# ROBUST WIDEBAND DOA ESTIMATION

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## ABSTRACT

Within the last decade there has been a growing interest in developing techniques for the estimation of the Direction Of Arrival (DOA) of wavefronts carrying wideband signals. The Coherent Signal-Subspace Method (CSM) is one of the most largely adopted technique and it is based upon the so-called focusing matrices. In the literature, several focusing matrices have been proposed and the most effective of them often require preliminary DOA estimation. Furthermore, classical design techniques do not take into account the fact that estimated DOAs may differ from the actual ones on which the manifold should be focused.

In this paper we propose a novel focusing matrix design technique. It is shown that some classical methods leave room for further optimization of the available degrees of freedom, that are exploited here to create a CSM robust against DOA estimation errors and which does not require any preliminary DOA estimation.

## 1. INTRODUCTION

Within the last decade there has been a growing interest in developing techniques for the estimation of the Direction Of Arrival (DOA) of wavefronts carrying wideband signals in order to locate the emitting sources.

One of the first approaches proposed in literature to cope with the wideband problem exploiting the concept of signal-subspace is referred to as *Incoherent Signal-Subspace Method* (ISM) [1]. Despite its simplicity, it has the main drawback of being unable to resolve coherent sources [2] which, in turn, are extremely likely to appear in all those propagation conditions wherein several delayed replica of a certain signal arrives at the array through different paths (multipath propagation).

Lately, Wang and Kaveh [2] proposed a technique referred to as *Coherent Signal-Subspace Method* (CSM) with the purpose of improving ISM performance by also handling wideband coherent sources. The main difficulty in developing coherent signal-subspace processing is due to the fact that the signal-subspace changes with frequency. The basic idea to overcome this problem is to apply a linear transformation to the array Power Spectral Density (PSD) matrix estimated at each frequency bin, with the purpose of removing the frequency dependence of the transformed signal-subspace and creating a single universal matrix having desired algebraic properties that can be exploited by subsequent processing stages to estimate the number of impinging wavefronts as well as their DOA. The linear transformation is performed by means of the so-called *focusing matrices*, whose name recalls the fact that they are designed to focus the signal subspace at each frequency bin into a certain reference frequency. Unfortunately, the focusing procedure often would require the knowledge of the exact DOAs, which are the final objective of the whole estimation procedure. Therefore, the DOAs are first roughly estimated and then focusing is applied onto an approximate signal-subspace. In this way a set of better estimates can be obtained and the procedure can be iteratively performed until stable DOA estimates are available. Among the proposals considered in [2] two relevant contributions are the *Auxiliary Direction Vector Augmentation* (ADVA) hereafter referred to as *Method 1* and a computationally lighter and simpler version referred to as *Method 2.*

After the work of Wang and Kaveh [2] many authors proposed improvements to the approach by investigating the characteristics of desirable focusing transformations and by suggesting better ways to design focusing matrices. In [3] Hung and Kaveh proposed a performance measure of focusing matrices referred to as *focusing loss*, which is defined as the ratio between the array signal to noise ratios after and before focusing operations. It is reasonable to assume that desirable focusing matrices should not have focusing loss, otherwise DOA estimation performed on focused signal-

This paper has been supported by the FIRB-PRIMO project founded founded by the Italian Ministero dell'Istruzione, Università e Ricerca (MIUR).

subspaces could produce poor results. On the basis of this definition, the authors [3] proposed a new class of focusing matrices referred to as *Rotational Signal-Subspace* (RSS), hereafter called *Type 3*. Lately Doron and Weiss [4] extended RSS to the more general class of *Signal-Subspace Transformation* (SST) matrices, referred to as *Type 4*, designed to generate sufficient statistics for maximum likelihood bearing estimation. In [5] Hung and Mao proposed a class of robust focusing matrices called *Unitary Constrained Array Manifold Focusing* (UCAM), and hereafter referred to as *Type 5*, which reduces the sensitivity of RSS-CSM to variations of initial focusing angles. The ideas underlying CSM have been slightly modified by Lee [6] which proposed a wideband DOA estimation scheme based upon the idea of frequency-invariant beamspace processing. The method, referred to as *Type 6*, is basically a fusion of beamspace processing with the classical CSM, but it is non-iterative, since the design of the beamspace matrices, which play the role of focusing matrices, does not take into account the spatial distribution of the sources. It is important to highlight that these focusing matrices are non-unitary and non-square.

Many of the most effective focusing techniques proposed in the literature so far share the same characteristic of requiring initial DOA estimates. The disadvantage caused by this initial step is twofold. Indeed, a pre-processing stage not relying upon the signal-subspace concept has to be included, which calls for additional computational burden. Furthermore, to keep the computational complexity low, the resulting lack of accuracy may lead to initial estimates too far from the true DOAs, thus intensifying the problem of bias and variance of certain types of focusing matrices. Lastly, the lower the initial accuracy, the longer the iterative DOA estimation procedure must be performed in order to get the final, possibly accurate, result.

On the basis of these considerations, it follows that good focusing should avoid initial DOA estimates and it should have some sort of robustness against DOA estimation errors. In this paper we propose a novel design procedure for focusing matrices. In particular, in the following it is shown that the classes of RSS or SST focusing matrices leave room for further optimization of the available degrees of freedom, that are exploited here to create a CSM robust against DOA estimation errors. For this reason, the proposed method is referred to as *Robust Coherent Signal-Subspace Method* (R-CSM). Thanks to the peculiarities of these novel focusing matrices, the initial preprocessing stage of the classical CSM is no longer required. Furthermore, the convergence speed is improved, because at each iteration the degrees of freedom are used to concentrate the focusing closer and closer about the estimated DOAs.

## 2. SIGNAL MODEL

In this section the mathematical model underlying the considered problem is detailed. In what follows, the symbols $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^*$ indicate the transpose, the Hermitian and the conjugate, respectively.

Let us consider $L$ spatially concentrated wide band sources emitting signals whose spectral content is assumed to be centered about a certain frequency $f_0$, and spread over a bandwidth $B$ comparable to $f_0$. The signals are supposed to be jointly stationary over an observation interval $T_1 = NT$, partitioned into $N$ non-overlapping subintervals of duration $T$ each. The signals emitted by the $L$ sources are collected by a linear array of $M > L$ wide-band sensors, uniformly spaced by half the minimum wavelength and located in the far field of all the sources.

According to the classical Coherent Signal Subspace Method (CSM), the array signal vector $\mathbf{x}(t)$ is first sampled at a sampling frequency $f_c$ and the resulting discrete-time array signal vector is partitioned into $N$ non-overlapping segments $\mathbf{X}_n \triangleq [\mathbf{x}_n[0],\, \mathbf{x}_n[1],\, \ldots\, \mathbf{x}_n[K-1]]$ of length $K = Tf_c$ samples each, where $\mathbf{x}_n[k] \triangleq \mathbf{x}[k + nK]$. Subsequently, a $K$-points Discrete Fourier Transform (DFT) is applied to each segment by producing $\tilde{\mathbf{X}}_n \triangleq \mathbf{X}_n\mathbf{F}_K^T = [\tilde{\mathbf{x}}_n[0],\, \tilde{\mathbf{x}}_n[1],\, \ldots\, \tilde{\mathbf{x}}_n[K-1]]$, where the matrix $\mathbf{F}_K$ is the $K$-points DFT matrix operator. It is not difficult to see that

$$\tilde{\mathbf{x}}_n[k] = \mathbf{A}\left(\mathbf{u}, f_k\right)\tilde{\mathbf{s}}_n[k] + \tilde{\boldsymbol{\eta}}_n[k], \qquad k = 1,\, 2 \ldots\, K-1 \tag{1}$$

where $f_k = \frac{k}{K} f_c$, $\tilde{\mathbf{s}}_n[k]$ and $\tilde{\boldsymbol{\eta}}_n[k]$ are the frequency-domain source and noise signal vectors, respectively, and $\mathbf{A}(\mathbf{u}, f_k)$ is a matrix whose columns are the steering vectors $\mathbf{a}(u_l, f_k)$, which depend on frequency $f_k$ and *spatial frequency* $u_l \triangleq d\sin(\theta_l)/\lambda_{\min}$, being $\theta_l$ the Direction Of Arrival (DOA), evaluated with respect to broadside. The column span of $\mathbf{A}(\mathbf{u}, f_k)$ identifies the *signal-subspace*, which is a function of frequency.

It can be shown [2] that the correlation matrix of $\tilde{\mathbf{x}}_n[k]$ can be written as

$$\mathbf{R}_{\tilde{\mathbf{x}}[k]\tilde{\mathbf{x}}[k]} = \mathbf{A}\left(\mathbf{u}, f_k\right)\mathbf{G_{ss}}(f_k)\mathbf{A}^H\left(\mathbf{u}, f_k\right) + G_{\eta\eta}(f_k)\mathbf{I}_M \tag{2}$$

where the matrix $\mathbf{G_{ss}}(f_k)$, referred to as *source signal Power Spectral Density (PSD) matrix*, may be singular in cases such as multipath propagation, the matrix $G_{\eta\eta}(f_k)$ is the noise PSD matrix and $\mathbf{I}_M$ is the $M \times M$ identity matrix.

By means of a properly chosen focusing matrix $\mathbf{T}_k$, whose purpose is to transform the signal-subspace at the frequency $f_k$ into a signal-subspace at the reference focusing frequency $f_0$, assumed to be the center frequency of the signal bandwidth, i.e.

$$\mathbf{T}_k\mathbf{A}\left(\mathbf{u}, f_k\right) = \mathbf{A}\left(\mathbf{u}, f_0\right) \tag{3}$$

each frequency domain array signal vector is then transformed into a new vector

$$\tilde{\mathbf{y}}_n[k] = \mathbf{T}_k \tilde{\mathbf{x}}_n[k] \tag{4}$$

and its correlation matrix assumes the form

$$\mathbf{R}_{\tilde{\mathbf{y}}[k]\tilde{\mathbf{y}}[k]} = \mathbf{A}\left(\mathbf{u}, f_0\right) \mathbf{G}_{\mathbf{ss}}(f_k) \mathbf{A}^H\left(\mathbf{u}, f_0\right) + G_{\eta\eta}(f_k) \mathbf{T}_k \mathbf{T}_k^H \tag{5}$$

This equation clearly shows the effect of focusing onto the signal-subspace. Its frequency-dependence is removed and each transformed frequency-domain array signal vector shares now the same signal-subspace, referred to as *coherent signal-subspace*.

The last step is to average all the correlation matrices $\mathbf{R}_{\tilde{\mathbf{y}}[k]\tilde{\mathbf{y}}[k]}$ to form a quantity called *Universal Spatial Correlation Matrix* which, under certain conditions, has the desired algebraic properties [2] that allow the application of any super-resolution subspace-based DOA estimation algorithm on it.

## 3. ROBUST COHERENT SIGNAL-SUBSPACE

Before considering the detailed analysis of the method, it is necessary to state the following result

**Theorem 3.1.** *Let* $\mathbf{A}$, $\mathbf{B} \in \mathbb{C}^{M,L}$, $L \leq M$ *be two arbitrary matrices with full column rank* $L$*. The following constrained minimization problem*

$$\mathbf{T}_{\text{opt}} = \arg\left\{\min_{\mathbf{T}} \|\mathbf{TA} - \mathbf{B}\|_{\text{F}}^2 \text{ subject to } \mathbf{T}^H\mathbf{T} = \mathbf{I}_M\right\} \tag{6}$$

*in the variable* $\mathbf{T} \in \mathbb{C}^{M,M}$*, admits a single solution for* $L > M-1$*, two solutions for* $L = M-1$ *and an infinite number of solutions for* $L < M-1$*. More specifically, being*

$$\mathbf{C} \triangleq \mathbf{AB}^H = \mathbf{U\Lambda V}^H = \begin{bmatrix}\mathbf{U}_1 & \mathbf{U}_2\end{bmatrix}\begin{bmatrix}\mathbf{\Lambda}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1^H\\ \mathbf{V}_2^H\end{bmatrix} \tag{7}$$

*the singular value decomposition of the matrix* $\mathbf{C}$*, the family of solutions can be written as*

$$\mathbf{T}_{\text{opt}} = \mathbf{VU}^H = \mathbf{V}_1\mathbf{U}_1^H + \mathbf{V}_2\mathbf{U}_2^H \tag{8}$$

*where* $\mathbf{U}_1, \mathbf{V}_1 \in \mathbb{C}^{M,L}$ *are univocally determined unitary matrices, while* $\mathbf{U}_2, \mathbf{V}_2 \in \mathbb{C}^{M,M-L}$ *are any matrices fulfilling the following properties*

$$\begin{cases}\mathbf{U}_2^H\mathbf{U}_2 = \mathbf{I}_{M-L}\\ \mathbf{U}_2^H\mathbf{U}_1 = \mathbf{0}_{M-L,L}\\ \mathbf{V}_2^H\mathbf{V}_2 = \mathbf{I}_{M-L}\\ \mathbf{V}_2^H\mathbf{V}_1 = \mathbf{0}_{M-L,L}\end{cases} \tag{9}$$

*where* $\mathbf{0}_{M-L,L}$ *is an* $(M-L) \times L$ *matrix of all zero entries.*

*Proof.* The proof of this theorem can be easily obtained by recalling the fact that Problem (6) is a Complex Orthogonal Procrustes Problem. Details are given in [9]. □

The ideas presented in [3] lead the way to an interesting and even more promising design criterion for focusing matrices. Indeed, by recognizing in Problem (6) the same optimization problem that defines Type 3 matrices, it is clear that whenever the number of sources $L$ is less than the number of sensors $M$, the RSS design does not produce univocally defined focusing matrices but, instead, it leads to a broad class of them. In other words, the optimization problem involved in the design of RSS matrices can be furthermore specified, and more requirements can be added to make the resulting focusing matrices unique and possibly better performing.

In [4] the authors pointed out that all the matrices of the SST and of the RSS class will have the same performance when the focusing angles are exactly equal to the true DOAs. Nonetheless, whenever the focusing angles are different, two focusing matrices belonging to the same class may perform quite differently, because they may present a different degree of sensitivity to focusing errors. Therefore, the sensitivity to DOA errors seems to be a relevant aspect that could be taken into account to improve the design of focusing matrices. Some sort of robustness against DOA estimation errors has to be included, so as to control the quality of focusing in the whole relevant portions of the visible region.

Finally, another important aspect of the whole iterative DOA estimation procedure concerns the initial preprocessing stage, which must be circumvented, somehow, and the very first DOA estimates must be accurate at possibly low computational costs.

Such considerations inspired the design of the focusing matrices presented in this section, leading to what is referred to as Robust Coherent Signal-Subspace Method (R-CSM). Two types of focusing matrices, to be applied in different moments of the iterative estimation procedure, are designed here. The first kind must be adopted at the very initial step and it is explicitly designed to work without requiring any initial guess whatsoever, while the second kind must be adopted subsequently and it is designed to work on the basis of the estimates available from previous iterations. Since focusing is applied from the very initial step, the same super-resolution signal-subspace-based DOA estimation algorithm can be applied from the beginning. This leads to improved initial estimates, with a possible reduction of the computational burden required to achieve a certain accuracy, as well as a reduction of the convergence time of the overall procedure.

The focusing matrices required in the very first step of the proposed R-CSM are designed according to the following optimization problem

$$\mathbf{T}_k[0] = \arg\min_{\mathbf{T}} \int_{-\frac{1}{2}}^{+\frac{1}{2}} \|\mathbf{T}\,\mathbf{a}(u, f_k) + \\ -\mathbf{a}(u, f_0)\|_{\mathrm{F}}^2\, w(u)\mathrm{d}u \text{ s. t. } \mathbf{T}^H\mathbf{T} = \mathbf{I}_M \quad (10)$$

in the variable $\mathbf{T} \in \mathbb{C}^{M,M}$. Here, $\mathbf{a}(u, f_k)$ is the array manifold evaluated at the frequency $f_k$ and at the spatial frequency $u$, $f_0$ is the focusing frequency, $w(u)$ is a generic weighting function and $\mathbf{I}_M$ is the $M \times M$ identity matrix.

As it can be observed, the form of the optimization problem (10) seems to be similar to the problem solved to obtain Type 6 focusing matrices, proposed in [6], but it differs in two aspects. First of all there is no reference focusing matrix and, more importantly, the resulting focusing matrices are constrained to be unitary, so as to avoid focusing loss. Other similarities could be found by looking at the definition of Type 3 focusing matrices, proposed in [3]. Nonetheless, Type 3 matrices focus the signal-subspace only in correspondence of some spatial frequencies, which are not necessarily close to the actual ones. In turn, since at this stage of R-CSM there is no *a priori* information concerning the DOA of impinging wavefronts, the focusing must be guaranteed to be robust with respect to the whole visible region $\mathcal{U} = [-1/2, 1/2]$.

**Theorem 3.2.** *The solution to problem (10) is given by*

$$\mathbf{T}_k[0] = \mathbf{V}_k\mathbf{U}_k^H \quad (11)$$

*where $\mathbf{V}_k$ and $\mathbf{U}_k$ are the matrices obtained from the singular value decomposition of the matrix*

$$\mathbf{Q}_k \triangleq \int_{-\frac{1}{2}}^{+\frac{1}{2}} \mathbf{a}(u, f_k)\mathbf{a}^H(u, f_0)w(u)\mathrm{d}u = \mathbf{U}_k\mathbf{\Lambda}_k\mathbf{V}_k^H \quad (12)$$

*Proof.* The proof is not reported here due to lack of space. The interested reader may want to refer to [9] for details. □

The matrices $\mathbf{T}_k[0]$ are adopted within the very first iteration of R-CSM and, after running a signal-subspace-based DOA estimation algorithm, such as MUSIC [7], to cite an example, a set of possibly reliable initial estimates $\hat{\mathbf{u}}[0] \triangleq [\hat{u}_1[0],\ \hat{u}_2[0],\ \ldots\ \hat{u}_L[0]]^T$ is available for refinements by subsequent iterations.

Within the next iterations of R-CSM, a second kind of focusing matrices is employed. In light of the results presented by Theorem 3.1 it is clear that only $L$ degrees of freedom are locked for the solution of problem (6), while the remaining $M - L$ can be exploited to add robustness against DOA estimation errors that might lead to signal-subspace misfocusing. First of all, due to Theorem 3.1, a *class* of matrices is obtained according to

$$\tilde{\mathbf{T}}_k[i] = \arg\Big\{\min_{\mathbf{T}} \|\mathbf{T}\mathbf{A}\left(\hat{\mathbf{u}}[i-1], f_k\right) + \\ -\mathbf{A}\left(\hat{\mathbf{u}}[i-1], f_0\right)\|_{\mathrm{F}}^2 \text{ s. t. } \mathbf{T}^H\mathbf{T} = \mathbf{I}_M\Big\} \quad (13)$$

where $\hat{\mathbf{u}}[i-1]$ is the vector of the spatial frequencies estimated at the $(i-1)$-th iteration and $\mathbf{A}\left(\hat{\mathbf{u}}[i-1], f_k\right)$ is a matrix whose columns are the steering vectors evaluated at the frequency $f_k$ and at each of the available spatial frequencies. Subsequently, the unique robust focusing matrix is obtained by solving

$$\mathbf{T}_k[i] = \arg\min_{\tilde{\mathbf{T}}_k[i]} \sum_{l=1}^{L} \int_{\mathcal{U}_l[i]} \left\|\tilde{\mathbf{T}}_k[i]\mathbf{a}(u, f_k) + \\ -\mathbf{a}(u, f_0)\right\|_{\mathrm{F}}^2 w(u)\mathrm{d}u \text{ s. t. } \tilde{\mathbf{T}}_k^H[i]\tilde{\mathbf{T}}_k[i] = \mathbf{I}_M \quad (14)$$

over the class of matrices provided by problem (13). The quantities $\mathcal{U}_l[i]$ are referred to as *robustness intervals* and define the region over which the robustness against DOA estimation errors needs to be mainly concentrated. In other words, after the first iteration, it may be no longer necessary to focus the array manifold over the whole visible region and the available degrees of freedom may be more effectively exploited by concentrating the robustness about the estimated spatial frequencies.

**Theorem 3.3.** *The solution to problems (13) and (14) is given by*

$$\mathbf{T}_k[i] = \mathbf{V}_1\mathbf{U}_1^H + \mathbf{Y}\mathbf{F}\mathbf{G}^H\mathbf{X}^H \quad (15)$$

*where:*

- *The matrices $\mathbf{U}_1$ and $\mathbf{V}_1$ are obtained from the singular value decomposition of the matrix*

$$\mathbf{A}\left(\hat{\mathbf{u}}, f_k\right)\mathbf{A}^H\left(\hat{\mathbf{u}}, f_0\right) = \\ = \begin{bmatrix}\mathbf{U}_1 & \mathbf{U}_2\end{bmatrix}\begin{bmatrix}\mathbf{\Lambda}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1^H\\ \mathbf{V}_2^H\end{bmatrix} \quad (16)$$

- *$\mathbf{X}$ is a matrix whose columns form an orthonormal basis for the orthogonal complement of the span of $\mathbf{U}_1$*
- *$\mathbf{Y}$ is a matrix whose columns form an orthonormal basis for the orthogonal complement of the span of $\mathbf{V}_1$*
- *The matrices $\mathbf{F}$ and $\mathbf{G}$ are obtained from the singular value decomposition of the matrix*

$$\mathbf{Y}^H\mathbf{Q}^H\mathbf{X} = \mathbf{F}\mathbf{\Delta}\mathbf{G}^H \quad (17)$$

*being*

$$\mathbf{Q} \triangleq \sum_{l=1}^{L} \int_{\mathcal{U}_l} \mathbf{a}(u, f_k)\mathbf{a}^H(u, f_0)w(u)\mathrm{d}u \quad (18)$$

*Proof.* The proof is not reported here due to lack of space. The interested reader may want to refer to [9] for details. □

The optimization problem (13) is coincident with the problem that defines RSS matrices. Nonetheless, as previously observed, the result of problem (13) is a class of matrices, rather than a single one. Therefore in order to extract from that class a matrix with desired properties, a further optimization is applied by solving problem (14). In this way it is possible to obtain a focusing matrix that not only avoids focusing loss, but also that exploits the previous information on the possible location of the sources in order to better concentrate the focusing. Thereby, robustness is added against DOA estimation errors.

The main critical point of the presented design could be the computational complexity when dealing with some particular array geometry. In fact, the matrices $\mathbf{Q}$, defined by equation (18) can be computed in closed form for some regular array geometry only, such as uniform and linear, to cite an example. In the case of more sophisticated geometries, the integrals involved in their definitions need to be computed numerically or approximated somehow, which could increase the computational burden of the overall estimation procedure.

## 4. SIMULATION RESULTS

In the first simulation, the behavior of the proposed R-CSM algorithm is investigated by considering $L = 3$ broadband sources emitting plane wavefronts from DOAs $\theta_1 = -30°$, $\theta_2 = -10°$ and $\theta_1 = 25°$, respectively. The spectral content of the source signals is centered about the frequency $f_0 = 70$ Hz, with a bandwidth $B = 100$ Hz.

The emitted wavefield, travelling at a speed $v = 300$ m/s, is sensed at the receiver by an uniform and linear antenna array, made by $M = 8$ wide-band omnidirectional sensors. The inter-elements spacing $d = v/(2f_0 + B)$ is set to half the smallest wavelength, to avoid DOA estimation ambiguity in the whole frequency range. At each sensor a noise process is present, whose power is such that the input SNR is 0 dB.

After reception, the source signal is sampled at a sampling frequency $f_c = 300$ Hz. $N = 10$ snapshots, of length $K = 256$ samples each, are collected and used by the proposed R-CSM algorithm to estimate the sources' DOA. The adopted narrowband DOA estimation algorithm is the well-known MUSIC algorithm [7].

In Figure 1 the focused MUSIC spatial power spectrum is depicted. It refers to the very first iteration of the proposed method, called *Iteration 0*, which plays the role of the initialization step of other focusing techniques. At this stage of the algorithm, no information concerning the sources' DOA is available and therefore the focusing matrices described by equation (11) must be adopted. As it can be seen, the focused MUSIC spatial spectrum shows a very promising shape even at this initial stage. Three narrow peaks are highlighted, each of which is centered about one of the actual DOAs, indicated by the dashed vertical lines.

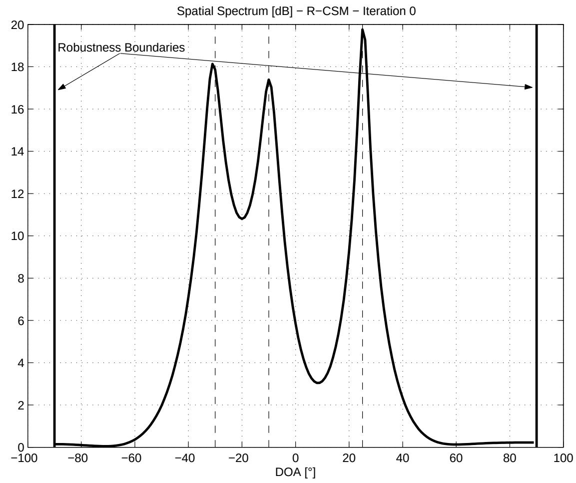


**Fig. 1**. Focused MUSIC spatial spectrum @ Iteration 0

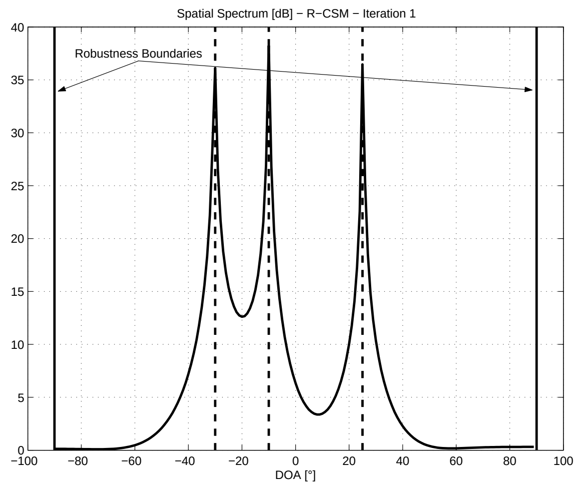


**Fig. 2**. Focused MUSIC spatial spectrum @ Iteration 1

The estimation accuracy is improved in subsequent iterations of the procedure, in which the information about previous DOA estimates is available and can be exploited by adopting the focusing matrices described by equation (15). In Figure 2 the focused MUSIC spatial spectrum at *Iteration 1* is shown. In this case, thanks to the peculiarity of the adopted focusing matrices, the shape of the MUSIC power spectrum is improved, since the peeks are now narrower and closer to the actual DOAs.

In the second simulation, the behavior of the proposed algorithm is compared with the other cited techniques, in

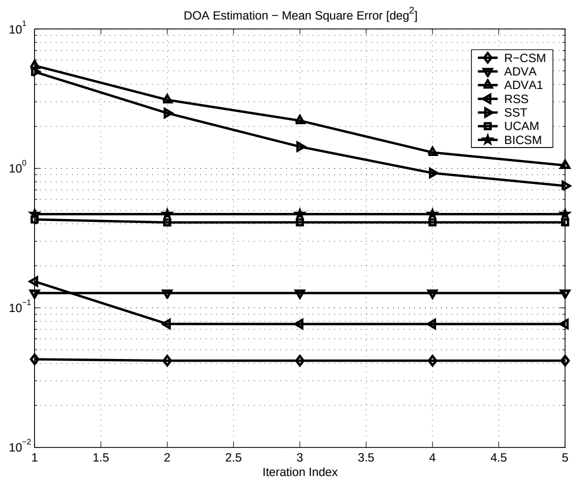


**Fig. 3**. Performance comparison: estimation mean square error.

terms of DOA estimation mean square error, measured by averaging the results over 50000 Monte Carlo runs. The simulation conditions are the same as for the aforementioned case, apart from the sources. In this second case only one source has been simulated whose DOA has been chosen randomly at each run in the range $[-60°, +60°]$. For those algorithms requiring initial DOA estimates, a value within a $\pm 2°$ about the actual DOA has been provided.

Figure 3 shows the comparison of the cited algorithms' performance in terms of DOA estimations mean square error, as a function of the number of iterations of the DOA estimation procedure. As it can be seen, the proposed R-CSM outperforms the other considered methods. Furthermore, since the R-CSM estimations are quite accurate from the very first iteration, the proposed method achieves convergence with a substantially lower number of iterations, which makes the procedure even more attractive under a computational viewpoint.

## 5. CONCLUSIONS

In this paper we have proposed a novel focusing matrices design technique aimed at counteracting some of the main disadvantages of other classical focusing matrices. The available degrees of freedom, are exploited to create a CSM robust against DOA estimation errors. Thanks to the peculiarities of these novel focusing matrices, the initial preprocessing stage of the classical CSM is no longer required.

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