CCA BASED ALGORITHMS FOR BLIND EQUALIZATION OF FIR MIMO SYSTEMS

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ABSTRACT

In this work the problem of blind equalization of multiple-input multiple-output (MIMO) systems is formulated as a set of canonical correlation analysis (CCA) problems. CCA is a classical tool that finds maximally correlated projections of several data sets, and it is typically solved using eigendecomposition techniques. Recently, it has been shown that CCA can be alternatively viewed as a set of coupled least squares regression problems, which can be solved adaptively using a recursive least squares (RLS) algorithm. Unlike other MIMO blind equalization techniques based on second-order statistics (SOS), the CCA-based algorithms does not require restrictive conditions on the spectral properties of the source signals. Some simulation results are presented to demonstrate the potential of the proposed CCA-based algorithms.

1. INTRODUCTION

Blind equalization of FIR MIMO channels is a common problem encountered in wireless and mobile communications. Although some higher-order statistics methods have been successfully developed [1], it is well known that, under mild assumptions on the source signals and the FIR channels, second-order statistics (SOS) are sufficient for blind equalization [2, 3]. However, a common drawback of all these methods is their restrictive conditions on the spectral properties of the source signals.

In this paper we consider the application of canonical correlation analysis (CCA) to blind equalization of FIR MIMO channels. CCA is a well-known technique in multivariate statistical analysis, which has been widely used in economics, meteorology, and in many modern signal processing problems [4]. It was developed by H. Hotelling [5] as a way of measuring the linear relationship between two multidimensional sets of variables and was later extended to several data sets [6].

Recently, the reformulation of CCA as a set of coupled least squares (LS) regression problems has been exploited to derive an adaptive RLS-CCA algorithm [7,8]. Furthermore, in [7,9] we have shown that maximizing the correlation among the outputs of the equalizers (i.e. CCA) provides a good equalization criterion for single-input multiple-output (SIMO) systems. In this work, we extend these ideas to the blind equalization of FIR MIMO systems, which is reformulated as a set of CCA problems. Provided that the lengths of the FIR channels are known, the source signals can be extracted unambiguously by a set of nested CCA problems. The constraint on the lengths of the channels avoids the requirement of colored input signals with different spectra in [10], which is very restrictive in practical wireless systems. Based on these ideas, we propose batch and adaptive equalization algorithms and evaluate their performance in comparison to other blind FIR-MIMO equalization techniques.

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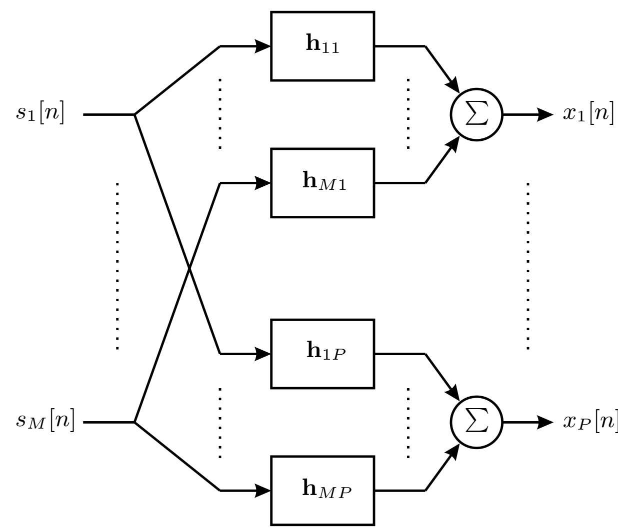

Fig. 1. MIMO system.

2. BLIND MIMO EQUALIZATION

Suppose the multiple-input multiple-output (MIMO) system shown in Fig. 1, where the signals $x_1[n], \ldots, x_P[n]$, are the outputs of an unknown M-input/P-output finite impulse response (FIR) system driven by M signals $s_1[n], \ldots, s_M[n]$. The channel input-output relationship can be expressed as

$$\tilde{\mathbf{x}}[n] = H[n] * \tilde{\mathbf{s}}[n] = \sum_{l=0}^{\infty} H[l]\tilde{\mathbf{s}}[n-l], \qquad (1)$$

where $\tilde{\mathbf{x}}[n] = [x_1[n], \ldots, x_P[n]]^T$, $\tilde{\mathbf{s}}[n] = [s_1[n], \ldots, s_M[n]]^T$, $\mathbf{h}_i[n] = [h_{i1}[n], \ldots, h_{iP}[n]]^T$, and $H[n] = [\mathbf{h}_1[n] \cdots \mathbf{h}_M[n]]$ is the channel response matrix, which is associated to the transfer function

$$\mathbf{H}(z) = \sum_{l=0}^{\infty} H[l] z^{-l}.$$

Denoting the length of the subchannel $\mathbf{h}_{ij}$ as L_{ij}, and considering the M single-input multiple-output (SIMO) channel lengths

$L_i = \max(L_{i1}, \ldots, L_{iP})$, we can rewrite (1) as

$$\mathbf{x}[n] = \sum_{i=1}^{M} \mathcal{T}(\mathbf{h}_i)\mathbf{s}_i[n] = \mathcal{T}\mathbf{s}[\mathbf{n}],$$

where

$$\mathbf{x}[n] = \left[\tilde{\mathbf{x}}^T[n], \ldots, \tilde{\mathbf{x}}^T[n-K+1]\right]^T,$$
$$\mathbf{s}_i[n] = [s_i[n], \ldots, s_i[n-K-L_i+2]]^T,$$
$$\mathbf{s}[n] = \left[\mathbf{s}_1^T[n], \ldots, \mathbf{s}_M^T[n]\right]^T,$$
$$\mathcal{T}(\mathbf{h}_i) = \begin{bmatrix} \mathbf{h}_i[0] & \cdots & \mathbf{h}_i[L_i-1] & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{h}_i[0] & \cdots & \mathbf{h}_i[L_i-1] \end{bmatrix},$$
$$\mathcal{T} = [\mathcal{T}(\mathbf{h}_1) \cdots \mathcal{T}(\mathbf{h}_M)],$$

and K is a parameter determining the dimensions of vectors and matrices. This formulation of the MIMO model, which is known as the slide-window formulation [3], can be used to develop a new equalization procedure based on the following conditions.

Condition 1 (MIMO Channel)

a) $P \geq M$.

b) The SIMO channel lengths $L_1, \ldots, L_M$ *are known.*

c) $\mathbf{H}(z)$ *is irreducible and column reduced.*

Condition 2 (Equalizer length) *The parameter* K *satisfies*

$$K \geq \frac{L-M}{P-M} \qquad \textit{with} \quad L = \sum_{i=1}^{M} L_i.$$

Condition 3 (Source signals) *The source signals are uncorrelated and* $(K+1)$*-th order persistently exciting.*

Based on conditions 1 and 2, it can be proved that $\mathcal{T}$ is a full column rank matrix [3], and then there exists a set of M matrices $\mathbf{W}_i$ of size $PK \times N_i$, with $N_i = K + L_i - 1$, such that

$$[\mathbf{W}_1 \cdots \mathbf{W}_M]^T \mathcal{T} = \mathbf{W}^T\mathcal{T} = c\mathbf{I},$$

where c is some nonzero constant. Denoting the k-th column of $\mathbf{W}_i$ as $\mathbf{w}_{ik}$ we can write, for $i = 1, \ldots, M$ and $k, l = 1, \ldots, N_i$

$$\mathbf{w}_{ik}^T\mathbf{x}[n+k-1] = \mathbf{w}_{il}^T\mathbf{x}[n+l-1]. \tag{2}$$

A good review of previously proposed methods for blind equalization of FIR MIMO systems can be found in [2, 3]. Some examples are the MIMO constant modulus algorithm (MIMO-CMA) [1], which is based on higher order statistics, and the column anchored zero forcing equalizers (CAZE) [2], which requires white source signals. Furthermore, the blind identification via decorrelation procedure presented in [3, 10] assumes source signals with different spectral properties to be able to work with less restrictive channel conditions.

The new equalization algorithms proposed in this paper avoid the restrictions on the spectral properties of the source signals, which are replaced by the condition 1.b. This condition requires the previous knowledge or estimation [11] of the SIMO channel lengths. Finally, we must point out that, under the proposed assumptions, multiple sources affected by channels of the same length can only be estimated up to a rotation (unitary) matrix, which is a common indeterminacy in blind MIMO equalization problems [2]. This indeterminacy can be solved by exploiting the finite alphabet property of the input signals [2], using higher order statistics, or assuming source signals with different spectral properties [3].

3. CCA-BASED BLIND MIMO EQUALIZATION

Let us start considering, without loss of generality, $L_1 \geq \ldots \geq L_M$, and defining the following data sets, for $i = 1, \ldots, M$, $k = 1, \ldots, N_1$ and $j = 1, \ldots, i$

$$\mathbf{S}_i[n] = [\mathbf{s}_i[n] \cdots \mathbf{s}_i[n+N-1]]^T,$$
$$\mathbf{X}_k[n] = [\mathbf{x}[n+k-1] \cdots \mathbf{x}[n+k+N-2]]^T,$$
$$\mathbf{Z}_{ij}[n] = [\mathbf{z}_j[n] \cdots \mathbf{z}_j[n-L_j+L_i]],$$
$$\mathbf{Z}_i[n] = [\mathbf{Z}_{i1}[n] \cdots \mathbf{Z}_{ii}[n]],$$

where $\mathbf{z}_i[n] = [s_i[n], \ldots, s_i[n+N-1]]^T$ is the desired output of the i-th equalizer. Based on these definitions, Eq. (2) can be rewritten as

$$\mathbf{X}_k[n]\mathbf{w}_{ik} = \mathbf{X}_l[n]\mathbf{w}_{il}, \qquad k, l = 1, \ldots, N_i. \tag{3}$$

In the Appendix we prove the following theorem

Theorem 1 *If the source signals are uncorrelated and* $(K+1)$*-th persistently exciting then, in the limiting case of* $N \to \infty$*, the solutions of (3) satisfy*

$$\mathbf{X}_k[n]\mathbf{w}_{ik} = \mathbf{Z}_i[n]\mathbf{a}_i, \qquad \textit{for} \quad i = 1, \ldots, M, \tag{4}$$

where $\mathbf{a}_i$ *is an arbitrary vector.*

Taking into account the channel noise, the solution of (3) can be obtained by minimizing

$$J_i = \sum_{k,l=1}^{N_i} \|\hat{\mathbf{z}}_{ik}[n] - \hat{\mathbf{z}}_{il}[n]\|^2,$$

subject to the nontrivial restriction

$$\frac{1}{N_i}\sum_{k=1}^{N_i} \mathbf{w}_{ik}^H\mathbf{R}_{kk}\mathbf{w}_{ik} = 1, \tag{5}$$

where $\hat{\mathbf{z}}_{ik}[n] = \mathbf{X}_k[n]\mathbf{w}_{ik}$ is the k-th estimate of $\mathbf{z}_i[n]$, and $\mathbf{R}_{kl} = \mathbf{X}_k^H[n]\mathbf{X}_l[n]$ is an estimate of the crosscorrelation matrix. Defining

$$\hat{\mathbf{z}}_i[n] = \frac{1}{N_i}\sum_{k=1}^{N_i} \hat{\mathbf{z}}_{ik}[n],$$

as the final estimate of $\mathbf{z}_i[n]$, we can avoid the interference between different source signals by means of the additional orthogonality restrictions

$$\hat{\mathbf{Z}}_i^H[n]\hat{\mathbf{z}}_i[n] = [0, \ldots, 0, \beta_i], \tag{6}$$

where $\hat{\mathbf{Z}}_i[n]$ denotes the estimate of $\mathbf{Z}_i[n]$ with the possible effect of a scale and unitary matrix indeterminacy, and $\beta_i = \|\hat{\mathbf{z}}_i[n]\|^2$. In this way, the above problem is equivalent to maximize

$$\beta_i = \frac{1}{N_i^2}\sum_{k,l=1}^{N_i} \hat{\mathbf{z}}_{ik}^H[n]\hat{\mathbf{z}}_{il}[n],$$

Initialize $i = 1$ and arrange $L_1 \geq \ldots \geq L_M$.
while $i \leq M$ **do**
Obtain the number of signals to extract N_{s_i}.
Obtain the number of restrictions to apply $N_{\perp_i}$.
Form the CCA problem with $\mathbf{X}_1[n], \ldots, \mathbf{X}_{N_i}[n]$.
Consider the $N_{\perp_i}$ main solutions $\left[\hat{\mathbf{Z}}_{i1}[n] \cdots \hat{\mathbf{Z}}_{i(i-1)}[n]\right]$.
Obtain the next N_{s_i} solutions $\hat{\mathbf{z}}_i[n], \ldots, \hat{\mathbf{z}}_{i+N_{s_i}-1}[n]$.
Update $i = i + N_{s_i}$.
end while

Algorithm 1: Summary of the CCA procedure for blind equalization of FIR MIMO channels.

subject to (5) and (6). This problem is the generalization of canonical correlation analysis to several complex data sets proposed in [7], which is equivalent to the Maximum Variance (MAXVAR) generalization [6].

Solving the CCA problem by the method of Lagrange multipliers we obtain the following GEV problem

$$\frac{1}{N_i}\mathbf{R}_i\mathbf{w}_i = \beta_i\mathbf{D}_i\mathbf{w}_i, \qquad (7)$$

where $\mathbf{w}_i = [\mathbf{w}_{i1}^T, \ldots, \mathbf{w}_{iN_i}^T]^T$,

$$\mathbf{R}_i = \begin{bmatrix} \mathbf{R}_{11} & \cdots & \mathbf{R}_{1N_i} \\ \vdots & \ddots & \vdots \\ \mathbf{R}_{N_i1} & \cdots & \mathbf{R}_{N_iN_i} \end{bmatrix}, \quad \mathbf{D}_i = \begin{bmatrix} \mathbf{R}_{11} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{R}_{N_iN_i} \end{bmatrix}.$$

Then, the main CCA solution is obtained as the eigenvector associated to the largest eigenvalue of (7), and the remaining eigenvectors and eigenvalues are the subsequent solutions of the CCA problem. Furthermore, by noting that $\mathbf{R}_{kk}^{-1}\mathbf{R}_{kl} = \mathbf{X}_k[n]^+\mathbf{X}_l[n]$, where $\mathbf{X}_k^+[n]$ is the pseudoinverse of $\mathbf{X}_k[n]$, the GEV problem (7) can be viewed as N_i coupled LS regression problems

$$J_{ik} = \|\hat{\mathbf{z}}_i[n] - \beta_i\mathbf{X}_k[n]\mathbf{w}_{ik}\|^2, \qquad k = 1, \ldots, N_i,$$

and then, the CCA solution can be obtained by solving these LS regression problems iteratively [7], which is equivalent to the well-known power method to extract the eigenvectors and eigenvalues of (7). In [7] we have developed an on-line procedure based on the direct application of the RLS algorithm to these N_i LS regression problems.

A direct consequence of (4) is that, in the noiseless case, $\mathbf{Z}_i[n]$ constitutes a basis for the subspace defined by the CCA solutions with canonical correlation equal to 1, of the CCA problem with data sets $\mathbf{X}_k[n]$, for $k = 1, \ldots, N_i$. Then, from this CCA problem we can extract, up to an unitary matrix, N_{s_i} source signals, where N_{s_i} is the number of inputs affected by a SIMO channel with length L_i. To this end, the previously extracted signals $\left[\hat{\mathbf{Z}}_{i1}[n] \cdots \hat{\mathbf{Z}}_{i(i-1)}[n]\right]$ are considered as the $N_{\perp_i}$ main solutions of the CCA problem, where

$$N_{\perp_i} = \sum_{j=1}^{i-1}(L_j - L_i + 1)$$

is the number of orthogonality restrictions to apply, and $\hat{\mathbf{Z}}_{ij}[n]$ denotes the estimate of $\mathbf{Z}_{ij}[n]$. Then, the N_{s_i} source signals are obtained as the subsequent N_{s_i} CCA solutions.

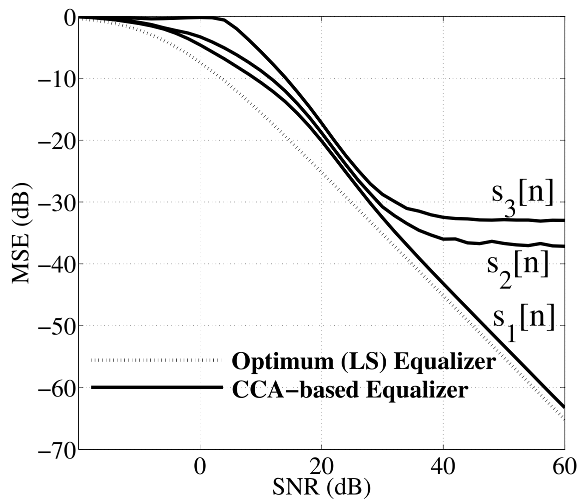

Fig. 2. Performance of the batch algorithm.

Algorithm 1 summarizes the batch procedure for the sequential extraction of the source signals. It is interesting to point out that, in the limiting case of $L_1 = \ldots = L_M = 1$, the CCA based procedure is equivalent to a prewhitening step, which is typically used in Blind Source Separation (BSS) and Independent Component Analysis (ICA) algorithms. Furthermore, the direct application of the RLS based adaptive CCA algorithm presented in [7] is used in the paper to develop an adaptive version of the MIMO blind equalization algorithm. Finally, straightforward modifications of this procedure can be used to develop blind equalization algorithms robust to outliers or impulsive noise [8], or to exploit the finite alphabet property of the source signals by means of a soft decision procedure [9].

4. SIMULATION RESULTS

In this section the performance of the proposed algorithms is evaluated. In all the simulations the results of 300 independent realizations are averaged. We consider 16-QAM source signals distorted by a MIMO channel and corrupted by zero-mean white Gaussian noise. The equalizer length parameter is selected as

$$K = \left\lceil \frac{L - M}{P - M} \right\rceil.$$

In the first example we compare the performance of the batch CCA procedure with the optimum equalizer, i.e. the informed (not blind) LS equalizer. The MIMO channel is a 3-input/6-output system with lengths $L_1 = 6$, $L_2 = 5$ and $L_3 = 4$; the channel impulse response is shown in Table 1. The performance of the algorithms as a function of the signal to noise ratio (SNR) is shown in Fig. 2. We can see that the equalization procedure for the first signal is close to the optimum, whereas the remaining signals show a noise floor that is due to the strict orthogonality constraints among the extracted signals and their delayed versions. This noise floor decreases when N increases ($N = 10000$ in Fig. 2).

In the second example we compare the performance of the adaptive algorithm with the MIMO-CMA [1]. Here, a 2×4 MIMO

i	n	$\mathbf{h}_{i1}[n]$	$\mathbf{h}_{i2}[n]$	$\mathbf{h}_{i3}[n]$	$\mathbf{h}_{i4}[n]$	$\mathbf{h}_{i5}[n]$	$\mathbf{h}_{i6}[n]$
1	0	$-1.223 + 0.896j$	$0.482 - 0.771j$	$-0.766 - 0.883j$	$0.961 + 0.415j$	$1.633 - 0.806j$	$0.855 + 0.076j$
	1	$1.669 + 0.438j$	$1.518 + 0.530j$	$1.484 - 0.692j$	$0.747 - 0.280j$	$0.709 + 0.683j$	$-0.119 + 0.132j$
	2	$-0.661 + 0.452j$	$-0.080 - 1.249j$	$0.269 + 0.509j$	$0.601 - 0.923j$	$1.010 - 1.120j$	$2.807 - 0.453j$
	3	$-0.089 - 1.179j$	$-0.453 + 0.939j$	$0.306 + 0.179j$	$1.688 - 0.741j$	$-0.153 - 0.669j$	$-0.136 + 2.480j$
	4	$0.311 + 0.516j$	$-1.126 - 0.403j$	$-1.707 - 0.045j$	$-0.658 + 0.547j$	$-0.723 - 0.781j$	$-1.212 + 0.833j$
	5	$-0.880 - 1.457j$	$-0.233 - 0.617j$	$-1.816 - 0.575j$	$1.734 - 0.749j$	$-1.367 + 1.217j$	$0.989 - 1.057j$
2	0	$1.749 + 0.811j$	$-0.527 + 0.213j$	$1.741 + 0.248j$	$-1.856 + 1.585j$	$-0.001 + 0.678j$	$1.801 + 0.197j$
	1	$0.133 + 0.699j$	$0.932 + 0.788j$	$0.487 + 0.060j$	$2.134 + 0.916j$	$-0.268 - 0.587j$	$-0.669 + 0.277j$
	2	$0.325 - 0.402j$	$1.165 + 0.897j$	$1.049 + 1.377j$	$1.436 - 0.557j$	$-0.983 - 0.520j$	$0.785 - 0.594j$
	3	$-0.794 + 1.269j$	$-2.046 - 0.187j$	$1.489 - 1.083j$	$-0.917 + 0.230j$	$-2.796 - 1.154j$	$0.629 - 0.019j$
	4	$0.315 - 0.784j$	$-0.644 + 1.013j$	$1.271 + 1.035j$	$-1.106 + 0.475j$	$-0.681 - 1.297j$	$-1.878 - 1.267j$
3	0	$0.192 - 0.404j$	$2.796 - 0.689j$	$-0.808 + 3.101j$	$-0.883 - 1.908j$	$-0.742 + 0.149j$	$-0.313 - 0.490j$
	1	$1.224 - 1.079j$	$-1.732 + 1.060j$	$-0.615 - 1.772j$	$-0.242 - 0.689j$	$1.099 + 1.094j$	$0.198 + 0.955j$
	2	$1.769 + 0.691j$	$0.008 + 0.614j$	$-0.775 - 0.892j$	$0.887 + 0.105j$	$-0.772 - 0.368j$	$0.465 - 0.076j$
	3	$1.359 + 0.451j$	$-1.040 + 0.934j$	$0.257 - 1.083j$	$-0.672 + 0.171j$	$-0.530 - 0.456j$	$2.762 + 2.176j$

Table 1. Impulse responses of the MIMO channel used in the simulation examples.

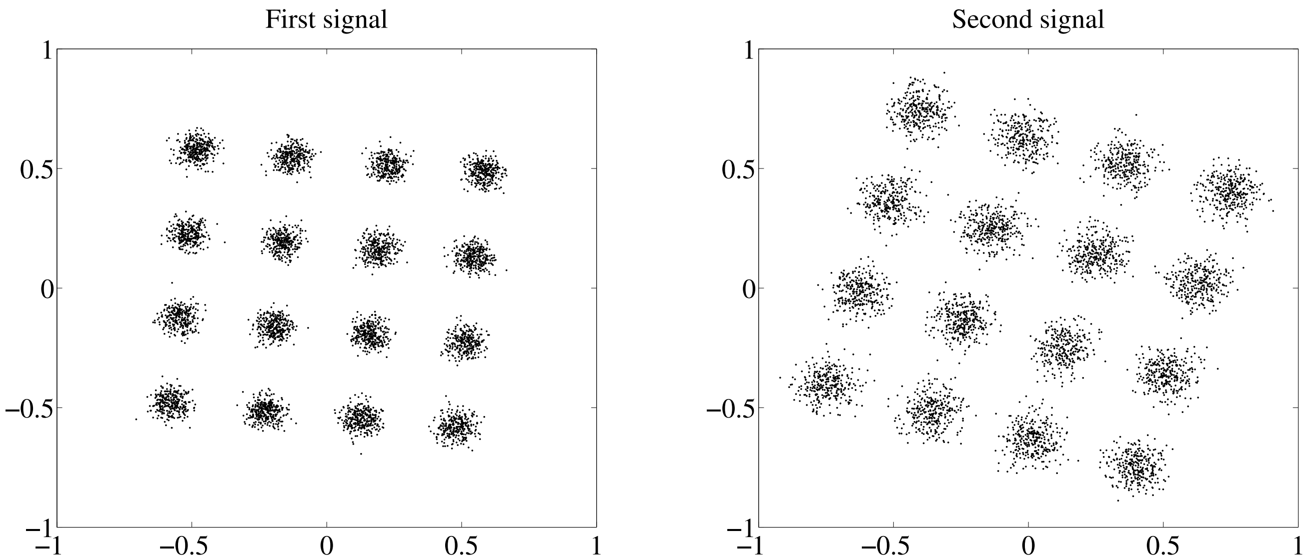

Fig. 3. Performance of the adaptive algorithm in a 2×4 MIMO system. Constellations of the equalized signals after 20000 iterations. SNR=30dB, $\lambda = 0.99$.

channel with $L_1 = 6$ and $L_2 = 5$ is constructed from the two first source signals and four first observations of Table 1. The RLS forgetting factor is $\lambda = 0.99$ and the signal to noise ratio is SNR=30dB. We consider separately the ISI and the co-channel interference, which can be added to obtain the total interference defined in [2]. The simulations have shown the strong dependency of the initialization parameters for the MIMO-CMA, which have failed in the extraction of the second signal in 24 out of the 300 independent experiments. Fig. 3 shows the eye pattern of the proposed adaptive algorithm after 20000 iterations and Fig. 4 shows the convergence for the two source signals. As we can see, the CCA-RLS converges faster than the MIMO-CMA, and, analogously to the batch case, the co-channel interference in the second signal depends on the RLS forgetting factor.

5. CONCLUSIONS

In this paper, new batch and adaptive algorithms for FIR MIMO blind equalization have been presented. The algorithms are based on the reformulation of the blind MIMO equalization procedure as a set of nested CCA problems. These CCA problems can be rewritten as different sets of coupled LS problems, which can be solved adaptively by means of the RLS. Unlike other SOS MIMO blind equalization methods, the CCA-based algorithms do not require strict conditions on the spectral properties of the source signals. By means of some simulation results we have shown that the proposed CCA algorithms consistently outperform other blind FIR-MIMO equalization techniques.

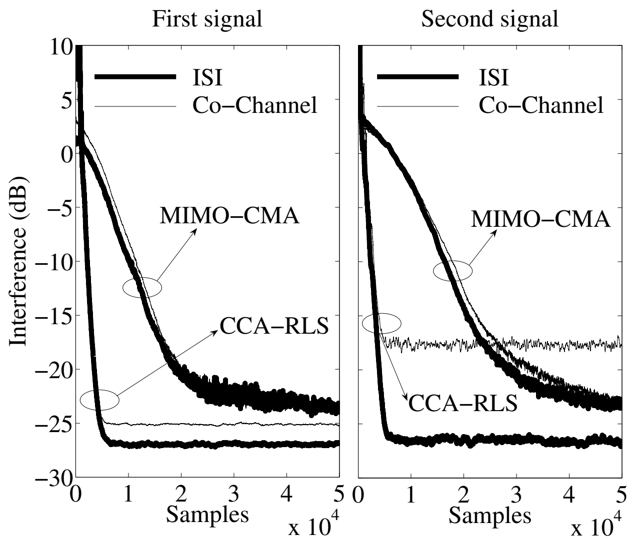

Fig. 4. Performance of the adaptive algorithm.

APPENDIX
PROOF OF THEOREM 1

In this appendix we show that, if the source signals are uncorrelated and $(K+1)$-th order persistently exciting, in the limiting case of $N \rightarrow \infty$, the solutions of (3) will satisfy (4) as well. Let us start rewriting (3) and (4), for $i = 1, \ldots, M$ and $k, l = 1, \ldots, N_i$, as

$$\sum_{j=1}^{M} \mathbf{S}_j[n+k-1]\mathbf{g}_{ikj} = \sum_{j=1}^{M} \mathbf{S}_j[n+l-1]\mathbf{g}_{ilj}, \qquad (8)$$

$$\sum_{j=1}^{M} \mathbf{S}_j[n+k-1]\mathbf{g}_{ikj} = \sum_{j=1}^{i} \mathbf{Z}_{ij}[n]\mathbf{a}_{ij}, \qquad (9)$$

respectively, where $\mathbf{a}_i = \left[\mathbf{a}_{i1}^T, \ldots, \mathbf{a}_{ii}^T\right]^T$ and $\mathbf{g}_{ikj} = \mathcal{T}^T(\mathbf{h}_j)\mathbf{w}_{ik}$ is the composite channel-equalizer response. Considering that, for uncorrelated signals and $i \neq j$

$$\lim_{N\rightarrow\infty} \frac{1}{N}\mathbf{S}_i^H[n+k-1]\mathbf{S}_j[n+l-1] = \mathbf{0},$$

the equality (8) can be decoupled and rewritten as

$$\mathbf{S}_j[n+k-1]\mathbf{g}_{ikj} = \mathbf{S}_j[n+l-1]\mathbf{g}_{ilj},$$

for $i = 1, \ldots, M$ and $k, l = 1, \ldots, N_i$, or equivalently

$$\mathbf{S}_j[n+k-1]\mathbf{g}_{ikj} = \mathbf{S}_j[n+k]\mathbf{g}_{i(k+1)j}, \qquad k = 1, \ldots, N_i - 1.$$

Taking into account that a sequence $s_i[n]$ is *K-th order persistently exciting* if and only if $\mathbf{S}_i[n]$ is of full column rank for some N, it can be deduced that the $K + L_j$ different columns in $\mathbf{S}_j[n+k-1]$ and $\mathbf{S}_j[n+k]$ are linearly independent and then, for $i, j = 1, \ldots, M$

$$g_{ikj}[l] = g_{i(k+1)j}[l+1],$$
$$g_{ikj}[K+L_j-1] = g_{i(k+1)j}[1] = 0,$$

which implies

$$g_{ikj}[l] = \begin{cases} a_{ij}[l-k] & \text{if} \quad 0 \leq (l-k) \leq (L_j - L_i), \\ 0 & \text{otherwise,} \end{cases}$$

where $a_{ij}[l-k]$ is some constant. Finally, it is straightforward to prove that the above equation yields, for $i = 1, \ldots, M$, $j = 1, \ldots, i$ and $k = 1, \ldots, N_i$

$$\mathbf{S}_j[n+k-1]\mathbf{g}_{ikj} = \mathbf{Z}_{ij}[n]\mathbf{a}_{ij},$$

where $\mathbf{a}_{ij} = [a_{ij}[0], \ldots, a_{ij}[L_j - L_i]]^T$. This implies (9) and concludes the proof.

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