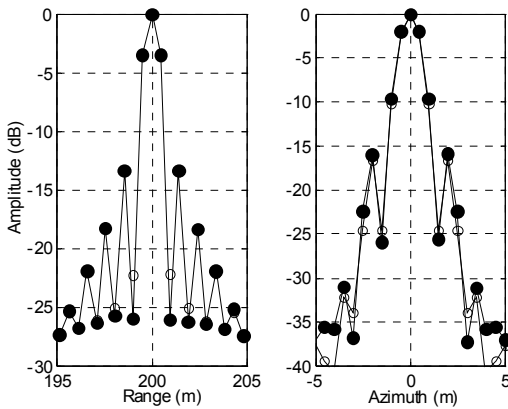


Fig. 4: The cut of reconstructed image around the scatterer (200,0) along range and azimuth direction (-●- : back-projection algorithm; -○- : range migration algorithm)

## 5. EXPERIMENTAL RESULTS ANALYSIS

### 5.1. Introduction of Field Experiment

For the study of FOPEN propagation and SAR imaging, an experimental radar system was developed by a joint group of NTU (Nanyang Technological University, Singapore) and ONERA (Office National d'Etudes et de

Recherches Aérospatiales, France). A series of field experiments had been conducted [4]. This system is based on a SAR device embarked in the basket of a boom lift whose motion can produce the synthetic aperture effect. Such system is also called Boom-SAR. With this system, the FOPEN imaging capability can be achieved by using low frequency bands (VHF/UHF) and the resolution of  $0.5 \times 0.5$  m can be obtained by using a wide band waveform. Fig. 5 shows the photo of experimental radar system and Fig. 6 shows an example area including many regularly planted tropical trees, that is similar to the trial site.



Fig. 5: The photo of experimental Boom-SAR system



Fig. 6: The photo of an example area including many regularly planted tropical trees

### 5.2. Experimental Results

The results presented in this section were obtained for VHF band (150-300 MHz). The crane altitude is 35 m. In this trial, a  $4 \times 4$  m trihedral corner reflector was set up under the foliage as a reference target. Fig. 7 and Fig. 8 show the reconstructed SAR image of a local region of trial site by using the back-projection and range migration algorithms respectively. In both these two images we can find a strong scatterer corresponding to the corner reflector locating under the foliage and many other regular remarkable points corresponding to the trunks.

To precisely analyze the range and azimuth resolution achieved, Fig. 9 shows the cut of corner reflector response in range and azimuth directions respectively when using the back-projection and range migration algorithms. Fig. 10 shows the corresponding results for an arbitrarily selected trunk response. It can be found that for both the corner reflector and trunk, the responses in range and azimuth directions quite coincide and the theoretical resolution can also be achieved when using different algorithm. These results verify both the success of experimental Boom-SAR system and the validity of back-projection and range migration algorithms in the application to FOPEN SAR imaging.

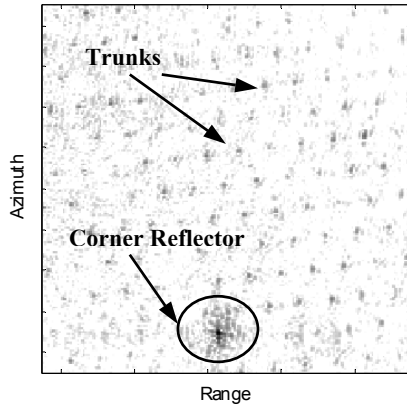


Fig. 7: Experimental result by using back-projection algorithm

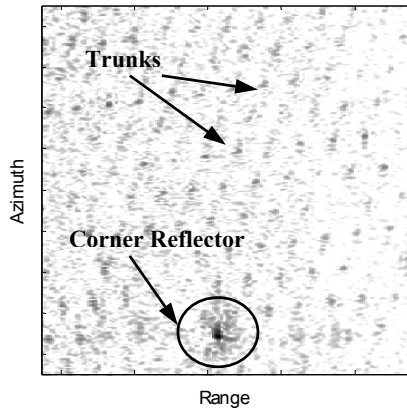


Fig. 8: Experimental result by using range migration algorithm

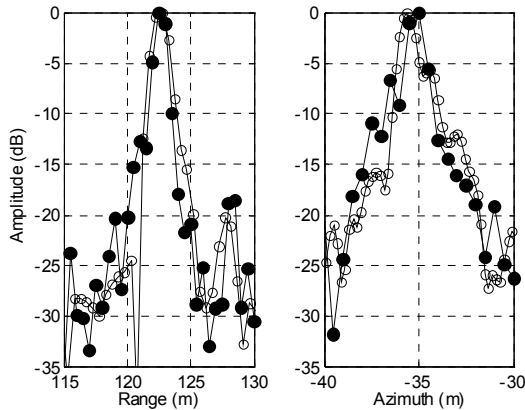


Fig. 9: The cut of corner reflector response in range and azimuth direction (-●- : back-projection algorithm; -○- : range migration algorithm)

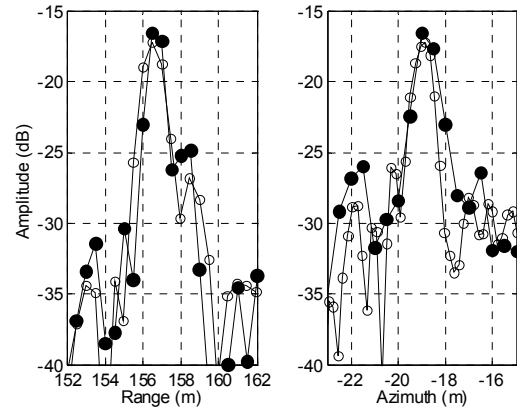


Fig. 10: The cut of trunk response in range and azimuth direction (-●- : back-projection algorithm; -○- : range migration algorithm)

## 6. CONCLUSION

In this paper, the simulation and experimental data processing results prove that both the back-projection and range migration algorithms can be applied in FOPEN SAR imaging when using UWB waveform and large integration angle, and the theoretical resolution performance can also be achieved. More theoretical analysis and comparison of these two algorithms will be made in the future paper.

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## 7. REFERENCES

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