A BERNOULLI-GAUSSIAN MODEL FOR MULTIPATH CHANNEL ESTIMATION IN DOWNLINK WCDMA

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ABSTRACT

This paper presents a new algorithm for channel estimation in downlink WCDMA-FDD mode where a Bernoulli-Gaussian (BG) model is introduced for multipath channel. We have investigated a deconvolution algorithm called Single Most Likely Replacement (SMLR) especially derived for complex baseband signals and we have experienced the failure of such tentative for multipath channel estimation. This leads us to develop a new algorithm noted E-SMLR which works as an interference cancelling technique to enhance the SMLR performance. E-SMLR performs very well, it gives good performance while retains a low complexity. Simulations are given that support these claims.

1. INTRODUCTION

This paper deals with multipath channel estimation in downlink WCDMA-FDD mode from the Common Pilot Channel (CPICH) [1]. The channel estimation is a crucial problem in wireless communications since some explicit or implicit knowledge on the channel is mandatory for the receiver's operations [2]. Channel estimation methods are often based on correlative techniques. These methods though simple have limited performances [3]. Indeed, multipath components closer than one time chip are not resolvable.

A different approach is taken here, where the channel estimation is formulated as a deconvolution problem. Regularization techniques must be used to cope with the ill-posedness nature of such an inverse problem. In bayesian framework the regularization is performed by incorporating a priori information on the expected solution. The BG process is a well established model for spiky signals encountered in geophysics and non-destructive evaluation areas [4]. The success of such a model is also due to the development of a powerful deconvolution algorithms, among which the SMLR [4]. Considering the similarities existing between multipath channel estimation and spiky signals restoration problems, the driving idea governing this work has been to apply the SMLR for multipath channel estimation.

Since the SMLR was originally derived for real signals only [4], first task was to extend it to complex baseband signals. However, poor performances are obtained when SMLR is directly applied to channel estimation. The failure of the SMLR is the consequence of the approximation done when trying to formulate the channel estimation as a deconvolution problem. Indeed, this causes the amount noise coloration. To overcome this drawback and enhance the SMLR performance an original algorithm noted E-SMLR for Enhanced SMLR is introduced. The proposed algorithm works as an interference canceller [5], where the term causing noise coloration is iteratively eliminated. It is showed that E-SMLR performs very well at a moderate extra computational cost.

The paper is organized as follows. Baseband system model and related assumptions are given in Section 2. SMLR algorithm derivation for baseband complex signals is considered in Section 3. In Section 4, the proposed E-SMLR algorithm is introduced. Section 5 is devoted to simulation results. Finally, Section 6 describes the conclusions.

2. SYSTEM MODEL

Hereafter we are interested to the downlink signal due to the CPICH only (i.e., no data channels are considered). The following baseband synchronous discrete-time convolution model is adopted

$$r(k) = \psi(k) * h(k) * s(k) + \eta(k). \quad (1)$$

The discrete-time received signal $r(k)$ is obtained by anti-alias filtering and sampling at rate $1/\triangle t = S/T_c$, where S is the number of samples per chip and T_c is the time chip duration ($T_c = 260.4$ ns) [1]. $\psi(k)$ is the normalized root-raised cosine transmit pulse shaping [2], with roll-off $\alpha = 0.22$. $h(k)$ is the downlink channel impulse response, considered as time-invariant over one time slot duration (i.e., 2560 chips) and has the following sparse multipath model [2]

$$h(k) = \sum_{l=1}^{L} a_l \delta(k - \tau_l), \quad (2)$$

where L is the number of propagation paths, a_l is the complex amplitude of the lth path and τ_l is the corresponding time-discretized propagation delay. $s(k)$ is the scrambling sequence code. Long scrambling codes are used in downlink, they constitute segments of a set of pseudo-noise Gold sequences with period truncated to one frame duration (i.e., 10 ms or 38400 chips) and belong to a QPSK modulation alphabet [1]. The scrambling code autocorrelation sequence corresponding to one time slot is depicted in Figure 1. The additive noise $\eta(k)$ is assumed to be complex zero-mean white Gaussian process with variance σ_η^2.

The channel profile signal is obtained as the output signal of the scrambling sequence matched filter

$$z(k) = r(k) * s^*(-k) = \psi(k) * h(k) * c_{ss}(k) + \nu(k), \quad (3)$$

with $\nu(k) \triangleq \eta(k) * s^*(-k)$ and where $c_{ss}(k)$ is the scrambling autocorrelation sequence. Since $s(k)$ is a pseudo-noise sequence (cf. Figure 1), its autocorrelation can be approximated as

$$c_{ss}(k) = c_{ss}(0)\delta(k). \quad (4)$$

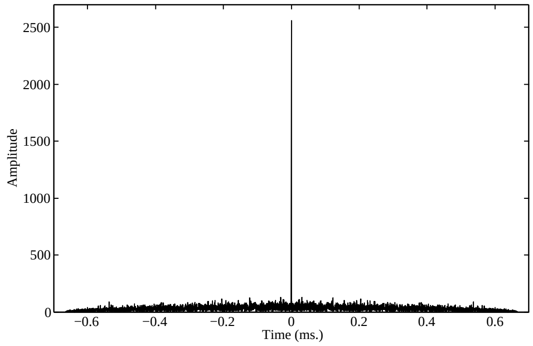

Fig. 1. Scrambling code autocorrelation sequence modulus for one time slot.

Considering this, (3) is simplified to

$$z(k) = g(k) * h(k) + \nu(k), \tag{5}$$

with $g(k) \triangleq c_{ss}(0) * \psi(k)$. It can be shown that the noise $\nu(k)$ is complex zero-mean white Gaussian process with variance $\sigma_\nu^2 = c_{ss}(0)\sigma_\eta^2$. From (5), it can be seen that the channel estimation is turned now to a deconvolution problem [4].

BG model is introduced to account for the spiky nature of the multipath channel. According to this model the optimal path amplitudes are found by linear methods. However, the time delays estimation or paths detection is a highly non-linear problem and is performed by SMLR algorithm [4]. The SMLR iterative search starts by comparing a reference sequence of the path time delays to a number of neighbours. The most likely sequence is used as the reference for the next iteration and the search is repeated until a reference sequence that is more likely than any of neighbours is obtained [4, 6].

3. SMLR DERIVATION FOR COMPLEX SIGNALS

Equation (5) can be expressed as

$$z(k) = \sum_{n=0}^{N-1} h(n)g(k-n) + \nu(k), \tag{6}$$

where $k = 0, 1, \ldots, M-1$. N stands for the channel impulse response length in samples and M represents the number of retained samples for the channel profile. Considering practical situations, it is assumed that $M \geq N + W - 1$, where W is the pulse shaping filter length in samples ($\psi(k)$ is defined for $k = 0, 1, \ldots, W-1$). Equation (6) can be rewritten in the matrix form

$$\boldsymbol{z} = \boldsymbol{G}\boldsymbol{h} + \boldsymbol{\nu}. \tag{7}$$

$\boldsymbol{G} \in \mathbb{R}^{M \times N}$ contains shifted versions of $g(k)$ and has a lower-triangular Toeplitz matrix structure. According to the last condition no truncated versions of $g(k)$ are contained in $\boldsymbol{G}$. Considering (2), (5) can be expressed as

$$z(k) = \sum_{l=1}^{L} a_l g(k-\tau_l) + \nu(k). \tag{8}$$

This expression is more convenient for the present SMLR derivation. It can be written in matrix form as

$$\boldsymbol{z} = \boldsymbol{G}_{\boldsymbol{\tau}}\boldsymbol{a} + \boldsymbol{\nu}, \tag{9}$$

where $\boldsymbol{a} = [a_1, a_2, \ldots, a_L]^{\mathrm{T}} \in \mathbb{C}^L$ is the channel complex amplitudes vector, $\boldsymbol{\tau} = [\tau_1, \tau_2, \ldots, \tau_L]^{\mathrm{T}} \in \mathbb{R}^L$ is the channel time delays vector and $\boldsymbol{G}_{\boldsymbol{\tau}} \in \mathbb{R}^{M \times L}$ is obtained from $\boldsymbol{G}$ by conserving the columns of index contained in $\boldsymbol{\tau}$. From (9), we remark that any columns permutation of $\boldsymbol{G}_{\boldsymbol{\tau}}$ does not change the channel estimation result provided that the elements of $\boldsymbol{a}$ undergo the same permutation. The noise $\boldsymbol{\nu}$ is a complex zero-mean white Gaussian vector with covariance matrix $\sigma_\eta^2 \boldsymbol{I}_M$ and it is assumed to be independent from $\boldsymbol{a}$ and $\boldsymbol{\tau}$. The time delays contained in $\boldsymbol{\tau}$ are controlled by a Bernoulli process with parameter λ and $\boldsymbol{a}$ is a complex zero-mean white Gaussian vector with covariance matrix $\sigma_a^2 \boldsymbol{I}_L$. The channel estimation is achieved by using a sequential approach [4], where paths detection is performed first through a marginal MAP estimator and then the amplitudes vector $\boldsymbol{a}$ is determined through a MAP estimator.

3.1. Amplitudes estimation

This is obtained by maximizing the posterior joint likelihood $p(\boldsymbol{a}, \boldsymbol{\tau}|\boldsymbol{z})$ with respect to $\boldsymbol{a}$ when $\boldsymbol{\tau}$ is given. Considering the statistical assumptions and from (9), the optimal solution of $\boldsymbol{a}$ is given by

$$\hat{\boldsymbol{a}} = \mu \boldsymbol{C}^{-1}\boldsymbol{G}_{\boldsymbol{\tau}}^{\mathrm{T}}\boldsymbol{z}, \tag{10}$$

with

$$\boldsymbol{C} \triangleq \mu \boldsymbol{G}_{\boldsymbol{\tau}}^{\mathrm{T}}\boldsymbol{G}_{\boldsymbol{\tau}} + \boldsymbol{I}_L, \tag{11}$$

and where $\mu \triangleq \sigma_a^2/\sigma_\nu^2$.

3.2. Time delays estimation : Paths detection

This is achieved by maximizing the posterior marginal likelihood $Pr(\boldsymbol{\tau}|\boldsymbol{z})$ with respect to $\boldsymbol{\tau}$. A logarithmic expression of the detection criterion can be derived easily

$$\mathcal{L}(\boldsymbol{\tau}) = -\sigma_\nu^{-2}\boldsymbol{z}^{\mathrm{H}}\boldsymbol{B}^{-1}\boldsymbol{z} - \ln(\det(\boldsymbol{B})) + L\ln(\lambda/(1-\lambda)), \tag{12}$$

where $\boldsymbol{B}$ is the normalized covariance matrix of $\boldsymbol{z}$ given $\boldsymbol{\tau}$

$$\boldsymbol{B} = \mu \boldsymbol{G}_{\boldsymbol{\tau}}\boldsymbol{G}_{\boldsymbol{\tau}}^{\mathrm{T}} + \boldsymbol{I}_M. \tag{13}$$

Let $\boldsymbol{\tau}_0$ be a reference sequence and $\boldsymbol{\tau}_k$ a neighbour sequence which differs from $\boldsymbol{\tau}_0$ by only one path, say at time delay τ_k. We seek a relationship between $\mathcal{L}(\boldsymbol{\tau}_0)$ and $\mathcal{L}(\boldsymbol{\tau}_k)$. Subscript k (resp. 0) will refer to any quantity related to $\boldsymbol{\tau}_k$ (resp. $\boldsymbol{\tau}_0$). Considering the remark above, we can write

$$\boldsymbol{G}_k = [\boldsymbol{G}_0 \quad \boldsymbol{g}_k], \tag{14}$$

if a path is added to $\boldsymbol{\tau}_0$ and

$$\boldsymbol{G}_0\boldsymbol{E} = [\boldsymbol{G}_k \quad \boldsymbol{g}_k], \tag{15}$$

if a path is removed from $\boldsymbol{\tau}_0$. In (15), $\boldsymbol{E}$ is a column elementary matrix. By using (11)-(15) and after some arrangements the algorithm can be summarized in term of $\boldsymbol{C}_0^{-1}$ as

$$\rho_k = \epsilon_k + \mu(\boldsymbol{g}_k^{\mathrm{T}}\boldsymbol{g}_k - \mu\boldsymbol{g}_k^{\mathrm{T}}\boldsymbol{G}_0\boldsymbol{C}_0^{-1}\boldsymbol{G}_0^{\mathrm{T}}\boldsymbol{g}_k), \tag{16}$$

$$\boldsymbol{g}_k^{\mathrm{T}}\boldsymbol{B}_0^{-1}\boldsymbol{z} = \boldsymbol{g}_k^{\mathrm{T}}\boldsymbol{z} - \mu\boldsymbol{g}_k^{\mathrm{T}}\boldsymbol{G}_0\boldsymbol{C}_0^{-1}\boldsymbol{G}_0^{\mathrm{T}}\boldsymbol{z}, \tag{17}$$

$$\mathcal{L}(\boldsymbol{\tau}_k) = \mathcal{L}(\boldsymbol{\tau}_0) + \mu\sigma_\nu^{-2}\rho_k^{-1}|\boldsymbol{g}_k^{\mathrm{T}}\boldsymbol{B}_0^{-1}\boldsymbol{z}|^2 - \ln(\epsilon_k\rho_k) + \epsilon_k\ln(\lambda/(1-\lambda)), \quad (18)$$

where ϵ_k takes 1 (resp. -1) when a path is added (resp. removed). To run the algorithm, update equations for $\boldsymbol{C}^{-1}$ must be derived.

When $\epsilon_k = 1$, from (11) and (14) and by invoking the inversion lemma for block matrices we obtain

$$\boldsymbol{C}_k^{-1} = \begin{bmatrix} \boldsymbol{C}_0^{-1} + \rho_k\boldsymbol{b}_k\boldsymbol{b}_k^{\mathrm{H}} & \boldsymbol{b}_k \\ \boldsymbol{b}_k^{\mathrm{H}} & \rho_k^{-1} \end{bmatrix}, \quad (19)$$

where $\boldsymbol{b}_k \triangleq -\mu\rho_k^{-1}\boldsymbol{C}_0^{-1}\boldsymbol{G}_0^{\mathrm{T}}\boldsymbol{g}_k$. The other case of $\epsilon_k = -1$ is a straightforward consequence of the former.

3.3. Implementation aspects

Quantities like $\boldsymbol{g}_k^{\mathrm{T}}\boldsymbol{z}$, $\boldsymbol{g}_k^{\mathrm{T}}\boldsymbol{g}_k$, $\boldsymbol{G}_0^{\mathrm{T}}\boldsymbol{z}$ and $\boldsymbol{G}_0^{\mathrm{T}}\boldsymbol{g}_k$ are directly extracted from the matrix $\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}$ and the vector $\boldsymbol{G}^{\mathrm{T}}\boldsymbol{z}$, which are computed and stored during the initialization. It can be shown that $\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}$ is Toeplitz and can be obtained by computing the pulse shaping autocorrelation sequence. Thus, it suffices to keep in the memory the first row only to store $\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}$. For computing savings, correlation can be formulated as convolution product and computed by FFT.

SMLR strategy consists to explore the whole neighbouring sequences by computing N detection criterion (18) values each iteration. Other exploration strategies can be found in [6] and references therein. A single evaluation of $\mathcal{L}(\boldsymbol{\tau}_k)$ and an update of $\boldsymbol{C}_0^{-1}$ both require $\mathcal{O}(L^2)$ complex multiplications. The amplitudes evaluation is obtained directly as a simple product of precomputed quantities and requires $\mathcal{O}(L^2)$ complex multiplications. The algorithm is initialized with the null sequence (i.e., no paths are contained in $\boldsymbol{\tau}_0$), which is a good choice since the channel is of sparse nature.

4. E-SMLR ALGORITHM

The SMLR failure for multipath channel estimation is explained as the consequence of the approximation in (4), which is not correct. Indeed, the scrambling autocorrelation sequence is written rather as

$$c_{ss}(k) = e(k) + c_{ss}(0)\delta(k), \quad (20)$$

where $e(k)$ is the scrambling autocorrelation sequence for lags different from zero (cf. Figure 1). The substitution of (20) in (3) leads to

$$z(k) = g(k) * h(k) + \nu'(k), \quad (21)$$

with

$$\nu'(k) \triangleq \psi(k) * e(k) * h(k) + \nu(k). \quad (22)$$

The first term in (22) which is neglected in (5) can be of $\nu(k)$ level's and even dominant in high signal-to-noise ratio conditions. Hence, the noise $\nu'(k)$ is not white any more as assumed in the SMLR derivation. This causes wrong detections and leads to poor performances. Colored noise extension of the SMLR [4] can be used in this case. However, this extension requires a high complexity for computing the initial quantities and hence it is not feasible in practice. The approach that we propose is an iterative noise cancellation method noted E-SMLR, where the SMLR is applied iteratively on

$$z^{(i)}(k) = z(k) - \psi(k) * e(k) * \hat{h}^{(i-1)}(k). \quad (23)$$

$i = 1, 2, \ldots, I$, where I stands for the number of iterations and $\hat{h}^{(i)}(k)$ is the channel estimate obtained at iteration i. The noise variance is set to $c_{ss}(0)\sigma_\eta^2$ for all iterations and $e(k)$ is computed and stored beforehand. The procedure is initialized by $\hat{h}^{(0)}(k)$ obtained when SMLR is directly applied to $z(k)$. To prevent a bad initial channel estimate, the noise variance is set to a value accounting for the first term in (22)

$$\sigma_{\nu'}^2 = c_{ss}(0)\sigma_\eta^2 + \lambda\sigma_a^2 \sum_{k=0}^{M-1} |\varepsilon(k)|^2, \quad (24)$$

where $\varepsilon(k) \triangleq \psi(k) * e(k)$. The iterative procedure is stopped by fixing beforehand the number of iterations I or by using some relative error criterion on the log-likelihood function (18). To fasten the convergence of the algorithm an original idea consists to use the channel estimate obtained at the last iteration to initialize the SMLR search at the next iteration instead of using the null sequence. Except for the first iteration where the channel estimate is of poor quality and if used can lead the algorithm to be trapped in local maxima. To run the algorithm the parameters λ and σ_a^2 must be specified somehow. As experienced, the SMLR algorithm results are less sensitive to the values given for these parameters and hence some error is allowed in their determination.

5. SIMULATION RESULTS

Since only the CPICH is considered, it is more convenient to evaluate performances by considering the energy per chip-to-noise spectrum density ratio

$$\frac{E_c}{N_0} = \frac{1}{2\sigma_\eta^2} \sum_{k=0}^{N+W-2} |y(k)|^2, \quad (25)$$

where $y(k) \triangleq \psi(k) * h(k)$ is the chip signature. One time slot is considered and the over-sampling factor S is set equal to 8. The root-raised cosine pulse shaping is truncated to five chip intervals on both sides and shifted to be causal. Two multipath channels containing each one four paths are considered. They are noted Channel A and Channel B and can be considered as representative of ITU [1] Vehicular and Indoor channels, respectively. Channel profile signals are given in Figure 2, where E_c/N_0 is fixed to 10 dB for Channel A and to 25 dB for Channel B.

For comparison purposes, we give first in Figure 3 the estimation results for Channel B when a traditional correlative method is used. This consists to passing the channel profile signal through the pulse shaping matched filter to enhance the resolution. It is important to note that the paths positions corresponding to the output signal peaks are localized by using visual inspection. The results in Figure 3 are of poor quality due to the method's resolution inherently limited to one time chip [3]. For Channel A, as it can be expected the method performs very well (results not reported here) since the paths are well resolved (cf. Figure 2(a)). However, even in this last case visual inspection is needed to locate the peaks and to adjust the detection threshold to prevent from false detections caused by the noise.

In Figures 4(a)-(b) and 5(a)-(b) are shown the results obtained when SMLR is applied. As aforementioned the channel estimates are of poor quality. There are false paths detections for Channel A, whereas miss paths detections are observed for Channel B. These channels estimates are used to initialize the E-SMLR. The results for the E-SMLR are given in Figures 4(c)-(d) and 5(c)-(d). These are obtained after one iteration for both channels. The channels

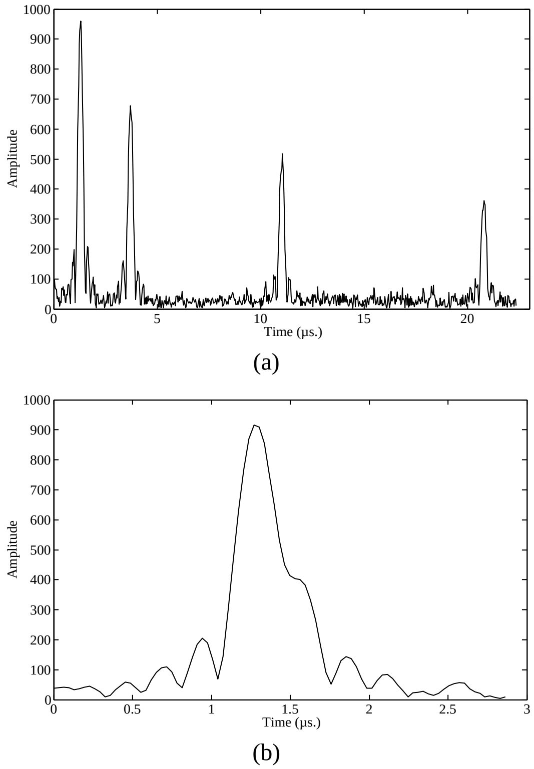

Fig. 2. Channel profile signals modulus. (a) Channel A, $E_c/N_0 = 10$ dB. (b) Channel B, $E_c/N_0 = 25$ dB.

estimates are of very good quality even for Channel B where the paths spacing is less than a quarter of the time chip. This shows clearly the superiority of the E-SMLR for very closed multipath channels estimation.

6. CONCLUSION

A new algorithm E-SMLR for downlink channel estimation in WCDMA-FDD mode has been proposed. E-SMLR is based on the SMLR algorithm and uses an interference cancelling-like technique to iteratively cancel the noise due to the scrambling autocorrelation sequence. Simulation results show that the E-SMLR performs very well while retains an acceptable complexity amount since a few number of iterations is often needed for the convergence. E-SMLR can be qualified as a high resolution technique since subchip-spaced paths can be resolved in a good signal-to-noise ratio conditions. Moreover, the proposed method is fully automated and does not need visual inspection or any other human intervention. In particular we believe that the E-SMLR is well adapted to be coupled with advanced receivers like interference cancelling techniques [5] and hence well suited to be used for High Speed Downlink Pack Access (HSDPA) in indoor conditions [1]. Moreover, extending the algorithm to account for the continuity of the channel variation from slot to slot can be done in a natural way in a bayesian framework.

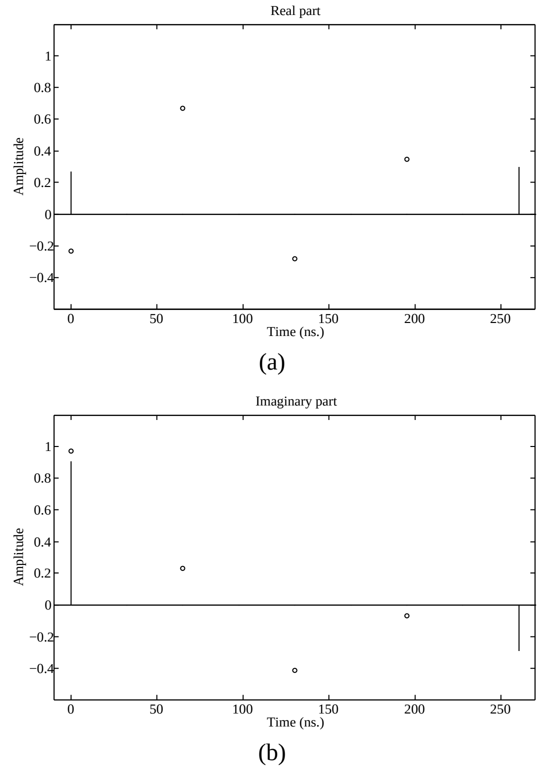

Fig. 3. Channel B estimation results, $E_c/N_0 = 25$ dB. Results obtained when a correlative method is applied. Bars depict estimates and circles depict true values.

7. REFERENCES

[1] H. Holma and A. Toskala, *WCDMA for UMTS*, Wiley, New Jersey, 2nd. edition, 2002.

[2] J. G. Proakis, *Digital communications*, McGraw-Hil, New York, 4th. edition, 2001.

[3] K. S. Gilhousen, I. M. Jacobs, R. Padovani, A. J. Viterbi, L. A. Weaver, and C. E. Wheatley, "On the capacity of cellular cdma system," *IEEE Transaction on Vehicular Technology*, vol. 40, no. 2, pp. 303–312, may 1991.

[4] F. Champagnat, Y. Goussard, and J. Idier, "Unsupervised deconvolution of sparse spike trains using stochastic approximation," *IEEE Transaction on Signal Processing*, vol. 44, no. 12, pp. 2988–2998, december 1996.

[5] K. Higuchi, A. Fujiwara, and M. Sawahashi, "Multipath interference canceller for high-speed packet transmission with adaptive modulation and coding scheme in w-cdma forward link," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 2, pp. 419–432, february 2002.

[6] L. Fathi, A. Ouamri, M. Keche, and M. Ouldmammar, "Deconvolution of sparse spike trains using local selection strategy," *Electronics Letters*, vol. 36, no. 4, pp. 364–365, february 2000.

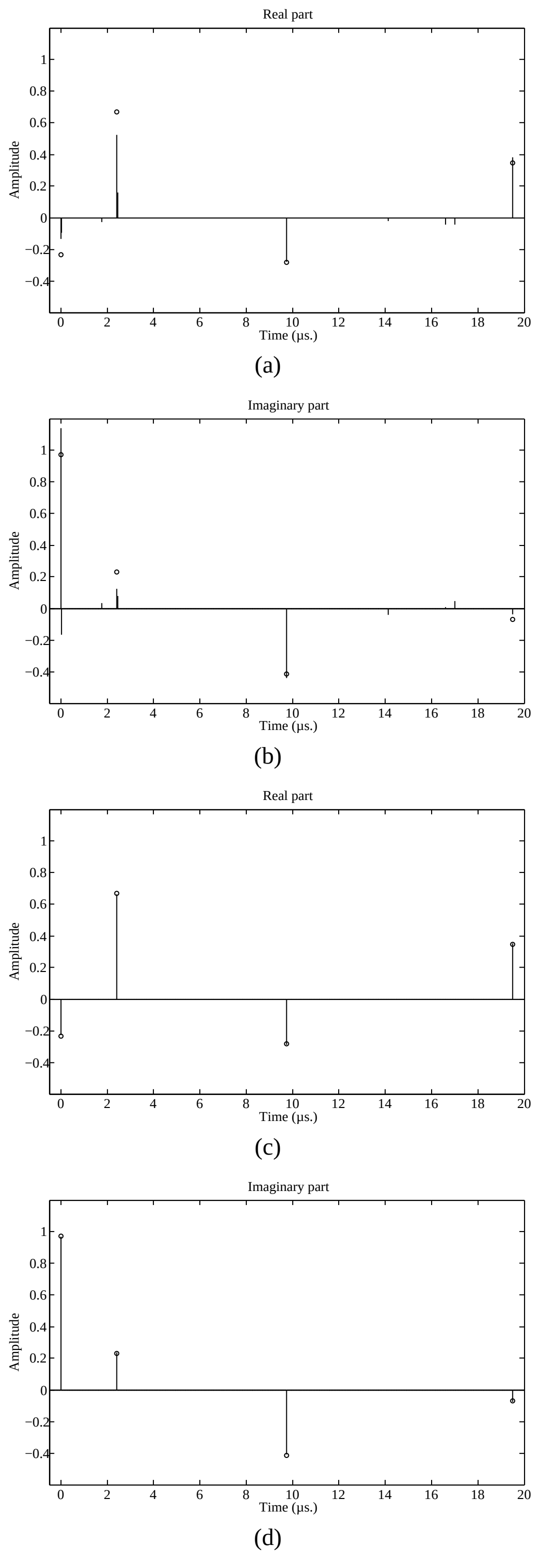

Fig. 4. Channel A estimation results, $E_c/N_0 = 10$ dB. (a)-(b) Results obtained by SMLR. (c)-(d) Results obtained by E-SMLR after one iteration. Bars depict estimates and circles depict true values.

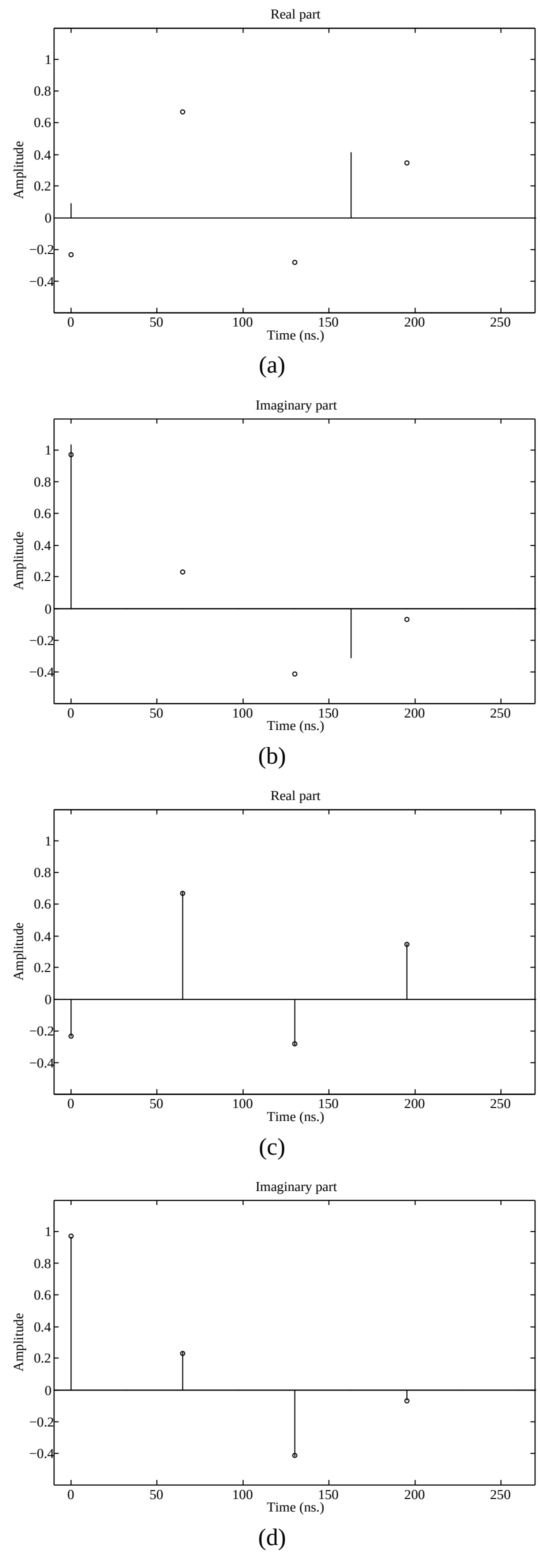

Fig. 5. Channel B estimation results, $E_c/N_0 = 25$ dB. (a)-(b) Results obtained by SMLR. (c)-(d) Results obtained by E-SMLR after one iteration. Bars depict estimates and circles depict true values.