BAYESIAN MODEL SELECTION FOR MULTISENSOR TRACK-TO-TRACK ASSOCIATION AND TRACK FUSION

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1. INTRODUCTION

This paper considers the problem of associating tracks represented by their local state estimates and covariances from M sources. These sources are processors that use data from the corresponding local sensor systems. While different sensors may have independent measurement errors, the local state estimation errors for the same target are in general correlated due to e.g., the common prior or process noise. This dependence is characterized by the crosscovariances of the local estimation errors [3]. The track association is traditionally treated as a hypothesis testing problem where local tracks are considered as from the same target if their kinematic states are close. By this token, a normalized distance is computed and compared with a threshold to obtain the desired test power [2]. However, there is in general no control of the misassociation probability, i.e., local tracks from different targets being considered as from the same target. In this paper we first consider the test of whether multiple local tracks are from the same target and derive a Bayesian test which, under the assumption of noninformative prior of the target state, coincides with the test obtained in [14, 2]. When M local track sources need to be associated with more than one target, we use the likelihood ratio test statistic to construct the cost function suitable for the use of a multidimensional assignment [3] to handle the general track association problem.

When local tracks are declared as from the same target, the local state estimates need to be fused to yield a more accurate global track. The optimal fusion rule within linear unbiased fusers has been studied in [3, 6, 12] and its performance limit compared with the centralized tracker in [6]. It is well known that the optimal linear fusion rule also yields the minimum mean square error under the linear Gaussian assumption [4]. However, if noninformative prior for the target state is assumed at the fusion center, then the optimal fusion rule in terms of minimizing the mean square error becomes nonlinear. We consider a regularized track fusion rule which is a biased estimator but has a smaller mean square estimation error than the optimal unbiased one. We use a three-sensor two-target tracking scenario to study the performance of the Bayesian track association and the regularized track fusion algorithm. Each sensor uses the state-of-the-art interacting multiple model (IMM) estimator to track the targets and occasionally sends the local track estimates to the fusion center. Both the estimation accuracy and the credibility of the distributed tracker are compared with those of the centralized one. The results show that the performance degradation is fairly small even during target maneuvers.

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2. ASSOCIATION OF MULTIPLE TRACKS TO A SINGLE TARGET

2.1. Bayesian Interpretation of the Test

Consider the problem of associating M local tracks from M local sensor systems. Each tracker provides the local target state estimate and possible target class information, i.e., target identity. We are interested in testing whether these M local tracks are from the same target. We first assume that only the target state information of M local trackers is available. Consider the hypothesis
H_0 : *all track estimates are from the same target.*
Traditional approaches reject H_0 only when the computed test statistic falls outside a predetermined confidence region, say an interval with 95% probability. However, there is no control of the misassociation probability since the alternative hypothesis is unspecified. Here we consider a typical alternative given by
H_1 : *all track estimates are from different targets.*
We use a representative hypothesis for H_1 which is simple and the likelihood function does not depend on the target state. Denote by $\hat{x}_i$ the local estimate from the i-th sensor. It is assumed that the estimation error is Gaussian with zero mean and covariance P_i. This is often the case when each local tracker uses a Kalman filter to recursively estimate the target state. The state prediction at the fusion center can also

be used as the prior if the fusion center assumes a kinematic model to propagate the target state [13]. However, the prediction error is also correlated with the state estimation error of each local tracker. The crosscovariance matrix is not readily available at the fusion center. To simplify the analysis, we assume that the target state has a noninformative prior, i.e., $p(x) = \frac{1}{V}$, $|V| \to \infty$, and thus $P_0^{-1} \to 0$. The likelihood ratio between the two hypotheses can be written as

$$\Lambda^*(\hat{\mathbf{x}}) = \frac{p(\hat{\mathbf{x}}|H_0)}{p(\hat{\mathbf{x}}|H_1)} = V^{M-1} \int_{\mathcal{V}} \mathcal{N}(\hat{\mathbf{x}}; Hx, \mathbf{P}) dx \tag{1}$$

and by completing the quadratic form, the log-likelihood ratio becomes

$$\ln\Lambda^*(\hat{\mathbf{x}}) = -\frac{1}{2}\hat{\mathbf{x}}'[\mathbf{P}^{-1} - \mathbf{P}^{-1}H(H'\mathbf{P}^{-1}H)^{-1}H'\mathbf{P}^{-1}]\hat{\mathbf{x}} + c^* \tag{2}$$

where

$$c^* = (M-1)\ln V - \frac{1}{2}\ln|2\pi\mathbf{P}| + \frac{1}{2}\ln|2\pi(H'\mathbf{P}^{-1}H)^{-1}| \tag{3}$$

If H_0 is accepted, the fused state estimate is

$$\bar{x}^{\mathrm{LS}} = (H'\mathbf{P}^{-1}H)^{-1}H'\mathbf{P}^{-1}\hat{\mathbf{x}} \tag{4}$$

with the covariance matrix given by

$$P^{\mathrm{LS}} = (H'\mathbf{P}^{-1}H)^{-1} \tag{5}$$

It is the weighted least squares solution to the problem

$$\min_x (\hat{\mathbf{x}} - Hx)'\mathbf{P}^{-1}(\hat{\mathbf{x}} - Hx) \tag{6}$$

The above fusion rule is also optimal in the maximum likelihood sense [6].

Another important point is that the log-likelihood ratio test statistic (1) can also be written as

$$\ln\Lambda^*(\hat{\mathbf{x}}) = -\frac{1}{2}(\hat{\mathbf{x}} - H\bar{x}^{\mathrm{LS}})'\mathbf{P}^{-1}(\hat{\mathbf{x}} - H\bar{x}^{\mathrm{LS}}) + c^* \tag{7}$$

Thus the hypothesis testing becomes a goodness-of-fit term being compared with a threshold. Under H_0, the test statistic

$$T_{\mathbf{x}}(\hat{\mathbf{x}}) = (\hat{\mathbf{x}} - H\bar{x}^{\mathrm{LS}})'\mathbf{P}^{-1}(\hat{\mathbf{x}} - H\bar{x}^{\mathrm{LS}}) \tag{8}$$

is chi-square distributed with $(M-1)n_x$ degrees of freedom where n_x is the dimension of target state x.

Theorem: For any $(M-1)n_x \times Mn_x$ matrix $\mathbf{K}$ with full row rank that satisfies $\mathbf{K}H = \mathbf{0}$ and $\hat{\mathbf{y}} = \mathbf{K}\hat{\mathbf{x}}$, the test statistic $T_{\mathbf{y}}(\hat{\mathbf{y}})$ is chi-square distributed with $(M-1)n_x$ degrees of freedom and $T_{\mathbf{y}}(\hat{\mathbf{y}}) = T_{\mathbf{x}}(\hat{\mathbf{x}})$.

The proof is sketched as follows. By definition we have

$$T_{\mathbf{y}}(\hat{\mathbf{y}}) = \hat{\mathbf{y}}'(\mathbf{P}_{\hat{\mathbf{y}}})^{-1}\hat{\mathbf{y}} = \hat{\mathbf{x}}'\mathbf{K}'(\mathbf{KPK}')^{-1}\mathbf{K}\hat{\mathbf{x}} \tag{9}$$

Denote by

$$\mathbf{Q} \triangleq \mathbf{K}'(\mathbf{KPK}')^{-1}\mathbf{K} \tag{10}$$

and

$$\mathbf{R} \triangleq \mathbf{P}^{-1} - \mathbf{P}^{-1}H(H\mathbf{P}^{-1}H')^{-1}H'\mathbf{P}^{-1} \tag{11}$$

It is easy to verify that

$$\mathbf{Q}H = \mathbf{R}H = \mathbf{0} \tag{12}$$

and

$$\mathbf{KPQ} = \mathbf{KPR} = \mathbf{K} \tag{13}$$

Thus

$$\mathbf{KP}(\mathbf{Q} - \mathbf{R})\mathbf{PK}' = \mathbf{0} \tag{14}$$

Since $\mathbf{KP}$, $\mathbf{Q}$ and $\mathbf{R}$ have the same rank and both $\mathbf{Q}$ and $\mathbf{R}$ are in the null space of H, we have $\mathbf{Q} = \mathbf{R}$ and thus

$$T_{\mathbf{x}}(\hat{\mathbf{x}}) = \hat{\mathbf{x}}'\mathbf{R}\hat{\mathbf{x}} = \hat{\mathbf{x}}'\mathbf{Q}\hat{\mathbf{x}} = T_{\mathbf{y}}(\hat{\mathbf{y}}) \tag{15}$$

The above result tells us that the association of multiple local tracks amounts to a chi-square test where the uniform prior is assumed for the target state.

2.2. Alternative Interpretation via Model Selection

The track association problem is a special case of the general model selection problem where different models can have unknown parameters of different dimensions. The conditional model estimator (CME) has been proposed in [11] to determine the model order where it is assumed that the sufficient statistics are available for all competing models. Here we derive the CME criterion for the linear model of the track association test. For hypothesis H_k, the minimum variance unbiased (MVU) estimate of $\mathbf{x}_k$ is

$$\hat{\mathbf{x}}_k = (\mathbf{H}_k'\mathbf{P}_k^{-1}\mathbf{H}_k)^{-1}\mathbf{H}_k'\mathbf{P}_k^{-1}\mathbf{y} \tag{16}$$

The CME criterion chooses hypothesis H_j if [11]

$$j = \arg\max_k p(\mathbf{y}|\hat{\mathbf{x}}_k) \tag{17}$$

It is equivalent to minimizing

$$\mathrm{CME}(k) = \frac{1}{2}\mathbf{y}'\mathbf{Q}_k\mathbf{y} - \frac{1}{2}\ln|2\pi\mathbf{P}_k| + \frac{1}{2}\ln|2\pi(\mathbf{H}_k'\mathbf{P}_k^{-1}\mathbf{H}_k)^{-1}| \tag{18}$$

where

$$\mathbf{Q}_k = \mathbf{P}_k^{-1} - \mathbf{P}_k^{-1}\mathbf{H}_k(\mathbf{H}_k'\mathbf{P}_k^{-1}\mathbf{H}_k)^{-1}\mathbf{H}_k'\mathbf{P}_k^{-1} \tag{19}$$

The underlying principle of CME is to decompose the data into two parts: one part summarizes the unknown parameter by its sufficient statistic and the other part does not depend on the parameter which is used to estimate the model likelihood. This part of data is sometimes called the ancillary statistic. The CME has a clear advantage over some other model selection criteria because it yields the MVU estimate of the model probability under the correct hypothesis [11].

Now consider the residue of the MVU estimate under hypothesis H_k given by

$$\nu_k = \mathbf{y} - \mathbf{H}_k \hat{\mathbf{x}}_k \tag{20}$$

It is clear that ν_k is zero mean with covariance

$$\mathbf{R}_k = \mathbf{P}_k - \mathbf{H}_k(\mathbf{H}_k' \mathbf{P}_k^{-1} \mathbf{H}_k)^{-1} \mathbf{H}_k' \tag{21}$$

The distribution of ν_k does not depend on the parameter $\mathbf{x}_k$. Thus the hypothesis H_k becomes simple when ν_k is used in the test statistic. The theorem in Section 2.1 indicates that

$$\mathbf{y}' \mathbf{Q}_k \mathbf{y} = \nu_k' \mathbf{P}_k^{-1} \nu_k \tag{22}$$

This implies that the CME is equivalent to a goodness-of-fit test given by

$$\mathrm{CME}(k) = \frac{1}{2}\nu_k' \mathbf{P}_k^{-1}\nu_k - \frac{1}{2}\ln|2\pi \mathbf{P}_k| + \frac{1}{2}\ln|2\pi(\mathbf{H}_k' \mathbf{P}_k^{-1} \mathbf{H}_k)^{-1}| \tag{23}$$

and there exist infinitely many equivalent tests of (18) based on the ancillary statistic γ_k.

3. GENERAL TRACK ASSOCIATION VIA MULTIDIMENSIONAL ASSIGNMENT

We want to consider the association of local tracks to more than one target. The problem can be formulated as finding the best hypothesis regarding the track-to-target correspondence which has been studied extensively in the framework of multiple hypothesis tracking (MHT) [5]. A promising approach to solve the MHT problem is via the multidimensional assignment [1, 15].

Assume there are S local sensors ($S \geq 3$) where sensor S_i provides a list of N_i tracks to the fusion center. Denote by $\chi_{i_1 i_2 \ldots i_S}$ the binary assignment variable and by $\lambda_{i_1 i_2 \ldots i_S}$ the likelihood ratio of the local tracks i_1, i_2, ..., i_S being from the same target vs. from different targets. The S-D assignment finds the most likely hypothesis by solving the following integer program.

$$\min_{\chi_{i_1 i_2 \ldots i_S}} \sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} \cdots \sum_{i_S=1}^{N_S} c_{i_1 i_2 \ldots i_S} \chi_{i_1 i_2 \ldots i_S} \tag{24}$$

subject to

$$\begin{array}{c} \sum_{i_2=0}^{N_2} \cdots \sum_{i_S=0}^{N_S} \chi_{j i_2 \ldots i_S} \leq 1, \quad j = 1, ..., N_1 \\ \sum_{i_1=0}^{N_1} \sum_{i_3=0}^{N_3} \cdots \sum_{i_S=0}^{N_S} \chi_{i_1 j i_3 \ldots i_S} \leq 1, \quad j = 1, ..., N_2 \\ \vdots \\ \sum_{i_1=0}^{N_1} \cdots \sum_{i_{S-1}=0}^{N_{S-1}} \chi_{i_1 i_2 \ldots i_{S-1} j} \leq 1, \quad j = 1, ..., N_S \end{array} \tag{25}$$

$$\chi_{i_1 i_2 \ldots i_S} \in \{0, 1\}, i_1 = 0, 1, ..., N_1, \cdots, i_S = 0, 1, ..., N_S \tag{26}$$

The above integer program is in general NP hard for $S \geq 3$. However, efficient algorithms exist to find a suboptimal solution with quantifiable upper and lower bound of the optimum via Lagrangian relaxation (see, e.g., [1, 7]).

4. REGULARIZED TRACK FUSION

When noninformative prior is used for the target state at the fusion center, equations (4)–(5) is optimal only within the class of linear unbiased estimators [12]. Note that the least squares solution minimizes the data fitting error rather than the mean square estimation error. This makes it possible to improve the estimation accuracy by exploring biased estimators. Various modifications of the least squares estimator have been proposed in [8, 9, 10]. The general guideline is that a biased estimator can reduce the mean square error when the variance being reduced is larger than the squared bias term. In the sequel, we present a scalar modification of the least squares estimator, which results in a regularized track fusion formula.

We introduce a multiplicative scalar bias term b and assume the regularized estimator (RE) has the form

$$\hat{x}^{\mathrm{RE}} = b\hat{x}^{\mathrm{LS}} \tag{27}$$

The RE class includes shrunken estimators, an example of which is the well-known James-Stein estimator [8]. To find the optimal bias, we want to solve the following problem which directly minimizes the trace of the mean square error matrix.

$$\min_b E[(b\hat{x}^{\mathrm{LS}} - x)'(b\hat{x}^{\mathrm{LS}} - x)] \tag{28}$$

The solution is given by

$$b = \frac{E[(\hat{x}^{\mathrm{LS}})' x]}{E[(\hat{x}^{\mathrm{LS}})' \hat{x}^{\mathrm{LS}}]} \tag{29}$$

where

$$E[(\hat{x}^{\mathrm{LS}})' x] = \mathrm{tr}[xx'] = ||x||^2 \tag{30}$$

$$\begin{aligned} E[(\hat{x}^{\mathrm{LS}})' \hat{x}^{\mathrm{LS}}] &= \mathrm{tr}[xx' + (H' \mathbf{P}^{-1} H)^{-1}] \\ &= ||x||^2 + \mathrm{tr}[(H' \mathbf{P}^{-1} H)^{-1}] \end{aligned} \tag{31}$$

Thus the optimal bias is

$$b = \frac{||x||^2}{||x||^2 + \mathrm{tr}[(H' \mathbf{P}^{-1} H)^{-1}]} \tag{32}$$

We can see that the optimal bias is always less than 1 which implies that it is always possible for the regularization to reduce the mean square error compared with the LS solution ($b = 1$). Note that the optimal bias depends on the unknown parameter x which can be replaced by the estimate $\hat{x}^{\mathrm{LS}}$. The resulting estimator is a nonlinear function of $\hat{\mathbf{x}}$. In practice, if we assume that x has a bounded norm, i.e., $||x||^2 \leq B$, then the bias can be chosen as

$$b = \frac{B}{B + \text{tr}[(H'\mathbf{P}^{-1}H)^{-1}]} \tag{33}$$

which makes the resulting regularized estimator linear. The mean square error of the regularized estimator is

$$P^{\text{RE}} = (1-b)^2 xx' + b^2 (H'\mathbf{P}^{-1}H)^{-1} \tag{34}$$

where the unknown state x can be replaced by its estimate. It is easy to verify that

$$\text{tr}[P^{\text{RE}}] = \frac{||x||^2 \text{tr}[(H'\mathbf{P}^{-1}H)^{-1}]}{||x||^2 + \text{tr}[(H'\mathbf{P}^{-1}H)^{-1}]} < \text{tr}[P^{\text{LS}}] \tag{35}$$

5. SIMULATION STUDY

We consider a ground target tracking scenario where three sensors are located at (-50,0)km, (0,187)km, and (50,0)km, respectively. All three sensors measure the target range and bearing with the same standard deviations of the measurement error given by $\sigma_r = 50$m and $\sigma_b = 2$mrad. The sampling interval of sensors 1 and 2 is $T_1 = T_2 = 2$s while the sampling interval of sensor 3 is $T_3 = 5$s.

The two targets are initially at (0,86.6)km and (0.4,86.6)km, respectively. Both targets move in parallel with a speed of 300m/s. They initially move toward south-east and then at $t = 15$s both targets make a course change with a constant turn rate of $4°$/s (acceleration of about 2.1g over a duration T_{man} of about 11s) and heads toward east. Both targets make a second course change at $t = 35$s with a constant turn rate of $4°$/s and heads toward north-east. The total time for the two targets to complete the designated trajectories is 60s. The true target motion has the random acceleration modeled by the white process noise spectrum $q = 1\text{m}^2/\text{s}^3$ in each realization. The trajectories of the two targets are shown in Fig. 1 where the true target positions are indicated at the time instances at which a measurement is made by one of the three sensors. The total time for the two targets to complete the designated trajectories is 60s. Note that the target ranges are around 100km at the beginning for all sensors, where the standard deviation of the crossrange measurement is around 200m.

Two tracking configurations for performance comparison are implemented as follows.

(i) A centralized estimator which uses an IMM with two models and sequentially updates the target state with measurements from sensors 1–3. This IMM estimator has a discretized continuous white noise acceleration (DCWNA) model [4] with a low process noise PSD q_l to capture the uniform target motion and a DCWNA model with a high

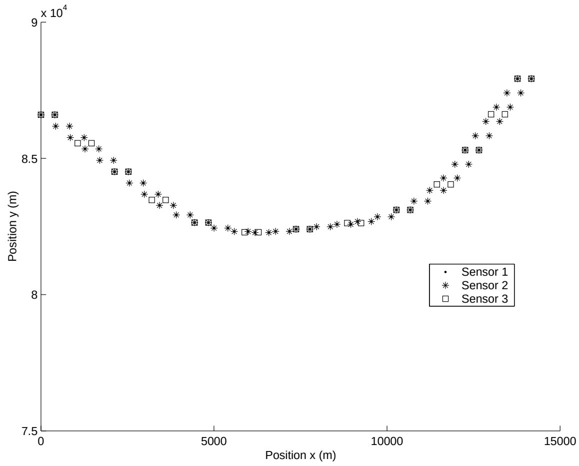

Fig. 1. Target trajectories with true positions at the times when measurements are made by the sensors.

process noise PSD q_h to capture the two turns. We use $q_l = 1\text{m}^2/\text{s}^3$ and $q_h = 8000\text{m}^2/\text{s}^3$.

(ii) In the decentralized tracking configuration each sensor uses an IMM estimator and the fusion center fuses the local estimates every $T_F = 10$s using all local state estimates and the corresponding covariances with approximate crosscovariances. For each local IMM estimator, 2-D assignment is used to solve the measurement-to-track association problem and the most likely hypothesis is chosen for the filter update. The track-to-track association is based on the most likely hypothesis obtained by solving the 3-D assignment among three local trackers. If the local tracks are declared as from the same target, then the track-to-track fusion is carried out using regularized track fusion with approximate cross covariance.

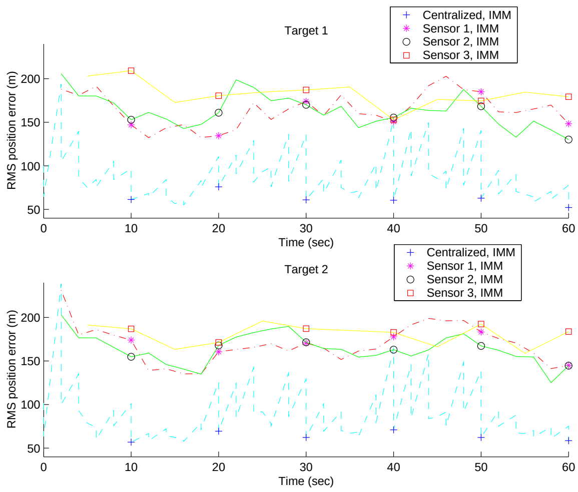

Fig. 2. Comparison of the RMS position errors for centralized IMM estimator (configuration (i)) and three local IMM estimators.

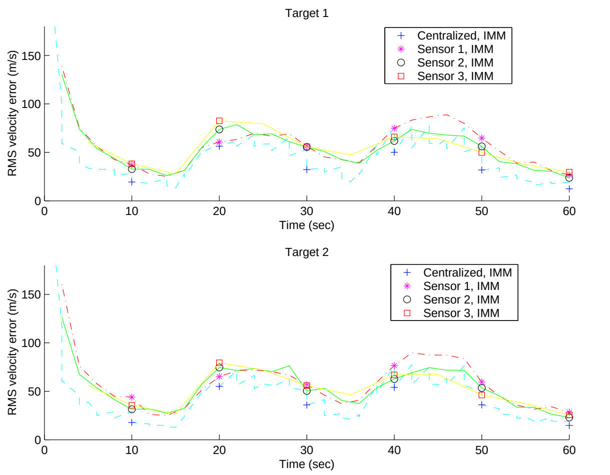

Fig. 3. Comparison of the RMS velocity errors for centralized IMM estimator (configuration (i)) and three local IMM estimators.

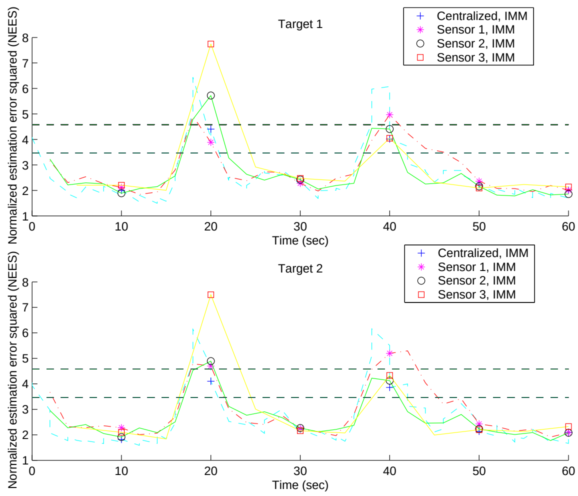

Fig. 4. Comparison of the NEES for centralized IMM estimator (configuration (i)) and three local IMM estimators.

Fig. 2 shows the RMS position errors at the fusion center for the centralized IMM estimator and three local IMM estimators. The improvement of the tracking accuracy is fairly large for both targets and during both non-maneuver and maneuver segments. The position errors do not increase significantly during target maneuver. Fig. 3 shows the corresponding RMS velocity errors for the centralized estimator and the local estimators. We can see that tracking accuracy does not improve too much because the sensors do not directly measure the velocity state. The velocity errors increase during the two maneuver segments with certain delay after the maneuver onset time. Fig. 4 shows the normalized estimation error squared (NEES) [4] at the fusion center for the above four estimators. The NEES is an indication of the filter credibility for linear Gaussian dynamics.

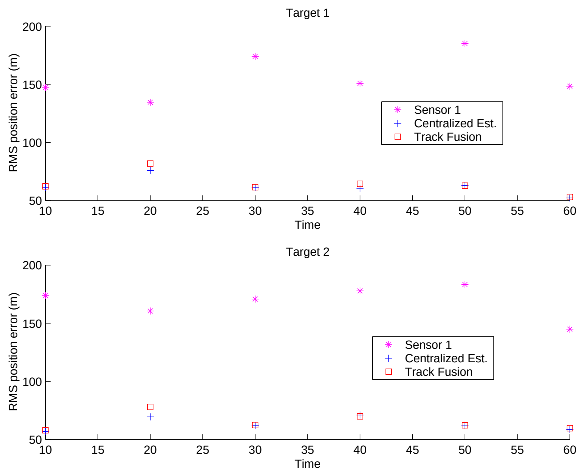

Fig. 5. Comparison of the RMS position errors for centralized IMM estimator (configuration (i)), track fusion from three IMM estimators (configuration (ii)); local IMM estimator from sensor 1 also shown.

The lower and upper bound based on the 95% confidence interval of the chi-square distribution is also shown in the figure. We can see that the IMM estimator is conservative about its mean square estimation error when the target has a nearly constant velocity, but optimistic during target maneuver. These plots provide the tracking performance of the ideal centralized tracker as well as the autonomous local trackers.

Fig. 5 shows the RMS position errors at the fusion center for the above two tracking configurations as well as that by sensor 1 alone served as a baseline. We can see that the track fusion of the three local IMM estimates (configuration (ii)) yields nearly the same RMS position error as that of the centralized estimator (configuration (i)). Fig. 6 compares the corresponding RMS velocity errors. We can see that the track fusion has smaller RMS velocity error when the targets have a nearly constant velocity but larger RMS velocity error after the targets start maneuver. Overall, the proposed assignment solution for the track-to-track association is very effective when the quality of the local tracks is good. Fig. 7 shows the NEES at the fusion center for the above two tracking configurations. The lower and upper bound based on the 95% confidence interval of the chi-square distribution is also shown in the figure. We can see that the distributed track fusion yield larger NEES than the centralized estimator during the target turns. This indicates that the fused track is optimistic about its mean square estimation error. Thus caution has to be exercised when fusing the local estimates that are not credible on their own NEES statistics.

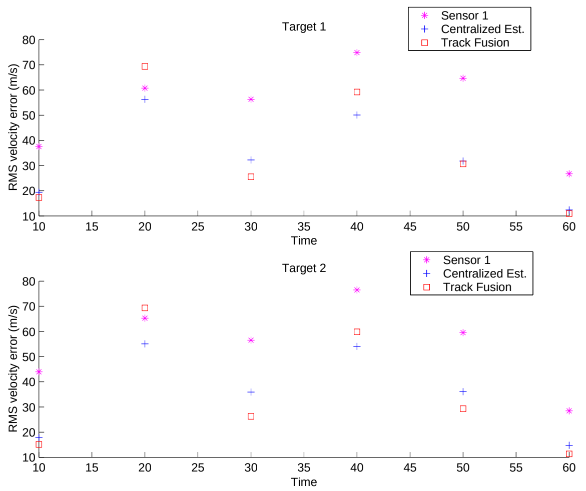

Fig. 6. Comparison of the RMS velocity errors for centralized IMM estimator (configuration (i)), track fusion from three IMM estimators (configuration (ii)); local IMM estimator from sensor 1 also shown.

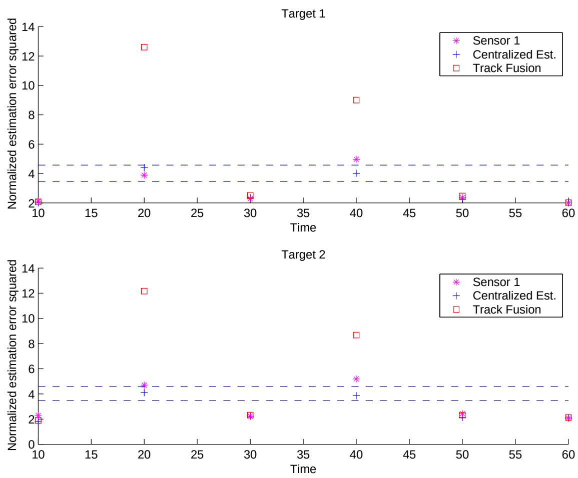

Fig. 7. Comparison of the NEES for centralized IMM estimator (configuration (i)), track fusion from three IMM estimators (configuration (ii)); local IMM estimator from sensor 1 also shown.

6. CONCLUSIONS

This paper derives a Bayesian procedure for track association that can solve a large scale distributed tracking problem where many sensors track many targets. When noninformative prior of the target state is assumed, the single target test becomes a chi-square test and it can be extended to the multiple target case by solving a multidimensional assignment problem. With the noninformative prior assumption, the optimal track fusion algorithm can be a biased one where the regularized estimate has smaller mean square estimation error. A regularized track fusion algorithm was presented which modifies the optimal linear unbiased fusion rule by a less-than-unity scalar. Simulation results indicate the effectiveness of the proposed track association and fusion algorithm through a three-sensor two-target tracking scenario.

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