A STATE SPACE APPROACH TO ROBUST ADAPTIVE BEAMFORMING

Amr El-Keyi† *Thia Kirubarajan*† *Alex B. Gershman*∗

†McMaster University
Department of Electrical and Computer Engineering
Hamilton, Ontario, Canada

∗Darmstadt University of Technology
Communication Systems Group
Darmstadt, Germany

ABSTRACT

In this paper, we present a novel approach to implement the robust minimum variance distortionless response (MVDR) beamformer of [1]. This beamformer has been shown to provide excellent robustness against arbitrary but norm bounded mismatches in the desired signal steering vector. However, existing algorithms to solve this problem do not have direct computationally efficient on-line implementations. We develop a new algorithm for the implementation of the robust MVDR beamformer based on state space modelling of the beamforming problem. Our algorithm can be implemented on-line via a second-order extended Kalman filter (EKF) with low computational cost compared to previous second-order cone programming (SOCP) based implementation.

1. INTRODUCTION

The need for robust adaptive beamforming arises in many practical applications where the desired signal is present in the beamformer training data and where the assumptions on the nature of the desired signal and/or interference are violated [2]. One of the main problems that occur in practical adaptive array problems is the mismatch between the desired signal steering vector and the actual steering vector [1]. Adaptive array techniques are known to be very sensitive to even slight mismatches of this type. Such mismatches may occur due to signal pointing errors, imperfect array calibration, source local scattering and/or wavefront distortions.

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Several *ad hoc* approaches exist to overcome arbitrary desired signal mismatches such as the diagonal loading of the sample covariance matrix [3] and the eigenspace-based beamformer [4]. One of the recent theoretically rigorous and powerful approaches to robust beamforming in the presence of an arbitrary unknown steering signal mismatch is based on worst-case performance optimization [1]. It obtains the weight vector by minimizing the output interference-plus-noise power while maintaining a distortionless response for the worst-case (mismatched) signal steering vector. The robust MVDR beamforming problem was formulated in [1] as a SOCP problem which can be solved in polynomial time using interior point methods. In further works [5]-[9], several extensions of the robust MVDR beamformer of [1] have been considered and alternative Newton-type iterative and closed-form algorithms have been developed for this beamformer and its extensions. However, the main shortcoming of the algorithms presented in these papers is that most of them do not have direct computationally efficient on-line implementations.

The use of the constrained Kalman filter to solve the linearly constrained MVDR beamforming problem was proposed in [10]. In this paper, we develop an alternative implementation of the robust MVDR beamformer. It is based on state space modelling of the beamforming problem where the robustness constraint of [1] is incorporated in the measurement equation instead of the distortionless-response constraint used in [10]. The beamformer weight vector is estimated adaptively though a second-order extended Kalman filter (EKF) with low computational cost. Our algorithm is shown to have a performance similar to that of the original SOCP-based implementation of the robust MVDR beamformer but with $\mathcal{O}(M^2)$ complexity per iteration as opposed to $\mathcal{O}(M^3)$ for the SOCP-based algorithm, where M is the number of array elements. This makes it more suitable for on-line implementation.

2. BACKGROUND

Consider a linear array of M sensors whose output at time k is given by

$$y(k) = \boldsymbol{x}^H(k)\boldsymbol{w} \tag{1}$$

where $\boldsymbol{x}(k) = [x_1(k), \ldots, x_M(k)]^T \in \mathcal{C}^M$ is the array observation vector, $\boldsymbol{w} = [w_1, \ldots, w_M]^T \in \mathcal{C}^M$ is the complex vector of beamformer weights, and $(\cdot)^H$ stands for the Hermitian transpose.

Let us consider the narrowband case when the observation vector can be written as

$$\boldsymbol{x}(k) = s(k)\boldsymbol{d} + \boldsymbol{i}(k) + \boldsymbol{n}(k) \tag{2}$$

where $s(k)$, $\boldsymbol{d}$, $\boldsymbol{i}(k)$, and $\boldsymbol{n}(k)$ are the desired signal waveform, the desired signal steering vector, the interference component, and the noise component, respectively. The desired signal and interferers are assumed to be uncorrelated and stationary.

The well-known MVDR beamformer minimizes the output interference-plus-noise power while maintaining a distortionless response to the desired signal [11]. The MVDR problem is given by

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w} \qquad \text{subject to} \qquad \boldsymbol{w}^H \boldsymbol{a} = 1 \tag{3}$$

where $\boldsymbol{R}_{xx} = E\{\boldsymbol{x}(k)\boldsymbol{x}^H(k)\}$ is the $M \times M$ covariance matrix and $\boldsymbol{a}$ is the presumed desired signal steering vector. Note that the presumed steering vector $\boldsymbol{a}$ may be an erroneous (mismatched) copy of the actual steering vector $\boldsymbol{d}$. The solution to the MVDR beamforming problem in (3) is given by [11]

$$\boldsymbol{w}_{\text{opt}} = \frac{\boldsymbol{R}_{xx}^{-1}\boldsymbol{a}}{\boldsymbol{a}^H \boldsymbol{R}_{xx}^{-1}\boldsymbol{a}}. \tag{4}$$

In practice, the exact covariance matrix $\boldsymbol{R}_{xx}$ is not available but has to be estimated from the received (training) data samples as [11]

$$\hat{\boldsymbol{R}}_{xx} = \frac{1}{N}\sum_{k=1}^{N} \boldsymbol{x}(k)\boldsymbol{x}^H(k) \tag{5}$$

where N is the number of snapshots available. The sample covariance matrix (5) is used in (4) instead of the true array covariance matrix, and the resulting solution is commonly referred to as the sample matrix inversion (SMI) algorithm [11]. If the signal is present in the beamformer training cell and the presumed signal steering vector $\boldsymbol{a}$ is different from the actual steering vector $\boldsymbol{d}$, then the desired signal is interpreted by the SMI beamformer as an interference signal and is cancelled out instead of being enhanced.

In practical applications, there may exist arbitrary unknown mismatches between the actual steering vector and the presumed one, yet the norm of the steering vector distortion can be usually bounded by some known constant $\varepsilon > 0$ [1]. Therefore, the actual desired signal steering vector belongs to the set

$$\mathcal{A}(\varepsilon) = \{\boldsymbol{c} \mid \boldsymbol{c} = \boldsymbol{a} + \boldsymbol{e}, \|\boldsymbol{e}\| \le \varepsilon\} \tag{6}$$

where $\|\cdot\|$ is the vector 2-norm.

The robust MVDR beamforming problem in [1] minimizes the total output power of the beamformer while requiring a distortionless response not only for the presumed steering vector $\boldsymbol{a}$, but for all the vectors that belong to $\mathcal{A}(\varepsilon)$. Thus, the robust formulation of the MVDR beamformer can be written as [1]

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \hat{\boldsymbol{R}}_{xx} \boldsymbol{w} \quad \text{subject to} \quad |\boldsymbol{w}^H \boldsymbol{c}| \ge 1 \quad \forall \boldsymbol{c} \in \mathcal{A}(\varepsilon). \tag{7}$$

The infinite number of nonconvex constraints in (7) was reformulated in [1] as a single convex constraint corresponding to the worst-case mismatch, and the original problem was converted to the following SOCP problem

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \hat{\boldsymbol{R}}_{xx} \boldsymbol{w} \qquad \text{subject to} \qquad \boldsymbol{w}^H \boldsymbol{a} \ge \varepsilon\|\boldsymbol{w}\| + 1. \tag{8}$$

Moreover, it can be easily proven that the constraint in (8) is satisfied with equality [1]. Based on this fact, Newton-type iterative procedures have been proposed in [6], [8], and [9] to solve (8) and some extensions of this problem.

3. KALMAN FILTER-BASED ROBUST BEAMFORMER

For the convenience of subsequent derivations, let us introduce the mean square error (MSE) between the zero signal and the beamformer output as

$$\text{MSE} = \text{E}[|0 - \boldsymbol{x}^H(k)\boldsymbol{w}(k)|^2] = \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w}. \tag{9}$$

Thus, minimizing the beamformer output power is equivalent to minimizing the MSE in (9). Define $h_2(\boldsymbol{w}(k))$ as

$$\begin{aligned} h_2(\boldsymbol{w}(k)) &= \varepsilon^2 \boldsymbol{w}^H(k)\boldsymbol{w}(k) - \boldsymbol{w}^H(k)\boldsymbol{a}\boldsymbol{a}^H\boldsymbol{w}(k) \\ &+ \boldsymbol{w}^H(k)\boldsymbol{a} + \boldsymbol{a}^H\boldsymbol{w}(k). \end{aligned} \tag{10}$$

Therefore, the robust beamforming problem (8) can be written as

$$\min_{\boldsymbol{w}} \text{MSE} \qquad \text{subject to} \qquad h_2(\boldsymbol{w}(k)) = 1. \tag{11}$$

The Kalman filter is a minimum mean square error estimator [12] and will be used to solve (11). An unknown dynamic system can be modelled as a filter whose weight vector $\boldsymbol{w}$ undergoes a first-order Markov process [13], i.e.,

$$\boldsymbol{w}(k+1) = \gamma \boldsymbol{w}(k) + \boldsymbol{v}_s(k) \tag{12}$$

where γ is a fixed parameter of the model, and $\boldsymbol{v}_s(k)$ is the process noise vector which is assumed to be white Gaussian with zero mean and covariance matrix $\boldsymbol{Q} = \sigma_s^2 \boldsymbol{I}$. Thus, the process equation of the optimal weight vector $\boldsymbol{w}$ is given by (12), whereas the measurement equation is given by

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}^H(k)\boldsymbol{w}(k) \\ h_2(\boldsymbol{w}(k)) \end{bmatrix} + \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix} \quad (13)$$

which can be written in matrix notation as

$$\boldsymbol{z} = \boldsymbol{h}(\boldsymbol{w}(k)) + \boldsymbol{v}_m(k) \quad (14)$$

where $v_1(k)$ and $v_2(k)$ are the residual and the constraint errors, respectively. They are modelled as zero-mean independent white noise sequences with the covariance matrix

$$\boldsymbol{R} = \text{diag}\{\sigma_1^2, \sigma_2^2\}. \quad (15)$$

Minimizing the mean square value of $v_1(k)$ will lead to minimizing the output power of the beamformer, while minimizing the mean square value of $v_2(k)$ will minimize the mean square error incurred in satisfying the robustness constraint.

Due to the nonlinearity of the measurement equation, the second-order EKF [12] will be used to find a recursion for the estimated weight vector $\hat{\boldsymbol{w}}(k)$. We start by evaluating the Jacobian, $\boldsymbol{H}_{\boldsymbol{w}}(k, \boldsymbol{w}(k))$, and the Hessian matrices, $\boldsymbol{H}_{\boldsymbol{ww}}^{(1)}$ and $\boldsymbol{H}_{\boldsymbol{ww}}^{(2)}$, of $\boldsymbol{h}(\boldsymbol{w}(k))$ as

$$\boldsymbol{H}_{\boldsymbol{w}}(k, \boldsymbol{w}(k)) = \begin{bmatrix} \boldsymbol{x}^H(k) \\ \varepsilon^2 \boldsymbol{w}^H(k) - (\boldsymbol{a}\boldsymbol{a}^H \boldsymbol{w}(k))^H + \boldsymbol{a}^H \end{bmatrix} \quad (16)$$

$$\boldsymbol{H}_{\boldsymbol{ww}}^{(1)} = \boldsymbol{0}, \qquad \boldsymbol{H}_{\boldsymbol{ww}}^{(2)} = \varepsilon^2 \boldsymbol{I} - \boldsymbol{a}\boldsymbol{a}^H. \quad (17)$$

The recursion for the estimated weight vector starts with an initial weight vector estimate $\hat{\boldsymbol{w}}(0)$ with the associated covariance matrix $\boldsymbol{P}(0|0)$, and updates the weight vector estimate through

$$\hat{\boldsymbol{w}}(k) = \hat{\boldsymbol{w}}(k-1) + \boldsymbol{G}(k)[\boldsymbol{z} - \hat{\boldsymbol{z}}(k|k-1)] \quad (18)$$

where the predicted measurement $\hat{\boldsymbol{z}}(k|k-1)$ and the filter gain $\boldsymbol{G}(k)$ are given by

$$\hat{\boldsymbol{z}}(k|k-1) = \begin{bmatrix} \gamma \boldsymbol{x}^H(k)\hat{\boldsymbol{w}}(k-1) \\ h_2(\gamma\hat{\boldsymbol{w}}(k-1)) + \frac{1}{2}\text{trace}\{\boldsymbol{H}_{\boldsymbol{ww}}^{(2)} \boldsymbol{P}(k|k-1)\} \end{bmatrix} \quad (19)$$

$$\boldsymbol{G}(k) = \boldsymbol{P}(k|k-1)\boldsymbol{H}_{\boldsymbol{w}}^H(k, \gamma\hat{\boldsymbol{w}}(k-1))\boldsymbol{S}^{-1}(k). \quad (20)$$

Here, the innovation covariance $\boldsymbol{S}(k)$ and the predicted weight vector covariance $\boldsymbol{P}(k|k-1)$ are given by

$$\begin{aligned} \boldsymbol{S}(k) &= \boldsymbol{H}_{\boldsymbol{w}}(k, \gamma\hat{\boldsymbol{w}}(k-1))\boldsymbol{P}(k|k-1)\boldsymbol{H}_{\boldsymbol{w}}^H(k, \gamma\hat{\boldsymbol{w}}(k-1)) \\ &+ \frac{1}{2}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \left(\varepsilon^4 \|\boldsymbol{P}(k|k-1)\|_F^2 - 2\varepsilon^2 \|\boldsymbol{P}(k|k-1)\boldsymbol{a}\|^2 \right. \\ &\left. + |\boldsymbol{a}^H \boldsymbol{P}(k|k-1)\boldsymbol{a}|^2\right) + \boldsymbol{R} \end{aligned} \quad (21)$$

$$\boldsymbol{P}(k|k-1) = \gamma^2 \boldsymbol{P}(k-1|k-1) + \boldsymbol{Q} \quad (22)$$

where $\|\cdot\|_F$ is the matrix Frobenius norm.

The updated covariance matrix can be expressed as

$$\boldsymbol{P}(k|k) = \boldsymbol{P}(k|k-1) - \boldsymbol{G}(k)\boldsymbol{S}(k)\boldsymbol{G}^H(k). \quad (23)$$

The consistency of the beamformer can be checked through the normalized innovation square (NIS) test [12]. In an on-line consistency check test and under the Gaussian assumption for measurement noise, the NIS

$$\epsilon_\nu(k) = [\boldsymbol{z} - \hat{\boldsymbol{z}}(k|k-1)]^H \boldsymbol{S}^{-1}(k)[\boldsymbol{z} - \hat{\boldsymbol{z}}(k|k-1)] \quad (24)$$

is chi-square distributed with three degrees of freedom and should be within acceptable limits with a certain probability if the beamformer is consistent. For example, using a 95% confidence region, the NIS should be less than 7.815 with probability 0.95.

For initialization of the iterative algorithm, a random weight vector estimate $\hat{\boldsymbol{w}}(0)$ can be used together with an initial covariance matrix estimate $\boldsymbol{P}(1|0) = \alpha\boldsymbol{I}$, where α is selected such that the NIS of the first iteration is acceptable. Hence, by ignoring the second-order terms and the measurement noise covariance matrix in (21), we can write

$$\begin{aligned} \alpha \approx\ & 0.33\big(\boldsymbol{z} - \hat{\boldsymbol{z}}(1|0)\big)^H \Big(\boldsymbol{H}_{\boldsymbol{w}}(1, \hat{\boldsymbol{w}}(0))\boldsymbol{H}_{\boldsymbol{w}}^H(1, \hat{\boldsymbol{w}}(0))\Big)^{-1} \\ & \big(\boldsymbol{z} - \hat{\boldsymbol{z}}(1|0)\big). \end{aligned} \quad (25)$$

The parameters γ and σ_s^2 of the state equation are chosen such that the model can track changes in the optimal weight vector due to changes in the operating environment. Their effect can be seen in (22); as they increase the predicted weight vector covariance, the Kalman filter puts more weight to recent data enabling better tracking of the environment. For a nonstationary environment, a typical choice for γ is slightly greater than 1. Although this choice makes the state equation (12) unstable, the stability of the filter can be guaranteed from the observability condition [12], [14]. Moreover, a value of $\sigma_s^2 = 10^{-4}$ indicates that each component of the optimal weight vector can change independently during one time step in the order of 10^{-2}. On the other hand, for a stationary environment, the optimal weight vector does not change with time and therefore $\gamma = 1$ and $\sigma_s^2 = 0$.

The mean square residual error σ_1^2 should be chosen in the order of the optimal output power of the array, which is roughly given by $\|\boldsymbol{w}\|^2(M\sigma_s^2 + \sigma_n^2)$, where σ_s^2 and σ_n^2 are the received desired signal power and white noise power in each sensor, respectively, and $\boldsymbol{w}$ is the beamformer optimal weight vector. On the other hand, σ_2^2 should be selected as small as possible (close to the machine epsilon) so that the robustness constraint is satisfied with high accuracy.

The proposed Kalman filter-based implementation has a computational complexity of $\mathcal{O}(M^2)$ per iteration, whereas

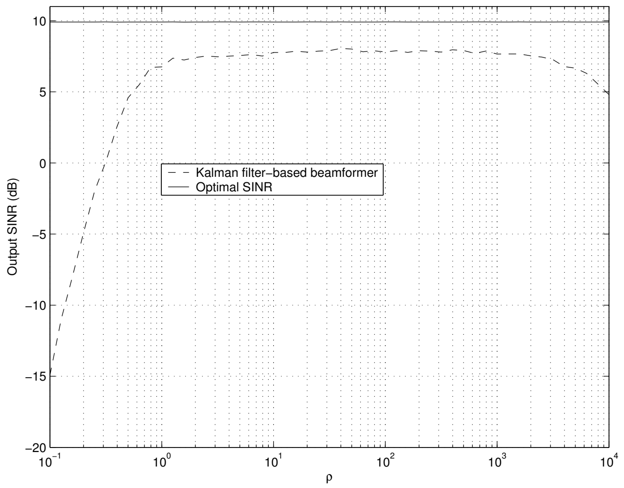

Fig. 1. Average output SINR versus ρ.

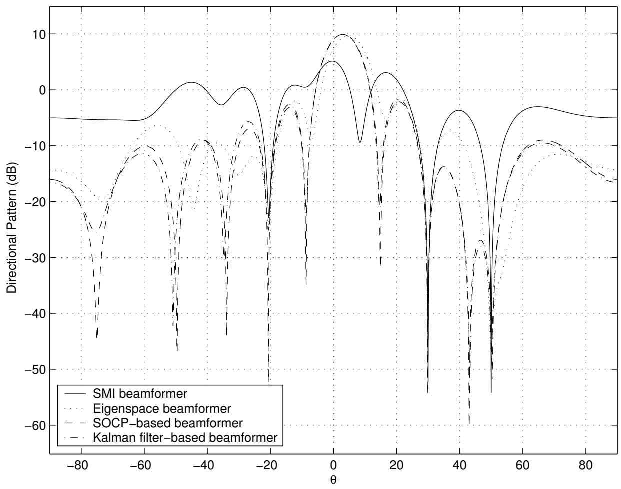

Fig. 2. Array beampattern after 100 iterations.

its SOCP-based and Newton algorithm-based implementations [1], [8], and [6], have a complexity of $\mathcal{O}(M^3)$. Moreover, an important advantage of the proposed Kalman filter-based algorithm is that it can be easily implemented without any need of specific built-in optimization software, as required by the SOCP-based beamformer.

4. SIMULATIONS

We consider a uniform linear array of ten sensors spaced half a wavelength apart. The array is steered towards the direction $\theta = 3^\circ$. The desired signal with signal-to-noise ratio (SNR) 0 dB is assumed to impinge on the array from the direction $\theta = 5^\circ$. Two interferers are assumed to impinge on the array from the directions $\theta = 30^\circ$ and $\theta = 50^\circ$, each with an interference-to-noise ratio (INR) equal to 30 dB. The desired signal is assumed to be always present in the test cell. The spatial signature of the desired signal is distorted by wave propagation effects in an inhomogeneous medium which are modelled as independent random phase increments drawn from a Gaussian random generator with variance 0.04. The robustness parameter $\varepsilon = 3$ is used both in the robust Kalman filter-based beamformer and the robust MVDR beamformer of [1]. The parameters of the Kalman filter are selected as $\sigma_2^2 = 10^{-12}$, $\gamma = 1$, $\sigma_s^2 = 0$, and a random initial vector is selected for the initialization of the Kalman filter beamformer with the associated initial covariance calculated from (25). The simulation results are averaged over a number of 200 runs.

We start by investigating the effect of the choice of the parameter σ_1^2 on the output signal-to-interference-plus-noise ratio (SINR) of the beamformer. Figure 1 shows the output SINR after 100 iterations versus different choices of the parameter $\rho = \sigma_1^2/(M\sigma_s^2 + \sigma_n^2)$. We can notice that the output SINR is high for a large range of ρ and that the filter is insensitive to the exact choice of σ_1^2. This behavior can be attributed to the change in the norm of the filter weight vector estimate to yield an output power of the beamformer matching the value of σ_1^2. At extremely high or low values of ρ, the weight vector norm can not match the value of ρ due to the robustness constraint that also controls the weight vector norm. This leads to a decreased SINR for these extreme values of ρ.

Next, the performance of the proposed Kalman filter-based beamformer (with $\rho = 50$) is compared with that of the SMI, eigenspace-based, and the robust MVDR beamformers. Figure 2 shows the directional patterns after 100 iterations for the four beamformers tested with all weight vectors normalized to have unit norm. From this figure, we can observe an essential similarity in the directional patterns of the robust MVDR and Kalman filter beamformers since they basically solve the same problem using different optimization techniques. Figure 3 displays the output power of the Kalman filter beamformer versus the iteration number (with the weight vector normalized to have unit norm after each iteration) along with the optimal output power under the unit norm weight vector constraint. From this figure, we can see that the convergence rate of the Kalman filter beamformer to the optimal output power is quite fast and that the steady state misadjustment is reasonably small.

Figure 4 shows the output SINR of the beamformers tested versus the iteration number (snapshot index). Additionally, the optimal SINR curve is shown in this figure. From this figure, it can be observed that the eignespace-based, Kalman filter-based, and robust MVDR beamformers have the best performances among the techniques tested although the performance of the eigenspace-based beamformer significantly drops if the number of snapshots is sm-

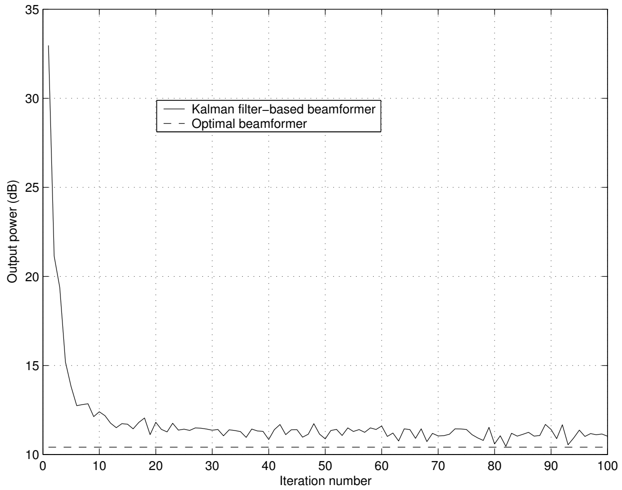

Fig. 3. Output power versus snapshot index.

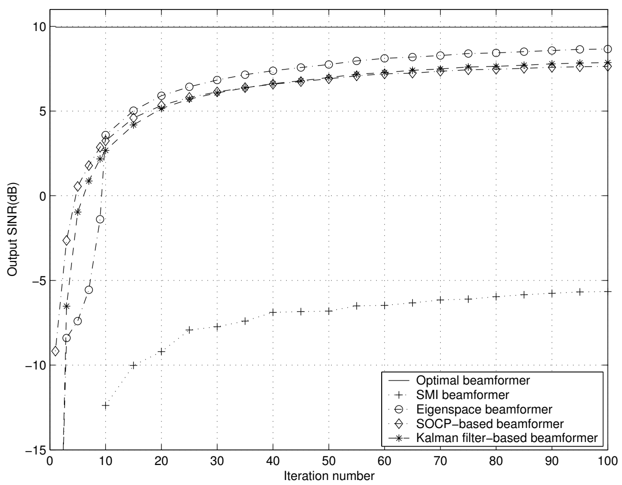

Fig. 4. Average output SINR versus snapshot index.

all. As expected, the performances of the Kalman filter-based and robust MVDR beamformers are nearly identical.

Finally, we investigate the effect of the SNR on the performance of the four beamformers tested. For the same scenario as in the previous example, the parameter σ_1^2 is selected as $20(M\sigma_d^2 + \sigma_n^2)$ and $N = 20$ snapshots are taken. The results are averaged over 200 independent runs. Figure 5 shows the output SINR of the beamformers tested versus the SNR. It can be seen that the performance of the Kalman filter-based beamformer is very similar to that of the robust MVDR beamformer. Both these techniques outperform the eigenspace-based beamformer (at low SNRs) and the SMI beamformer (at all values of SNR).

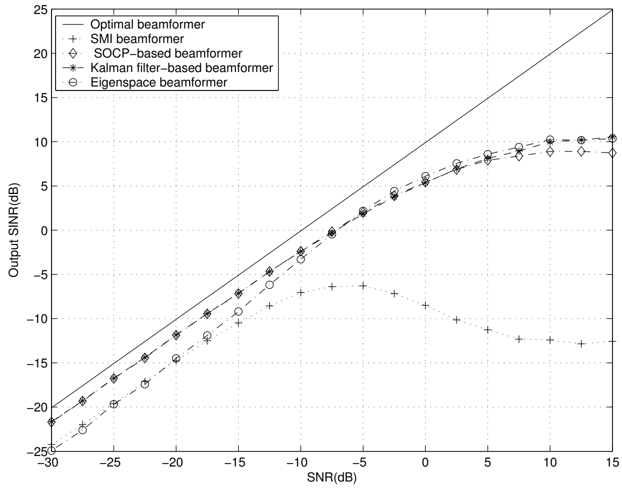

Fig. 5. Average output SINR versus SNR.

5. CONCLUSION

A novel computationally efficient constrained Kalman filter-based algorithm for the robust MVDR beamformer of [1] has been proposed. This algorithm is suitable for on-line implementation and has performance similar to that of the robust MVDR beamformer but with reduced complexity per iteration. Simulation results have been presented to assert the robustness of the proposed beamformer. Further extensions to nonstationary scenarios will be considered in future work.

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