BURIED OBJECTS LOCALIZATION IN PRESENCE OF CORRELATED SIGNALS

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ABSTRACT

In this paper a MUSIC algorithm for locating buried cylindrical shells using the scattering field at a wideband frequency is developed. The algorithm presented here modifies the conventional direction finding array precessing techniques, it uses the spatial complexities of the fields combined to the focusing operator to decorrelate the signals that allows us to estimate both the range and the bearing of the objects in the region of interest. Finally, the performances of the proposed method are validated on simulated data and experimental data recorded during an underwater acoustics experiments.

1. INTRODUCTION

MUSIC algorithm is widely used in underwater acoustics for sources bearing estimation problem. Typically this technique assumes that the wavefront model is known and usually the considered sources are far from the array [1]. Consequently, the measured wavefronts are all planar in nature. In many applications, where this assumption is not valid, the objects localization needs both, bearing and range estimation.

The existing techniques to estimate both the range and the bearing of objects do not take into account the interactions between objects which induce correlated signals, and consider the localization and the range estimation separately which leads to suboptimal solution, and the other methods are validated only on simulated data [2] [3] [4]. In our study, we have developed a method that solves the objects localization problem in the underwater acoustics environment and in presence of interactions between objects. The employed signals are wideband. This choice is made in order to decorrelate the signals by means of an average of the focused spectral matrices. Therefore the objects can be localized even if the received signals are totally correlated. This would have not been possible with the narrowband signals without the spatial smoothing. Moreover in our method the exact solution of the scattered field by an elastic cylindrical shell (object) is considered. So the main goal of this processing is the localization of these objects with they known structures. In addition, the performances of the proposed method are validated on experimental data recorded during an underwater acoustics experiments.

We begin by presenting an overview, and describing a simple modification of the MUSIC algorithm for narrowband signals, in section 2. In section 3, we will treat wideband signals. Thus, we will describe how to use the bilinear focusing operator in order to decorrelate the signals. We will present numerical examples on localization of multiple objects in section 4. In this section, we treat two cases, when the interactions between objects are ignored (uncorrelated signals) and when the all multiple scattering effects are taken into consideration (correlated signals). The developed method is validated on experimental data and the experimental set up will be presented in section 5. The obtained results will be shown and discussed in section 6. The conclusion will be drawn in section 7.

2. MODIFICATION OF THE MUSIC ALGORITHM FOR NARROWBAND SIGNALS

2.1. Overview of the MUSIC algorithm

Consider K narrowband signals, scattered from radiating point sources and impinging on a uniform linear array of N sensors. The output signals of these N sensors, in the frequency domain, can be arranged to form an $N \times 1$ vector $\mathbf{y}$ as follows

$$\mathbf{y} = \mathbf{A}(\theta)\mathbf{s} + \mathbf{b}, \tag{1}$$

where $\mathbf{s}$ is a $K \times 1$ vector of source signals, $\mathbf{b}$ is a $N \times 1$ noise vector and θ is the direction of arrival. The noise at each sensor is assumed to be uncorrelated with the signals, uncorrelated from sensor to sensor, and to have variance σ^2. We assume that each sensor have a unit gain. $\mathbf{A}(\theta)$ is a $N \times K$ direction matrix, $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), ..., \mathbf{a}(\theta_K)]$ and $\mathbf{a}(\theta_k)$ $(k = 1, 2, ..., K)$, represents the plane wave direction vector given by,

$$\mathbf{a}(\theta_k) = \left[1,\ e^{-2j\pi f \frac{d \sin\theta_k}{c}}\ ,\ \dots,\ e^{-2j\pi f (N-1) \frac{d \sin\theta_k}{c}}\right]^T,$$

where the superscript "$[.]^T$" represents the transpose, c is the sound speed in the medium, d is the distance between two adjacent sensors, f is the signal frequency.
The spectral matrix $\mathbf{Y}$, $(N \times N)$, is given by

$$\mathbf{Y} = E\left[\mathbf{y}\mathbf{y}^+\right] = \mathbf{A}(\theta)E\left[\mathbf{s}\mathbf{s}^+\right]\mathbf{A}^+(\theta) + \sigma^2\mathbf{I}, \quad (2)$$

where $\mathbf{I}$ is an $N \times N$ identity matrix and the superscript "$(.)^+$" represents the Hermitian transpose. Then, a singular value decomposition of the $\mathbf{Y}$ matrix is done to separate signal and noise subspaces,

$$\mathbf{Y} = [\mathbf{V_s}\mathbf{V_b}]\begin{bmatrix} \mathbf{\Lambda_s} & 0 \\ 0 & \mathbf{\Lambda_b} \end{bmatrix}[\mathbf{V_s}\mathbf{V_b}]^+, \quad (3)$$

where $\mathbf{V_s}$ and $\mathbf{V_b}$ are respectively, the signal and noise eigenvectors, $\mathbf{\Lambda_s}$ and $\mathbf{\Lambda_b}$ the associated eigenvalues.
The MUSIC algorithm uses these noise eigenvectors to estimate the bearings of these plane wavefronts associated to these point sources. Thus the spatial spectrum of the MUSIC algorithm is given by

$$\mathbf{p}_{MUSIC}(\theta) = \frac{1}{\mathbf{a}(\theta)^+\mathbf{V_b}\mathbf{V_b}^+\mathbf{a}(\theta)}, \quad (4)$$

It is important to realize that the formulation of the array processing presented in this section implicitly assumes that the object is infinitely distant so that the scattered field has planar wavefronts at the sensor array. therefore, the performances of the MUSIC method are lost for the applications where the steering vector model is not valid. Thus we propose, in the following section, to use the exact solution to fill the direction vector instead to use the plane wave model, scattered from point sources, in order to estimate both the range and the bearing of the objects. This method remains valid when the objects are in the far-field and in the near-field region of the sensor array.

2.2. Objects localization

The MUSIC algorithm outlined in the previous section can be modified so that the direction vector is filled with the type of the wavefront impinging on the sensor array. Furthermore, instead of radiating point sources, the radiators in this section scatter the incident plane wave. Thus, the scattering model is exactly incorporated in the processing in this section. By using the exact solution of the scattered field [5] [6], we can fill the direction vector in the MUSIC algorithm with non planar scattered field to locate the objects. By modifying the spatial MUSIC spectrum in (4), we form the following spectrum

$$\mathbf{p}_{\mathbf{MUSICnb}}(r,\theta) = \frac{1}{\mathbf{p_s}(r,\theta)^+\mathbf{V_b}\mathbf{V_b}^+\mathbf{p_s}(r,\theta)}, \quad (5)$$

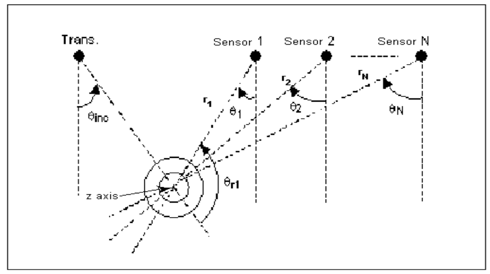


Fig. 1. Cylinder localization associated to each sensor.

where the new direction vector $\mathbf{p_s}(r,\theta)$ is now filled with the scattered field, written as,

$$\mathbf{p_s}(r,\theta) = [p_s(r_1,\theta_1),\ p_s(r_2,\theta_2),\ \dots,\ p_s(r_N,\theta_N)]^T,$$

observed for an object located at (r,θ). Then the location $(\hat{r},\hat{\theta})$ maximizing the MUSIC spectrum in (5) is selected as the estimated object center.
Because a two dimensional search requires that the exact scattered field be calculated at each point. Here we consider the case of infinitely long cylindrical shell of outer radius a, inner radius b, in a free space, located at (r_1,θ_1) the range and the bearing object respectively, associated to the first sensor of the array (figure 1). The two fluids outside and inside the shell are labeled by 1 and 3, respectively, sound velocities $c_{1,3}$, the wavenumbers $k_{1,3}$. Suppose that the incident field is a plane wave with an angle θ_{inc}. In order to calculate the exact solution for the scattered field $p_s(r_1,\theta_1)$ a decomposition of the different fields is used, according to the Bessel (J_m, N_m) and Hankel (H_m) functions [6]. We adopt cylindrical coordinates.
In medium 1, the pressure is taken as $p = p_i + p_s$, with a given incident plane wave pressure

$$p_i = p_0 \sum_{m=0}^{\infty} j^m \epsilon_m J_m(k_1 r_1)\cos(m\theta_{r1}), \quad (6)$$

where $\theta_{r1} = \theta_1 - \theta_{inc}$, p_0 a constant
and a scattered pressure

$$p_s = p_0 \sum_{m=0}^{\infty} i^m \epsilon_m b_m H_m^{(1)}(k_1 r_1)\cos(m\theta_{r1}), \quad (7)$$

we use $\epsilon_0 = 1, \epsilon_1 = \epsilon_2 = \dots = 2$, $b_m, \dots, g_m$ are coefficients and m is the number of modes.
In medium 2, the displacement vector **u** is written as

$$\mathbf{u} = -\nabla\psi + \nabla\phi, \quad (8)$$

where ψ is the scalar potential and ϕ is the vector potential. The solutions are written

$$\psi = p_0 \sum_{m=0}^{\infty} j^m \epsilon_m [c_m J_m(k_l r_1) + d_m N_m(k_l r_1)] \cos(m\theta_{r1}), \tag{9}$$

$$\phi = p_0 \sum_{m=0}^{\infty} j^m \epsilon_m [e_m J_m(k_t r_1) + f_m N_m(k_t r_1)] \cos(m\theta_{r1}), \tag{10}$$

where k_t and k_l are respectively the transversal and the longitudinal wavenumber.
In medium 3, one has again a compressional wave, which must be regular at the origin:

$$p_z = p_0 \sum_{m=0}^{\infty} j^m \epsilon_m g_m J_m(k_3 r_1) \cos(m\theta_{r1}). \tag{11}$$

Both at $r_1 = a$ and $r_1 = b$, the following boundary conditions have to be satisfied:

- the pressure in the fluid equals the normal component of stress in the solid,
- the normal component of displacement is continuous,
- the tangential component of shearing stress are zero.

$b_m, ..., g_m$ are six unknown coefficients which can be estimated from equations (6) - (11) [6]. Thus, equation (7) gives $p_s(r_1, \theta_1)$ and in a similar manner $p_s(r_n, \theta_n)$, associated to the nth sensor, can be determined. Where the couple (r_n, θ_n) is calculated using the general pythagore theorem, as shown in figure 1. The obtained r_n, θ_n are given by

$$r_n = \sqrt{r_{n-1}^2 - d^2 - 2r_{n-1} d \cos(\frac{\pi}{2} + \theta_{n-1})} \tag{12}$$

$$\theta_n = \cos^{-1}[\frac{d^2 + r_n^2 - r_{n-1}^2}{2r_{n-1} d}], \quad n = 2, ..., N. \tag{13}$$

The modified MUSIC algorithm presented in this section, is limited to one or multiple objects localization where the interactions are ignored. So, the localization problem is approached as if these objects are independently scattering the incident plane wave. Numerical examples are presented in section 4, in order to show the performance and the limit of this method in presence of interactions between objects (correlated signals) and when they are ignored (uncorrelated signals).

3. MODIFICATION OF THE MUSIC ALGORITHM FOR WIDEBAND SIGNALS

In the previous section, the modified MUSIC method has been developed for a narrowband signals. In this section, the employed signals are wideband. This choice is made in order to decorrelate the signals by means of an average of the focused spectral matrices. Therefore the objects can be localized even if the received signals are totally correlated. This would have not been possible with the narrowband signals without the spatial smoothing. We propose to apply the bilinear focusing operator [7], technique which divides the frequency band into L narrowbands. This technique transforms the received signals in the L bands into the focusing frequency f_0 and consequently decorrelates the signals [8][9]. Here, f_0 is the center frequency of the spectrum of the received signal and it is chosen as the focusing frequency.
The following is the step-by-step description of the technique:

1. using an ordinary beamformer to find an initial estimate of r, θ and the number of objects K,
2. filling the directional matrix,

   $$\hat{\mathbf{P}}_s(f_l) = [\mathbf{p_{s1}}(r, \theta, f_l), \mathbf{p_{s2}}(r, \theta, f_l), ..., \mathbf{p_{sK}}(r, \theta, f_l)],$$

   where each component of the directional vector $\mathbf{p_{sk}}(\mathbf{r}, \theta, \mathbf{f_l})$, $1 \leq k \leq K$, is filled using equation (7), $1 \leq l \leq L$,
3. estimating the spectral matrix output sensors data $\mathbf{\Gamma}(f_l)$ at frequency f_l,
4. calculating objects spectral matrix at each frequency f_l using:

   $$\begin{aligned} \mathbf{\Gamma}_s(f_l) \quad = \quad & (\hat{\mathbf{P}}_\mathbf{s}^+(f_l)\hat{\mathbf{P}}_\mathbf{s}(f_l))^{-1}\hat{\mathbf{P}}_\mathbf{s}^+(f_l) \\ & [\mathbf{\Gamma}(f_l) - \hat{\sigma}^2(f_l)\mathbf{I}]\hat{\mathbf{P}}_\mathbf{s}(f_l) \\ & (\hat{\mathbf{P}}_\mathbf{s}^+(f_l)\hat{\mathbf{P}}_\mathbf{s}(f_l))^{-1}, \end{aligned} \tag{14}$$

   where, $\mathbf{I}$ is the identity matrix and $\hat{\sigma}^2$ the estimated noise variance.
5. calculating the average of the spectral matrices associated to the objects:

   $$\mathbf{\Gamma}_s(f_0) = \frac{1}{L}\sum_{l=1}^{L} \mathbf{\Gamma}_s(f_l), \tag{15}$$

6. calculating $\hat{\mathbf{\Gamma}}(f_0) = \hat{\mathbf{P}}_\mathbf{s}(f_0)\mathbf{\Gamma}_s(f_0)\hat{\mathbf{P}}_\mathbf{s}^+(f_0)$ and $\hat{\mathbf{\Gamma}}(f_l) = \mathbf{\Gamma}(f_l) - \hat{\sigma}^2(f_l)\mathbf{I}$,
   then, the noise variance is estimated by :

   $$\hat{\sigma}^2(f_l) = \frac{1}{N-K}\sum_{i=K+1}^{N} \lambda_i(f_l), \tag{16}$$

   where $\lambda_i(f_l)$ is the ith eigenvalue of $\mathbf{\Gamma}(f_l)$,

7. estimating the bilinear focusing operator:

$$\mathbf{T}(f_0, f_l) = \mathbf{V}(f_0)\mathbf{V}^{+}(f_l), \qquad (17)$$

where $\mathbf{V}(f_0)$ and $\mathbf{V}(f_l)$ are the eigenvector matrices of $\hat{\mathbf{\Gamma}}(f_0)$ and $\hat{\mathbf{\Gamma}}(f_l)$, respectively,

8. calculating the focused spectral matrix:

$$\hat{\mathbf{\Gamma}}(f_0) = \frac{1}{L}\sum_{l=1}^{L}\mathbf{T}(f_0, f_l)\hat{\mathbf{\Gamma}}(f_l)\mathbf{T}^{+}(f_0, f_l), \qquad (18)$$

9. using a detection method (AIC or MDL) to estimate the number of objects [10].

The modified spatial spectrum of MUSIC method for wideband correlated signals is given by

$$\mathbf{p}_{MUSICwb}(r, \theta) = \frac{1}{\mathbf{p_s}(r, \theta, f_0)^{+}\mathbf{V_{b0}}\mathbf{V_{b0}}^{+}\mathbf{p_s}(r, \theta, f_0)}, \qquad (19)$$

where $\mathbf{V_{b0}}$ is the eigenvector matrix of $\hat{\mathbf{\Gamma}}(f_0)$ associated to the smallest eigenvalues.

4. NUMERICAL EXAMPLES

In this section, we present numerical examples on localization of multiple objects. In order to simplify the scattering field associated with the detection problem, the objects are modeled as simple, cylindrical shells with diameters of 0.02 m. The scattered field due to an incident plane wave is observed along a uniform, linear receiver array of 10 sensors. The distance between two adjacent sensors is 0.002 m. The objects are placed in a lossy, homogeneous space. In order to show the performance of the developed method, we consider three objects geometries. In the first case, the objects are located quite far from each other ($\theta_1 = 50^\circ$, $r_1 = 2$ m, $\theta_2 = 25^\circ$, $r_2 = 0.6$ m). For this case, the interactions between the objects are very weak (figure 2-a). In the second case, the objects are located closely ($\theta_1 = 50^\circ$, $r_1 = 2$ m, $\theta_2 = 48^\circ$, $r_2 = 2.2$ m), (figure 2-c). The employed signals, in the first and the second cases, are narrowband. The third case is similar to the second but wideband signals are used instead narrowband signals.

- uncorrelated signals are simulated by setting $E\left[\mathbf{s}\mathbf{s}^{+}\right] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,
- correlated signals are simulated by setting $E\left[\mathbf{s}\mathbf{s}^{+}\right] = \begin{bmatrix} 1 & 1e^{-j\varphi} \\ 1e^{+j\varphi} & 1 \end{bmatrix}$,

where φ is the dephasing between the signals scattered from the two objects. Then, we have applied our method. The obtained results are shown on figures 2-b, 2-d and 2-e. These figures show, the effects of the interactions between objects on the narrowband MUSIC algorithm, presented in section 2 and how the performances are completely degraded when the signals are correlated. Thus the use of the frequency diversity of the wideband signals by means of an average of the focused spectral matrices is necessary to locate multiple objects in presence of interactions.

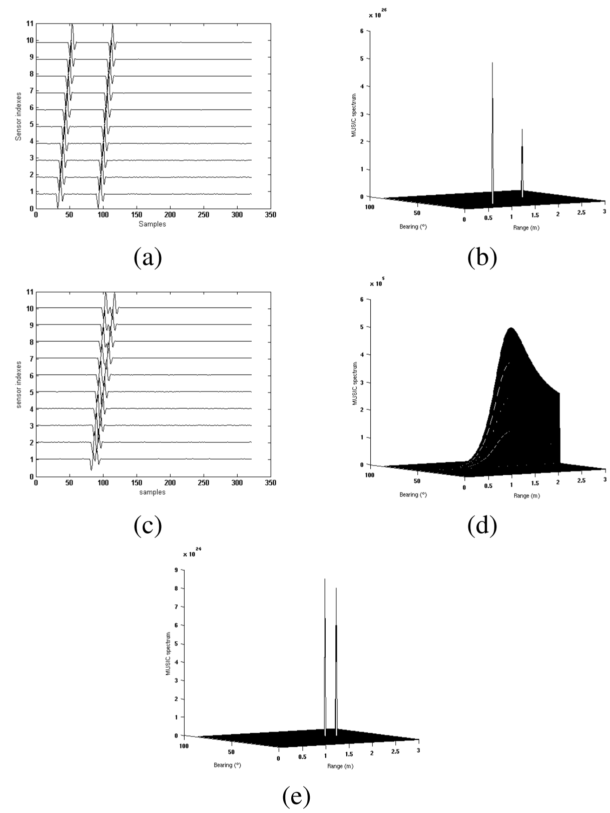


Fig. 2. The scattered field and the spectrum of the modified MUSIC algorithm for (a) and (b) narrowband uncorrelated signals. (c) and (d) narrowband correlated signals. (e) wideband correlated signals.

5. EXPERIMENTAL DATA

5.1. Experimental conditions

Time domain measurements in an experimental water tank have been used in order to evaluate the performances of the developed method. The experimental set up is shown on figure 3 where all the dimensions are given in meter. The bottom of the tank is full of fine and homogeneous sand where are buried six cylindrical shells, between 0 and 0.005 m, of different dimensions (table 1). We have done six experiments. The transmitter is fixed at an incident angle 60° and the receiver moves horizontally from the initial to the final position with a step size $d = 0.002$ m. The distance,

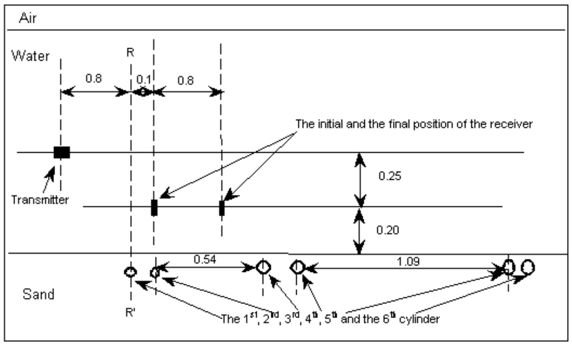


Fig. 3. Experimental set up

	1^{st} couple	2^{nd} couple	3^{rd} couple
inner radius a (m)	0.01	0.018	0.02
Filled of	air	water	air
Separated by (m)	0.13	0.16	0.06

Table 1. characteristics of the various cylinders (the inner radius $b = a - 0.001$ m)

between the transmitter, the RR' axis and the receiver, remains the same for all the experiments. For the three first experiments, we have fixed the receiver axis at 0.2 m from the bottom of the tank and the RR' axis is positioned on the 1^{st}, the 2^{nd} and the 3^{rd} cylinders couple. For the three last experiments, the receiver axis is fixed at 0.4 m and we have repeated the same experiments as in the first time.

5.2. Experimental data

The received signals come from the reflections on the buried objects thus these signals are correlated. It appears clearly that it is necessary to apply any preprocessing to decorrelate the signals. We propose to apply the bilinear focusing operator technique which divides the frequency band into L narrowbands. This technique transforms the received signals in the L bands into the focusing frequency f_0 and consequently decorrelates the signals . Here, f_0 is the center frequency of the spectrum of the received signal and it is chosen as the focusing frequency. Typical sensor output signals corresponding to one experiment are shown in figure 4. Sensor output signal in time and frequency domain are presented in figure 5. The frequency Band is $[fmin = 150, fmax = 250]$ kHz, the center frequency is $f_0 = 200$ kHz and the sampling rate is 2 MHz.

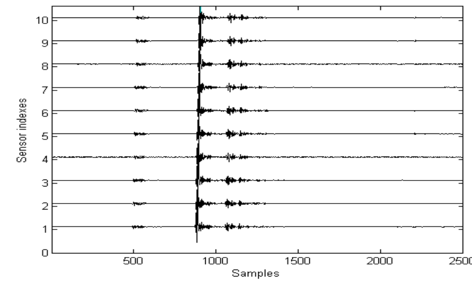
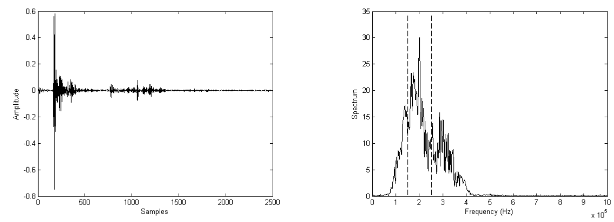


Fig. 4. Observed sensor output signals

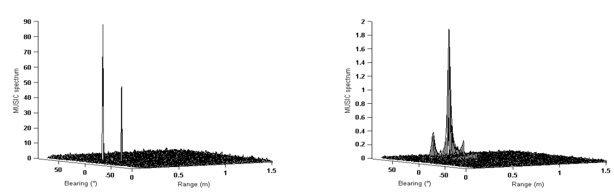


(a) (b)

Fig. 5. Typical sensor output signal during the experiment. (a) Temporal signal. (b) Spectrum of the signal

6. RESULTS AND DISCUSSIONS

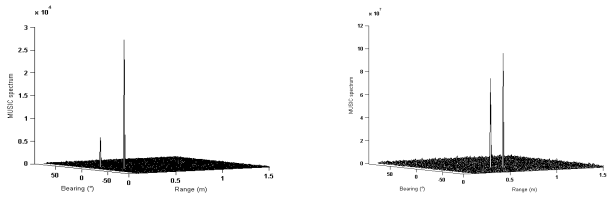
The considered cylinders are buried under the sand which has geoacoustic characteristics near to those of water. Then, we can make the assumption that the cylinders are in a free space. The eight steps, listed above, are applied on each experimental data. A sweeping on r and θ have been applied ($[0.2, 1.5]$ m for r and $[-90^\circ, 90^\circ]$ for θ) with $N = 10$ sensors. We have then applied the bilinear focusing operator in order to decorrelate the signals before applying the developed method. Thus the frequency wideband $B = [150, 250]$ kHz is divided into 100 narrowbands. The obtained spatial spectrum of the modified MUSIC method are shown in figures 6, 7 and 8. For each experiment, only one cylinders couple is radiated by the transmitter. Table 2 gives the real



(a) (b)

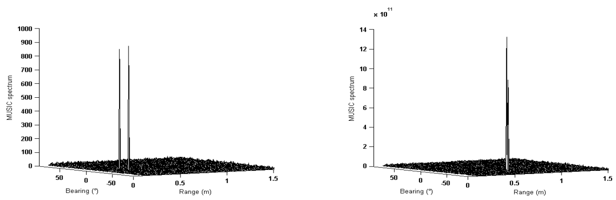
Fig. 6. Spectrum of the modified MUSIC method associated to the first cylinders couple. (a) The receiver is at 0.2 m from the bottom (Exp.1). (b) The receiver is at 0.4 m from the bottom (Exp.4)

and the estimated range and bearing values obtained with



(a) (b)

Fig. 7. Spectrum of the modified MUSIC method associated to the second cylinders couple. (a) The receiver is at 0.2 m from the bottom (Exp.2). (b) The receiver is at 0.4 m from the bottom (Exp.5)



(a) (b)

Fig. 8. Spectrum of the modified MUSIC method associated to the third cylinders couple. (a) The receiver is at 0.2 m from the bottom (Exp.3). (b) The receiver is at 0.4 m from the bottom (Exp.6)

the modified MUSIC method. The indexes 1 and 2 in table 2 are the 1^{st} and the 2^{nd} cylinder of each couple of cylinders. Note that, the difference between the estimated value $(r_{1,2est}, \theta_{1,2est})$ and the expected value $(r_{1,2exp}, \theta_{1,2exp})$ is very small and only two cylinders that have not been detected in Exp.4 and Exp.6, because, the received echo, associated to these cylinders, is rather weak.

	Exp.1	Exp.4	Exp.2	Exp.5	Exp.3	Exp.6
r_{1exp}(m)	0.24	0.65	0.26	1.24	0.26	0.65
$\theta_{1exp}(^\circ)$	-25	-50	-34	-70	-34	-50
r_{1est}(m)	0.25	0.63	0.29	1.21	0.28	0.63
$\theta_{1est}(^\circ)$	-23	-52	-33	-70	-32	-52
r_{2exp}(m)	0.22	0.56	0.24	1.17	0.22	0.64
$\theta_{2exp}(^\circ)$	8	-41	-22	-69	4	-49
r_{2est}(m)	0.25	$-$	0.25	1.2	0.23	$-$
$\theta_{2est}(^\circ)$	9	$-$	-20	-65	6	$-$

Table 2. (r, θ) expected (exp) and estimated (est) values with the modified MUSIC method (negative θ is clockwise from the vertical)

7. SUMMARY AND CONCLUSION

The array processing approaches, as the MUSIC method and the focusing operator, are combined with the exact solution of the scattered field in order to estimate both the range and the bearing objects. The performances of this method are investigated through real data associated to many cylinders buried under the sand. The proposed method is superior in terms of performance to the conventional method when the objects are in the far field and even in the near field. The range and the bearing objects are estimated with a significantly good accuracy due to the free space assumption. The proposed method is superior in terms of performance to the conventional method when the objects are in the far field and even in the near field The range and the bearing objects are correctly estimated.

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