CYCLOSTATIONARITY-BASED PARAMETER ESTIMATION OF WIDE-BAND SIGNALS IN MOBILE COMMUNICATIONS

Valentina De Angelis [(a)] *Luciano Izzo* [(a)] *Antonio Napolitano* [(b)] *Mario Tanda* [(a)]

[(a)] Università di Napoli "Federico II", Dipartimento di Ingegneria Elettronica e delle Telecomunicazioni
via Claudio 21, I-80125 Napoli, Italy
[vdeangel izzoluc tanda]@unina.it

[(b)] Università di Napoli "Parthenope", Dipartimento per le Tecnologie
via Acton 38, I-80133 Napoli, Italy
antonio.napolitano@uniparthenope.it

ABSTRACT

In this paper, the problem of blind estimation of signal parameters of wide-band signals in mobile communications systems is addressed. It is shown that the wide-band assumption implies that a nonunit time-scale factor needs to be accounted for in the received-signal model even for moderate data-record lengths. Cyclostationarity and conjugate cyclostationarity properties are exploited to estimate the time-scale factor, frequency offset, scaling amplitude, phase shift, and time delay of the received signal. Simulation results show the effectiveness of the proposed algorithm.

1. INTRODUCTION

In communication systems, the problem of the estimation of the received-signal parameters is of great interest for synchronization, equalization, and power-control purposes. In narrow-band or moderately wide-band systems, parameters of interest are amplitude, phase, time delay, and frequency offset. The amplitude accounts for the transmitted power and the attenuation introduced by the channel, the phase shift is due to the phase rotation introduced by the channel and the possible phase mismatch between the transmitter and receiver oscillators, the time delay is consequence of the propagation delay, and the frequency offset is due to Doppler effect and mismatch between the frequencies of the transmitter and receiver oscillators.

Both data-aided and non data-aided (blind) algorithms have been proposed to estimate amplitude, phase, time delay, and frequency offset of the received signal. In particular, some blind algorithms exploit the cyclostationarity properties exhibited by almost all modulated signals. Cyclostationary signals have statistical functions such as the autocorrelation function, moments and cumulants that are almost-periodic functions of time [3]. The frequencies of the Fourier series expansion of such almost-periodic functions are called cycle frequencies and are related to parameters such as the carrier frequency and the baud rate. Unlike second-order stationary statistics, second-order cyclic statistics (e.g., the cyclic spectrum and the conjugate cyclic spectrum) preserve phase information and, hence, are suitable for developing blind estimation algorithms. Cyclostationarity-exploiting blind algorithms have been proposed in both single-user [2], [4], [12] and multi-user [7], [8] scenarios and their performance has been assessed.

In wide-band mobile communication systems, however, the channel can also introduce a non unit time-scale factor in the received signal [13]. Specifically, this non unit time-scale factor is observed when there is a relative motion between the transmitter and the receiver and the relative radial speed is such that the product between signal-bandwidth and data-record length is not much smaller than the ratio between the medium propagation speed and the radial speed [13]. Therefore, such a model is appropriate in modern communication systems where wider and wider bandwidths are considered to get higher and higher bit rates and, moreover, large data-record lengths are used for blind channel identification or equalization algorithms or for detection techniques in highly noise- and interference-corrupted environments. For an application in space communications, see [9].

In the present paper, a cyclostationarity-based algorithm is presented to estimate time-scale factor, frequency offset, scaling amplitude, phase shift, and time delay of the received signal. The time-scale factor and the frequency offset modify the cycle frequencies and the conjugate cycle frequencies of the transmitted signal. Thus, the proposed algorithm performs the time-scale factor and frequency offset estimations by estimating a cycle frequency and a conjugate cycle frequency of the received signal. The estimated (conjugate) cycle frequencies are then filled in the second-order cyclic statistics exploited to estimate the remaining parameters. In order to estimate amplitude, time delay, and phase, the minimization of two mean-square errors (MSEs) is performed. The former is the MSE between the measured and the actual cyclic spectrum and the latter is the MSE between the measured and the actual conjugate cyclic spectrum. Under the assumption that the transmitted signal exhibits at least one (conjugate) cycle frequency not too close to those of the disturbance signal, the proposed algorithm provides estimates of the unknown parameters that are intrinsically immune to the effects of noise and interference. Therefore, it is highly tolerant to noise and interference in practice, provided that a sufficiently long data record is utilized for the estimates. Finally, it is worthwhile to emphasize that the proposed algorithm is not based on the assumption of white and/or Gaussian noise. Thus,

This work was partially supported by "Centro Regionale di Competenza sull'ICT".

it can be suitably used in the presence of unknown narrow-band interference and/or in impulsive noise environments encountered in many physical channels such as urban and indoor radio channels [10].

2. RECEIVED-SIGNAL MODEL

In mobile communications, if the relative radial speed between transmitter and receiver can be assumed constant within the observation interval, then the transmitted complex envelope $x(t)$ is received as [13, pp. 240-242]

$$\begin{aligned} r(t) &= y(t) + n(t) \\ &= A e^{j\varphi}\, x(st-d)\, e^{j2\pi\nu t} + n(t) \end{aligned} \tag{1}$$

where $n(t)$ is additive noise, A is the scaling amplitude, φ the phase shift, d the time delay, s the time-scale factor, and ν the frequency offset. In (1), $s = c/(c+v)$ and $\nu = -f_c v/(c+v) + \nu_{\rm osc}$, where c is the medium propagation speed, v the relative radial speed, f_c the carrier frequency of the transmitted signal, and $\nu_{\rm osc}$ the frequency mismatch between the transmitter and receiver oscillators. Moreover, $\varphi = \varphi_{\rm ch} + \varphi_{\rm osc}$, where $\varphi_{\rm ch}$ is the phase shift introduced by the channel and $\varphi_{\rm osc}$ is the phase mismatch between transmitter and receiver oscillators.

It can be shown that the time-scale factor s can be considered unitary provided that the condition

$$BT \ll c/v + 1 \tag{2}$$

is fulfilled, where B is bandwidth of $x(t)$ and T is the data-record length [13, pp. 240-242]. In such a case, the channel between transmitter and receiver can be modelled as linear periodically time variant.

There are practical situations, however, where the BT product does not satisfy (2) and, hence, the time-scale factor cannot be considered unitary [5]. For example, let us consider the transmitted signal in a code-division multiple access (CDMA) system with chip period T_c, number of chip per bit N_c, bit period $T_b = N_c T_c$, and T_c-duration rectangular chip-pulse. By assuming an approximate bandwidth $B \simeq 1/T_c$, it results that condition (2) becomes

$$BT \simeq N_b N_c \ll c/v + 1 \tag{3}$$

where $N_b = T/T_b$ is the number of processed bits in the data-record of length T. Therefore, if $c \simeq 3{\cdot}10^8$ m s^{-1}, v=100 km h^{-1}, and $N_c = 512$, then (3) leads to $N_b \ll 2{\cdot}10^4$. That is, if the maximum number of processed bits exceeds few hundreds, the non-unit time-scale factor should be accounted for. Further examples where condition (2) is not fulfilled occur in ultra wide-band communications [11] and space communications [9].

The signal model (1) is adopted in radar/sonar applications where $x(t) \in L^2([0,T])$, $[0,T]$ being the observation interval. In this case, the signal parameters are estimated by resorting to the wide-band ambiguity function (see, e.g., [6]). In communications applications, however, for the purpose of signal-parameter estimation or synchronization, the transmitted and received signals are modelled as cyclostationary and, hence, with infinite energy. Consequently, their wide-band ambiguity function can be defined only resorting to generalized functions.

3. THE PROPOSED METHOD

The proposed estimation method is based on the computation of second-order cyclic statistics of the received signal $r(t)$. The cyclic autocorrelation function (at cycle frequency α) and the conjugate cyclic autocorrelation function (at conjugate cycle frequency β) of $r(t)$ are defined as

$$R^{\alpha}_{rr^*}(\tau) \triangleq \left\langle r(t+\tau) r^*(t)\, e^{-j2\pi\alpha t} \right\rangle \tag{4}$$

and

$$R^{\beta}_{rr}(\tau) \triangleq \left\langle r(t+\tau) r(t)\, e^{-j2\pi\beta t} \right\rangle \tag{5}$$

respectively, where superscript $*$ denotes complex conjugation and $\langle\cdot\rangle$ infinite-time average [3]. The Fourier transforms of $R^{\alpha}_{rr^*}(\tau)$ and $R^{\beta}_{rr}(\tau)$, denoted by $S^{\alpha}_{rr^*}(f)$ and $S^{\beta}_{rr}(f)$, are referred to as the cyclic spectrum and the conjugate cyclic spectrum, respectively [3].

Under the assumption that both $x(t)$ and $n(t)$ in (1) are zero-mean and statistically independent, the cyclic spectrum and the conjugate cyclic spectrum of the received signal $r(t)$ are given by

$$S^{\alpha}_{rr^*}(f) = \frac{A^2}{|s|}\, e^{-j2\pi\alpha d/s} S^{\alpha/s}_{xx^*}\left(\frac{f-\nu}{s}\right) + S^{\alpha}_{nn^*}(f) \tag{6}$$

$$\begin{aligned} S^{\beta}_{rr}(f) &= \frac{A^2}{|s|}\, e^{j2\varphi}\, e^{-j2\pi(\beta-2\nu)d/s} \\ &\qquad S^{(\beta-2\nu)/s}_{xx}\left(\frac{f-\nu}{s}\right) + S^{\beta}_{nn}(f) \end{aligned} \tag{7}$$

respectively, where $S^{\alpha}_{xx^*}(f)$ and $S^{\beta}_{xx}(f)$ are the cyclic spectrum and the conjugate cyclic spectrum, respectively, of $x(t)$ and $S^{\alpha}_{nn^*}(f)$ and $S^{\beta}_{nn}(f)$ those of $n(t)$.

Let α_0 and β_0 be a cycle frequency and a conjugate cycle frequency of $x(t)$, respectively. Moreover, assume that the values of the time-scale factor s and the frequency offset ν are such that, for some $\Delta\alpha$ and $\Delta\beta$, there is only one cycle frequency of $y(t)$ in the set $J(\alpha_0, \Delta\alpha) \triangleq [\alpha_0 - \Delta\alpha/2, \alpha_0 + \Delta\alpha/2]$ and only one conjugate cycle frequency of $y(t)$ in the set $J(\beta_0, \Delta\beta)$, and, moreover $S^{\alpha}_{nn^*}(f) = 0$ for $\alpha \in J(\alpha_0, \Delta\alpha)$ and $S^{\beta}_{nn}(f) = 0$ for $\beta \in J(\beta_0, \Delta\beta)$. Thus, accounting for (6) and (7), estimates of s and ν can be obtained by

$$\widehat{s} = \frac{\widehat{\alpha}}{\alpha_0} \qquad\qquad \widehat{\nu} = \frac{1}{2}(\widehat{\beta} - \widehat{s}\beta_0) \tag{8}$$

where

$$\widehat{\alpha} = \arg \max_{\alpha\in J(\alpha_0,\Delta\alpha)} \int_{\mathbb{R}} \left|\widehat{S}^{\alpha}_{rr^*}(f)\right|^2 \mathrm{d}f \tag{9}$$

$$\widehat{\beta} = \arg \max_{\beta\in J(\beta_0,\Delta\beta)} \int_{\mathbb{R}} \left|\widehat{S}^{\beta}_{rr}(f)\right|^2 \mathrm{d}f \tag{10}$$

with $\widehat{S}^{\alpha}_{rr^*}(f)$ and $\widehat{S}^{\beta}_{rr}(f)$ being estimates of the cyclic spectrum and the conjugate cyclic spectrum of $r(t)$, respectively. Note that the above mentioned assumptions can be easily verified provided that $|\alpha_0 - \alpha_I| > \Delta\alpha/2$ and $s\alpha_0 \in J(\alpha_0, \Delta\alpha)$ with α_I the closest to α_0 cycle frequency of $n(t)$ and, in addition, $|\beta_0 - \beta_I| > \Delta\beta/2$ and $s\beta_0 + 2\nu \in J(\beta_0, \Delta\beta)$ with β_I the closest to β_0 conjugate cycle frequency of $n(t)$.

Estimates of the remaining parameters A, φ, and d can be obtained by two minimum mean-square error procedures.

From equation (6) it follows that the estimates of the amplitude and time-delay can be obtained by minimizing with respect to γ the function

$$g(\gamma,\gamma^*) \triangleq \int_{\mathbb{R}} \left| \widehat{S}^{\widehat{s}\alpha_0}_{rr^*}(f) - \gamma\, S^{\alpha_0}_{xx^*}\left(\frac{f-\widehat{\nu}}{\widehat{s}}\right)\right|^2 \mathrm{d}f\,. \tag{11}$$

In the ideal case of perfect measurements ($\widehat{S}^{\alpha}_{rr^*}(f) \equiv S^{\alpha}_{rr^*}(f)$), according to (6) with $S^{\alpha}_{nn^*}(f) \equiv 0$, the solution of the minimization problem is

$$\gamma^{(\mathrm{opt})} = \frac{A^2}{|s|} e^{-j2\pi\alpha_0 d} \tag{12}$$

from which it follows $g(\gamma^{(\mathrm{opt})}, \gamma^{(\mathrm{opt})*}) = 0$. In the real case of finite data-record length, the function $g(\gamma,\gamma^*)$ can be minimized by solving the equation

$$\left.\frac{\partial}{\partial\gamma} g(\gamma,\gamma^*)\right|_{\gamma=\gamma^{(\mathrm{opt})}} = 0 \tag{13}$$

with γ and γ^* considered as independent variables [1]. This leads to

$$\gamma^{(\mathrm{opt})} = \int_{\mathbb{R}} \widehat{S}^{\widehat{s}\alpha_0}_{rr^*}(f)\, S^{\alpha_0}_{xx^*}\left(\frac{f-\widehat{\nu}}{\widehat{s}}\right)^* \mathrm{d}f \cdot \left[\int_{\mathbb{R}} \left|S^{\alpha_0}_{xx^*}\left(\frac{f-\widehat{\nu}}{\widehat{s}}\right)\right|^2 \mathrm{d}f\right]^{-1}. \tag{14}$$

Consequently, according to (12), the estimates of amplitude and time-delay are obtained as

$$\widehat{A} = \left|\gamma^{(\mathrm{opt})}\widehat{s}\right|^{1/2} \tag{15}$$

$$\widehat{d} = -\frac{1}{2\pi\alpha_0} \angle\left[\gamma^{(\mathrm{opt})}\right] \tag{16}$$

where $\angle[\cdot]$ denotes the angle of a complex quantity.

From equation (7) it follows that the estimate of the phase can be obtained by minimizing with respect to $\bar{\gamma}$ the function

$$h(\bar{\gamma},\bar{\gamma}^*) \triangleq \int_{\mathbb{R}} \left| \widehat{S}^{\widehat{s}\beta_0+2\widehat{\nu}}_{rr}(f) - \bar{\gamma}\, S^{\beta_0}_{xx}\left(\frac{f-\widehat{\nu}}{\widehat{s}}\right)\right|^2 \mathrm{d}f\,. \tag{17}$$

In the ideal case of perfect measurements, ($\widehat{S}^{\beta}_{rr}(f) \equiv S^{\beta}_{rr}(f)$), according to (7) with $S^{\beta}_{nn}(f) \equiv 0$, the solution of the minimization problem is

$$\bar{\gamma}^{(\mathrm{opt})} = \frac{A^2}{|s|}\, e^{j2\varphi}\, e^{-j2\pi\beta_0 d} \tag{18}$$

from which it follows $h(\bar{\gamma}^{(\mathrm{opt})}, \bar{\gamma}^{(\mathrm{opt})*}) = 0$. In the real case of finite data-record length, the function $h(\bar{\gamma},\bar{\gamma}^*)$ can be minimized by solving the equation

$$\left.\frac{\partial}{\partial\bar{\gamma}} h(\bar{\gamma},\bar{\gamma}^*)\right|_{\bar{\gamma}=\bar{\gamma}^{(\mathrm{opt})}} = 0 \tag{19}$$

which leads to

$$\bar{\gamma}^{(\mathrm{opt})} = \int_{\mathbb{R}} \widehat{S}^{\widehat{s}\beta_0+2\widehat{\nu}}_{rr}(f)\, S^{\beta_0}_{xx}\left(\frac{f-\widehat{\nu}}{\widehat{s}}\right)^* \mathrm{d}f \cdot \left[\int_{\mathbb{R}} \left|S^{\beta_0}_{xx}\left(\frac{f-\widehat{\nu}}{\widehat{s}}\right)\right|^2 \mathrm{d}f\right]^{-1}. \tag{20}$$

Thus, according to (12) and (18), the estimate of the phase is obtained as

$$\widehat{\varphi} = \frac{1}{2} \angle\left[\frac{\bar{\gamma}^{(\mathrm{opt})}}{\gamma^{(\mathrm{opt})}}\, e^{-j2\pi(\alpha_0-\beta_0)\widehat{d}}\right]. \tag{21}$$

Finally, note that it can be straightforwardly verified that

$$g(\gamma,\gamma)^*|_{\gamma=\gamma^{(\mathrm{opt})}+\epsilon} = g(\gamma,\gamma^*)|_{\gamma=\gamma^{(\mathrm{opt})}} + S|\epsilon|^2 \tag{22}$$

where

$$S \triangleq \int_{\mathbb{R}} \left|S^{\alpha_0}_{xx^*}\left(\frac{f-\widehat{\nu}}{\widehat{s}}\right)\right|^2 \mathrm{d}f\,. \tag{23}$$

Thus, $\gamma^{(\mathrm{opt})}$ is a global minimum for (11) since $S > 0$. Analogously, $\bar{\gamma}^{(\mathrm{opt})}$ is a global minimum for (17).

4. SIMULATION RESULTS

Monte Carlo simulations have been carried out to corroborate the effectiveness of the proposed estimators. In the experiments, $x(t)$ is a binary pulse-amplitude-modulated (PAM) signal with Nyquist-shaped pulse with 0.85 excess bandwidth and bit period $T_b = 9T_s$, T_s being the sampling period. The noise $n(t)$ is a stationary circular white Gaussian noise and the signal-to-noise ratio in the band $(-1/2T_s, 1/2T_s)$ is 10 dB. The received-signal parameters are $s = 1.0013$, $\nu = 0.00047/T_s$, $A = 1$, $\varphi = 0.78$, and $d = 2.7T_s$. The cycle frequency and conjugate cycle frequency of $x(t)$ are $\alpha_0 = \beta_0 = 1/T_b$ [3]. The (conjugate) cyclic spectra are estimated by the frequency-smoothed (conjugate) cyclic periodogram [3] with a rectangular frequency-smoothing window with width $\Delta f = 0.027/T_s$, on the basis of a number of bits N_b ranging from 2^6 to 2^{11}. The performance of the algorithm is determined in terms of the sample root mean-square error (rmse), evaluated over 400 runs, for time-scale factor, frequency shift, amplitude, delay, and phase shift normalized to $|s|$, $1/T_b$, A, T_b, and 2π, respectively (see Fig. 1). The results show that the proposed algorithm largely outperforms the one that estimates the remaining parameters neglecting the time-scaling effect (i.e., that assumes $s = 1$ in (1)). Moreover, for a sufficiently large data-record length, the performance of the proposed estimator for amplitude, delay, and phase is very close to that of the algorithm that assumes perfect knowledge of frequency shift and time-scale factor.

5. REFERENCES

[1] D. H. Brandwood, “A complex gradient operator and its application in adaptive array theory,” *IEE Proc. Part F and H*, vol. 130, pp. 11-16, February 1983.

[2] P. Ciblat, P. Loubaton, E. Serpedin, G. B. Giannakis, “Performance of blind carrier offset estimators for non circular transmissions through frequency-selective channels,” *IEEE Trans. on Signal Processing*, vol. 50, pp. 130-140, January 2002.

[3] W.A. Gardner, *Statistical Spectral Analysis: A Nonprobabilistic Theory*. Englewood Cliffs, NJ, Prentice-Hall, 1988.

[4] F. Gini and G. B. Giannakis, “Frequency offset and symbol timing recovery in flat-fading channels: a cyclostationary approach,” *IEEE Trans. on Communications*, vol. 46, March 1998.

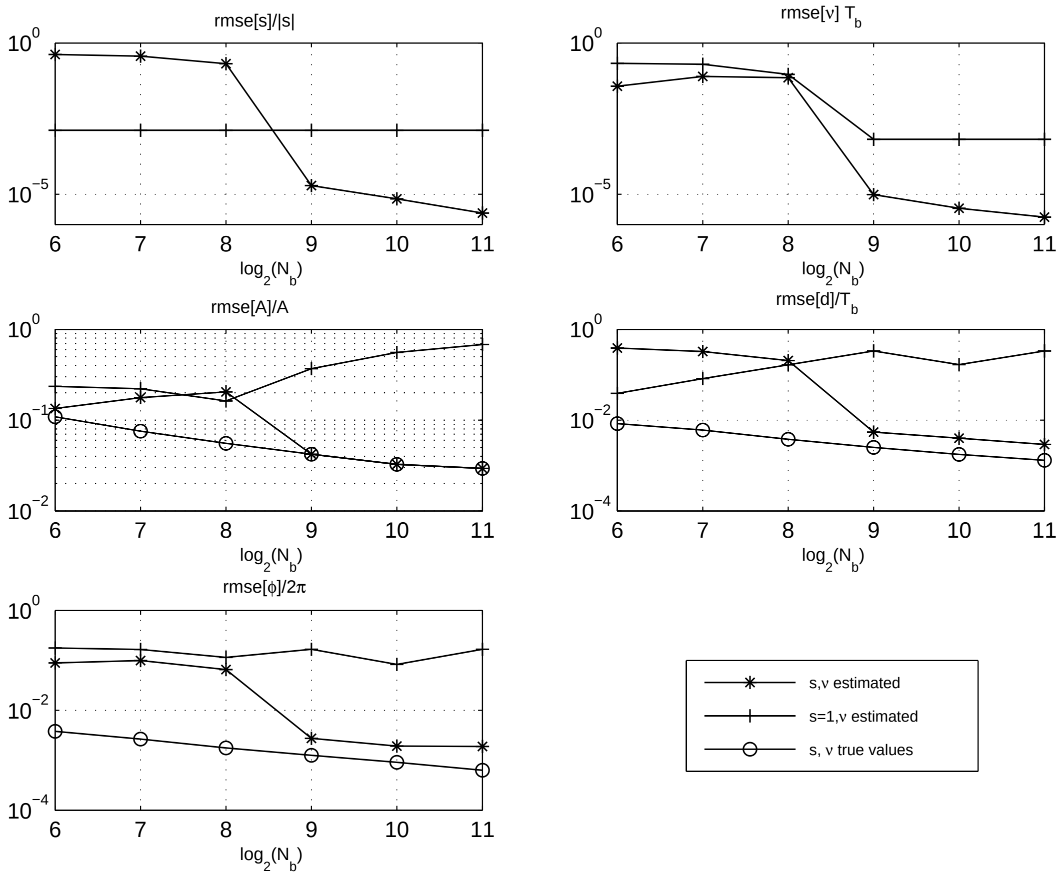

Fig. 1. Sample root mean-square error for estimates of time-scale factor s, frequency shift ν, amplitude A, delay d, and phase shift φ, normalized to $|s|$, $1/T_b$, A, T_b, and 2π, respectively, as a function of the processed number of bits N_b.

[5] L. Izzo and A. Napolitano, "Linear time-variant transformations of generalized almost-cyclostationary signals, Part II: Development and applications", *IEEE Transactions on Signal Processing*, vol. 50, pp. 2962-2975, December 2002.

[6] Q. Jin, K. M. Wong, and Z.-Q. Luo, "The estimation of time delay and Doppler stretch of wideband signals," *IEEE Transactions on Signal Processing*, vol. 43, pp. 904-916, April 1995.

[7] A. Napolitano and M. Tanda, "Blind parameter estimation in multiple-access systems," *IEEE Trans. on Communications*, vol. 49, pp. 688-698, April 2001.

[8] A. Napolitano and M. Tanda, "Doppler-channel blind identification for non-circular transmissions in multiple-access systems," *IEEE Trans. on Communications*, vol. 52, pp. 2073-2078, December 2004.

[9] J. Oberg, "Titan calling," *IEEE Spectrum*, vol. 41, pp. 28-33, October 2004.

[10] H. V. Poor and M. Tanda, "Multiuser detection in flat fading non-Gaussian channels," *IEEE Trans. on Communications*, vol. 50, pp. 1769-1777, November 2002.

[11] R. A. Scholtz and J.-Y. Lee, "Problems in modeling UWB channels," in Proc. of *Thirty-Sixth Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, 3-6 November 2002.

[12] D. A. Straight, G. K. Lot, and W. A. Brown, "Maximum likelihood estimates of time and frequency differences of arrival of weak cyclostationary digital communications signals," in *Proc. of 21st Century Military Communications Conference (MILCOM 2000)*, 2000.

[13] H. L. Van Trees, *Detection, Estimation, and Modulation Theory, Part III*, John Wiley & Sons, Inc., New York, 1971.