TRACKING ANALYSIS OF SOME SPACE-TIME BLIND EQUALIZATION ALGORITHMS

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ABSTRACT

Although space-time blind equalization algorithms have been widely studied in the last decade, their convergence and tracking behavior are not yet completely characterized. In this paper, we address the tracking analysis of space-time blind quasi-Newton algorithms, using the energy conservation relation. We obtain an expression to adjust the algorithms to reach the same steady-state mean-square error. Assuming a low degree of nonstationarity and high signal-to-noise ratio, close agreement between analytical and simulation results is observed.

1. INTRODUCTION

Blind adaptive algorithms are usually employed for signal recovery in several digital communication applications. In nonstationary environments, it is important to study their ability to track channel changes. The literature contains some steady-state analysis for CMA (Constant Modulus Algorithm) based on the energy conservation relation [1, 2] and on the stochastic averaging method [3]. Although the energy conservation relation method tends to require stronger assumptions than the latter, it can bypass many of the difficulties encountered in the nonlinear algorithm analysis [4].

The CMA tracking analysis of [2] was extended in [5] to blind quasi-Newton algorithms that minimize the Godard cost function [6]. It was also verified that the ratio between the minimum mean-square error for the Shalvi-Weinstein Algorithm (SWA) [7] and CMA is the same obtained between the RLS and LMS algorithms [8]. In this paper, we extend these results to quasi-Newton space-time blind algorithms that minimize the multiuser Godard cost function [9]. We consider the energy conservation relation method and some assumptions of the MU-CMA (Multiuser-CMA) MSE analysis presented in [10].

In the sequel, data model is presented and some space-time blind algorithms are revisited. Then, we present the tracking analysis, followed by simulation results. We close the paper, making some concluding remarks.

2. DATA MODEL

A MIMO system with N sources and with an antenna array which has $L > N$ sensors is depicted in Fig. 1. The source sequences $a_i(n)$, $i = 1, \ldots, N$ satisfy the circularity condition and are assumed independent and identically distributed (i.i.d.), with the same statistics, independent from one another, non Gaussian, and zero-mean. The transmitted signals suffer intersymbol and co-channel interferences. The channel $H_{ij}(z)$ from the i^{th} source to the j^{th} sensor is modeled by an FIR filter with K_c coefficients and η_i, $i = 1, 2, \ldots, L$ represent additive white Gaussian noise. The outputs of the L sensors are processed with N parallel space-time FIR equalizers, each with K_t time diversity and $M = L\,K_t$ taps. The i^{th} equalizer's output can be written as

$$y_i(n) = \mathbf{u}^T(n)\mathbf{w}_i(n-1),$$

where

$$\mathbf{u}(n) = [\,\mathbf{u}_1^T(n)\ \mathbf{u}_2^T(n)\ \cdots\ \mathbf{u}_L^T(n)\,]^T,$$
$$\mathbf{u}_p(n) = [\,u_p(n)\ u_p(n-1)\cdots u_p(n-K_t+1)\,]^T,$$

with $p = 1, 2, \ldots, L$, and $\mathbf{w}_i(n-1)$ is the equalizer weight vector at time $n-1$.

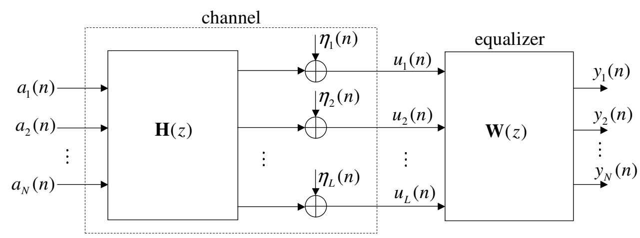

Fig. 1. MIMO equivalent system model.

In the case of joint blind simultaneous recovery of all input signals, the multiuser Godard cost function is given by [9]

$$J_G = \sum_{i=1}^{N}\left[J_{Gi} + \alpha \sum_{j=1, j\neq i}^{N} \sum_{\delta=0}^{\delta_1} |r_{ij}(\delta)|^2\right], \qquad (1)$$

in which

$$\begin{aligned} J_{Gi} &= \mathrm{E}\{(|y_i(n)|^2 - R_2^a)^2\}, \\ R_2^a &= \mathrm{E}\{|a(n)|^4\}/\mathrm{E}\{|a(n)|^2\}, \\ r_{ij}(\delta) &= \mathrm{E}\{y_i(n)y_j^*(n-\delta)\}, \\ \delta_1 &= K_t + K_c - 1, \end{aligned}$$

α is a positive constant, and $*$ stands for complex conjugate. The instantaneous gradient estimate of this cost function related to the i^{th} equalizer is given by

$$\widehat{\boldsymbol{\nabla}}_i J_G = \bar{e}_i(n)\mathbf{u}^*(n), \tag{2}$$

where

$$\bar{e}_i(n) = e_i(n) + \epsilon_i(n), \tag{3}$$

$$e_i(n) = (|y_i(n)| - R_2^a)y_i(n), \tag{4}$$

and

$$\epsilon_i(n) = \frac{\alpha}{2}\,\hat{\mathbf{r}}_i^T(n)\mathbf{y}_i(n). \tag{5}$$

The column vectors $\hat{\mathbf{r}}_i(n)$ and $\mathbf{y}_i(n)$ have $(N-1)(\delta_1+1)$ elements and are defined by

$$\hat{\mathbf{r}}_i(n) = \left[\ \hat{\mathbf{r}}_{i,1}^T \ \cdots \ \hat{\mathbf{r}}_{i,i-1}^T \ \ \hat{\mathbf{r}}_{i,i+1}^T \ \cdots \ \hat{\mathbf{r}}_{i,N}^T\ \right]^T \tag{6}$$

and

$$\mathbf{y}_i(n) = \left[\breve{\mathbf{y}}_1^T(n) \cdots \breve{\mathbf{y}}_{i-1}^T(n) \ \ \breve{\mathbf{y}}_{i+1}^T(n) \cdots \breve{\mathbf{y}}_N^T(n)\right]^T, \tag{7}$$

with

$$\hat{\mathbf{r}}_{i,j} = \left[\ \hat{r}_{ij}(0) \ \ \cdots \ \ \hat{r}_{ij}(\delta_1)\ \right]^T \tag{8}$$

and

$$\breve{\mathbf{y}}_j(n) = \left[\ y_j(n) \ \ \cdots \ \ y_j(n-\delta_1)\ \right]^T, \tag{9}$$

for $1 \le j \le N$ and $j \ne i$.

The cross-correlation vetor $\hat{\mathbf{r}}_i(n)$ can be updated by the following recursive equation

$$\hat{\mathbf{r}}_i(n) = \lambda_r \hat{\mathbf{r}}_i(n-1) + (1-\lambda_r)y_i(n)\mathbf{y}_i^*(n), \tag{10}$$

where $0 \ll \lambda_r < 1$ is a forgetting factor.

3. THE ALGORITHMS

Gradient and quasi-Newton methods have been exploited to minimize J_G, leading to different algorithms (see e.g. [9, 11, 12]). These algorithms can be described by the following general expression

$$\mathbf{w}_i(n) = \mathbf{w}_i(n-1) - \rho(n)\widehat{\mathbf{F}}^{-1}(n)\mathbf{u}^*(n)\bar{e}_i(n), \tag{11}$$

for $i = 1, 2, \cdots, N$, where $\rho(n)$ is a positive step-size, $\bar{e}_i(n)$ is given by (3), and $\widehat{\mathbf{F}}(n)$ is a symmetric and nonsingular matrix, which represents an approximation of the Hessian estimate. All adaptive schemes of the form (11) obey the energy conservation relation [4].

The autocorrelation matrix $\mathbf{R} = \mathrm{E}\{\mathbf{u}^*(n)\mathbf{u}^T(n)\}$ is usually estimated as

$$\widehat{\mathbf{R}}(n) = \sum_{l=1}^{n} \lambda^{n-l}\mathbf{u}^*(l)\mathbf{u}(l),$$

where $0 \ll \lambda < 1$ is a forgetting factor. If $\widehat{\mathbf{R}}(n)$ is assumed to be an approximation of the Hessian matrix of J_G, the Multiuser-SWA (MU-SWA) can be interpreted as a quasi-Newton algorithm [12]. The same occurs with MU-CMA [9] which uses the identity matrix $\mathbf{I}$.

4. THE TRACKING ANALYSIS

In a nonstationary environment, the variation in the zero-forcing solution $\mathbf{w}_{\mathrm{o},i}$ of the i^{th} equalizer is assumed to follow the model [8, 4]

$$\mathbf{w}_{\mathrm{o},i}(n) = \mathbf{w}_{\mathrm{o},i}(n-1) + \mathbf{q}_i(n), \tag{12}$$

where $\mathbf{q}_i(n)$ is an i.i.d sequence with positive definite autocorrelation matrix $\mathbf{Q}_i = \mathrm{E}\{\mathbf{q}_i^*(n)\mathbf{q}_i^T(n)\}$ and is independent of the initial conditions $\{\mathbf{w}_{\mathrm{o},i}(-1), \mathbf{w}_i(-1)\}$ and of $\{\mathbf{u}(l)\}$ for all $l < n$ [4, Sec. 7.4].

Defining

$$\widetilde{\mathbf{w}}_i(n) \triangleq \mathbf{w}_{\mathrm{o},i}(n) - \mathbf{w}_i(n)$$

as the weight-error vector, (11) can be rewritten for the nonstationary case as

$$\widetilde{\mathbf{w}}_i(n) - \mathbf{q}_i(n) = \widetilde{\mathbf{w}}_i(n-1) + \rho(n)\widehat{\mathbf{F}}^{-1}(n)\mathbf{u}^*(n)\bar{e}_i(n). \tag{13}$$

Let $e_{a,i}(n) = \mathbf{u}^T(n)\widetilde{\mathbf{w}}_i(n-1)$ be the *a priori* error. The steady-state MSE, defined as

$$\zeta_i \triangleq \lim_{n\to\infty} \mathrm{E}\{|e_{a,i}(n)|^2\}$$

is a measure of filter performance. The MSE and tracking analysis of the algorithms of the form (11) are based on the following assumptions:

A1. $\mathrm{E}\{a(n)\} = 0$, $\gamma \triangleq \beta\mathrm{E}\{|a(n)|^2\} - R_2^a > 0$, where $\beta = 3$ (resp., $\beta = 2$) for the real (resp., complex) case, and for complex data, $\mathrm{E}\{a^2(n)\} = 0$ (circularity condition). This assumption holds for most constellations used in digital communications [7, 4].

A2. Algorithms of the form (11) obey the steady-state ($n \to \infty$) condition given by [4, Sec.6.9.3]

$$\mathrm{E}\left\{\|\widetilde{\mathbf{w}}_i(n)\|^2_{\widehat{\mathbf{F}}}\right\} = \mathrm{E}\left\{\|\widetilde{\mathbf{w}}_i(n-1)\|^2_{\widehat{\mathbf{F}}}\right\},$$

where

$$\|\widetilde{\mathbf{w}}_i(n)\|^2_{\widehat{\mathbf{F}}} \triangleq \widetilde{\mathbf{w}}_i^T(n)\widehat{\mathbf{F}}(n)\widetilde{\mathbf{w}}_i^*(n).$$

A3. $a_i(n-\tau_d)$ and $e_{a,j}(n)$ for $j=1,2,\cdots,i,\cdots,N$ are independent at steady-state, where τ_d is a delay [10]. This assumption is a generalization of that given in [2], where the case $i=j$ is considered [10].

A4. $\|\mathbf{u}(n)\|^2_{\widehat{\mathbf{F}}^{-1}}$ and $\bar{e}_i(n)$ are independent at steady-state [10]. This requires that the weighted energy of the input vector to be independent of the i^{th} equalizer output. Similar assumptions are considered in [1, *As.I.2*, p.84],[10, 2].

A5. Equation (4) can be rewritten using the approximation $y_i(n) \approx a_i(n-\tau_{d,i}) - e_{a,i}(n)$, since at the steady-state $a_i(n-\tau_{d,i}) \approx \mathbf{u}^T(n)\mathbf{w}_{\text{o},i}(n-1)$.

A6. The estimation errors at the outputs of different equalizers are assumed uncorrelated, that is,

$$\text{E}\{e_{a,i}(n)e_{a,j}(n-k)\} = 0, \quad \text{for} \quad i \neq j.$$

According to [10], this assumption is realistic when the equalizer is long enough to ensure that its output converges asymptotically to the correspondent source estimate at steady-state.

A7. The estimate of crosscorreltion $\text{E}\{y_i(n-1)y_j^*(n-1-k)\}$, defined as $\hat{r}_{ij,k}(n-1)$, is assumed to tend to a constant at steady-state. Consequently, it is uncorrelated with the equalizer outputs $y_i^*(n)$ and $y_j(n-k)$. This assumption is reasonable when the crosscorrelation is estimated by an exponential window with forgetting factor λ_r close to unity [10].

A8. The matrices $\hat{\mathbf{r}}_i^*(n)\hat{\mathbf{r}}_i^T(n)$ and $\mathbf{y}_i^*(n)\mathbf{y}_i^T(n)$ are uncorreltated at steady-state. This assumption is a consequence of A7, since $\hat{\mathbf{r}}_i(n)$ tends to a constant vector at steady-state [10].

A9. $\lim_{n\to\infty} \text{E}\{\hat{\mathbf{r}}_i^T(n)\mathbf{y}_i^*(n)\} = 0$, which can be justified similarly to A7 and A8 [10].

A10. $\lim_{n\to\infty} \text{E}\{e_{a,i}^*(n)\bar{e}_i(n) + e_{a,i}(n)\bar{e}_i^*(n)\} \approx -2\gamma\zeta_i$. This assumption is obtained by using A3, A5, A9, and considering $\gamma\text{E}\{|e_{a,i}(n)|^2\} \gg \text{E}\{|e_{a,i}(n)|^4\}$ [1, *Th.*3 and *Th.*4].

A11. $\lim_{n\to\infty} \text{E}\{|\bar{e}_i(n)|^2\} \approx \bar{\xi} \triangleq \xi + \mathcal{K}\text{E}\{|a(n)|^2\}^3$, where [1, 10]

$$\xi = \text{E}\{|a(n)|^6 - (R_2^a)^2|a(n)|^2\} \tag{14}$$

and

$$\mathcal{K} = \frac{\alpha^2}{4}(\delta_1 - 1)(N-1)\frac{1-\lambda_r}{1+\lambda_r}. \tag{15}$$

To obtain this assumption, besides the use of A1, A3, A5, A7, and A8, the terms which depend on $\text{E}\{|e_{a,i}(n)|^l\}$ with $l = 2,4,6$, were considered to be small when compared to ξ [1, *Th.*3 and *Th.*4]. It can also be obtained by considering $\text{E}\{|y_i(n)|^l\} \approx \text{E}\{|a(n)|^l\}$ with $l=2,4,6$, in mean-square of (4).

Now the Theorem 1 can be established.

Theorem 1 *Under the assumptions* A1−A11*, the steady-state MSE* $(n\to\infty)$ *of the algorithms of the form* (11) *can be approximated by*

$$\zeta_i \approx \frac{1}{2\gamma\rho(n)}\left(\rho^2(n)\text{E}\left\{\|\mathbf{u}(n)\|^2_{\widehat{\mathbf{F}}^{-1}}\right\}\bar{\xi} + \text{E}\left\{\|\mathbf{q}_i(n)\|^2_{\widehat{\mathbf{F}}}\right\}\right). \tag{16}$$

Proof: By equating the squared and weighted norms on both sides of (13), using $\widehat{\mathbf{F}}(n)$ as a weighting matrix, we obtain

$$\begin{aligned}\|\widetilde{\mathbf{w}}_i(n) - \mathbf{q}_i(n)\|^2_{\widehat{\mathbf{F}}} =& \|\widetilde{\mathbf{w}}_i(n-1)\|^2_{\widehat{\mathbf{F}}} \\ &+ \rho(n)\left[\bar{e}_i(n)e_{a,i}^*(n) + \bar{e}_i^*(n)e_{a,i}(n)\right] \\ &+ \rho^2(n)\|\mathbf{u}(n)\|^2_{\widehat{\mathbf{F}}^{-1}}|\bar{e}_i(n)|^2.\end{aligned} \tag{17}$$

By taking expectations of both sides of (17), using A2, we arrive at

$$\begin{aligned}&-\text{E}\left\{e_{a,i}^*(n)\bar{e}_i(n) + e_{a,i}(n)\bar{e}_i^*(n)\right\} \\ &\approx \rho(n)\text{E}\left\{\|\mathbf{u}(n)\|^2_{\widehat{\mathbf{F}}^{-1}}|\bar{e}_i(n)|^2\right\} + \rho^{-1}(n)\text{E}\left\{\|\mathbf{q}_i(n)\|^2_{\widehat{\mathbf{F}}}\right\}.\end{aligned} \tag{18}$$

Using A4, A10, and A11 in (18), we obtain (16), which completes the proof. ■

It is relevant to note that the scalar ξ given by (14) appears in the tracking analysis of [5, 2]. Considering just one user, that is, making $N=1$ in (15), we get $\mathcal{K}=0$ and $\bar{\xi}=\xi$. In this case, Theorem 1 reduces to the one obtained in the tracking analysis of [5]. The MSE from (16) has two terms: one for the stationary environment and another that appears in the nonstationary case. The expressions for the steady-state MSE for MU-CMA and MU-SWA are derived from Theorem 1 by considering usual approximations for $\text{E}\{\|\mathbf{u}(n)\|^2_{\widehat{\mathbf{F}}^{-1}}\}$ and $\text{E}\{\|\mathbf{q}_i(n)\|^2_{\widehat{\mathbf{F}}}\}$ [8], [4, pp.320,382]. In a noise free environment, there exist optimal adaptation factors, denoted by $\nu_{\text{o},i}$, that minimize the steady-state MSE. These factors are $\mu_{\text{o},i}$ and $\lambda_{\text{o},i}$ for MU-CMA and MU-SWA, respectively. The corresponding minima, denoted by $\zeta_{\text{o},i}$, can also be evaluated. All these results are summarized in Table 1.

Comparing the minimum MSE value of MU-CMA and MU-SWA, we obtain the following ratio

$$\frac{\zeta_{\text{o},i}^{\text{MU-SWA}}}{\zeta_{\text{o},i}^{\text{MU-CMA}}} = \sqrt{\frac{M\text{Tr}(\mathbf{Q}_i\mathbf{R})}{\text{Tr}(\mathbf{R})\text{Tr}(\mathbf{Q}_i)}}, \tag{19}$$

Algorithm	MU-CMA	MU-SWA
$\rho(n)$	μ	γ^{-1}
$\widehat{\mathbf{F}}(n)$	$\mathbf{I}$	$\sum_{l=1}^{n} \lambda^{n-l}\mathbf{u}^*(l)\mathbf{u}(l)$
$\mathrm{E}\left\{\Vert\mathbf{u}(n)\Vert^2_{\widehat{\mathbf{F}}^{-1}}\right\}$	$\mathrm{Tr}(\mathbf{R})$	$(1-\lambda)M$
$\mathrm{E}\left\{\Vert\mathbf{q}_i(n)\Vert^2_{\widehat{\mathbf{F}}}\right\}$	$\mathrm{Tr}(\mathbf{Q}_i)$	$\dfrac{\mathrm{Tr}(\mathbf{Q}_i\mathbf{R})}{1-\lambda}$
ζ_i	$\dfrac{\mu\bar{\xi}\mathrm{Tr}(\mathbf{R})}{2\gamma} + \dfrac{\mathrm{Tr}(\mathbf{Q}_i)}{2\gamma\mu}$	$\dfrac{M(1-\lambda)\bar{\xi}}{2\gamma^2} + \dfrac{\mathrm{Tr}(\mathbf{Q}_i\mathbf{R})}{2(1-\lambda)}$
$\nu_{\mathrm{o},i}$	$\mu_{\mathrm{o},i} = \sqrt{\dfrac{\mathrm{Tr}(\mathbf{Q}_i)}{\bar{\xi}\mathrm{Tr}(\mathbf{R})}}$	$1-\lambda_{\mathrm{o},i} = \sqrt{\dfrac{\gamma^2\mathrm{Tr}(\mathbf{Q}_i\mathbf{R})}{M\bar{\xi}}}$
$\zeta_{\mathrm{o},i}$	$\sqrt{\gamma^{-2}\bar{\xi}\mathrm{Tr}(\mathbf{R})\mathrm{Tr}(\mathbf{Q}_i)}$	$\sqrt{\gamma^{-2}M\bar{\xi}\mathrm{Tr}(\mathbf{Q}_i\mathbf{R})}$

Table 1. Steady-state MSE (ζ_i), optimum adaptation factors ($\nu_{\mathrm{o},i}$), and minimum steady-state MSE ($\zeta_{\mathrm{o},i}$) for MU-CMA and MU-SWA.

which is similar to the one obtained in [8, 4] for the RLS and LMS algorithms. This means that, as in the comparison between RLS and LMS, there are situations where MU-SWA presents superior tracking capability compared to MU-CMA and vice-versa.

Considering the MSE in a stationary environment, it follows that

$$\frac{\zeta_i^{\text{MU-SWA}}}{\zeta_i^{\text{MU-CMA}}} = \frac{M(1-\lambda)}{\mu\gamma\mathrm{Tr}(\mathbf{R})}. \tag{20}$$

We may adjust the adaptation factors of MU-CMA and MU-SWA to reach the same steady-state MSE. Let σ_u^2 be the variance of the input signal. From (20), with $\mathrm{Tr}(\mathbf{R}) = M\sigma_u^2$, we obtain

$$\mu = \frac{1-\lambda}{\gamma\sigma_u^2}. \tag{21}$$

5. SIMULATION RESULTS

To verify the tracking analysis for MU-CMA and MU-SWA, we assume $\mathbf{Q}_i = \sigma_q^2\mathbf{I}$, 4-QAM with statistics $\mathrm{E}\{|a|^2\} = \mathrm{E}\{|a|^4\} = \mathrm{E}\{|a|^6\} = 1$, and 16-QAM with statistics $\mathrm{E}\{|a|^2\} = 1$, $\mathrm{E}\{|a|^4\} = 1.32$, $\mathrm{E}\{|a|^6\} = 1.96$. We consider a MIMO system with $N=2$ users, $L=3$ sensors, and the channel model $H_{ij}(z) = h_0 + h_1 z^{-1}$ of Table 2 [10]. In the initialization of the $M=9$ taps, the non-null element of the equalizer 1 (Eq. 1) is the 3^{rd} coefficient and of the Eq. 2 is the 7^{th} one.

The steady-state MSE is measured with different values of forgetting factor λ for MU-SWA and with different values of step-size μ for MU-CMA. Fig. 2 shows that simulation results are in good agreement with the analysis in the convergence region of the algorithms for constant and nonconstant modulus sources. The theoretical minimum MSE $\zeta_{o,i}$ and the optimal adaptation factors $\mu_{o,i}$ and $\lambda_{o,i}$, predicted by the expressions of Table 1 are close to the experimental values.

In order to observe the distance between analysis and simulation results in presence of noise, we consider different values of sinal-to-noise ratio (SNR), as shown in Fig. 3. As expected, the higher the noise level, the greater the difference between analysis and simulation. It is relevant to note that, although the analysis is valid in absence of noise, the optimal adaptation factors can be predicted by the expressions of Table 1, independently of SNR.

Fig. 4 shows the theoretical and simulation results of steady- state MSE for different values of $\mathrm{Tr}(\mathbf{Q}_i)$. The good match between analysis and simulation can be observed for $\mathrm{Tr}(\mathbf{Q}_i) \leq 10^{-4}$. However, for $\mathrm{Tr}(\mathbf{Q}_i) > 10^{-4}$, there is a relatively large gap between them due to the high degree of nonstationarity.

	H_{ij} of Channel 1	
ij	h_0	h_1
11	$-0.6 + j0.4$	$+1.2 - j0.2$
21	$-0.6 + j0.8$	$+0.9 - j0.1$
12	$+0.1 + j0.7$	$-0.2 - j0.5$
22	$+0.4 - j0.3$	$-0.2 + j0.2$
13	$+0.5 + j0.4$	$-1.0 + j0.3$
23	$-0.1 + j0.8$	$+0.4 + j0.1$

Table 2. Communication channel model.

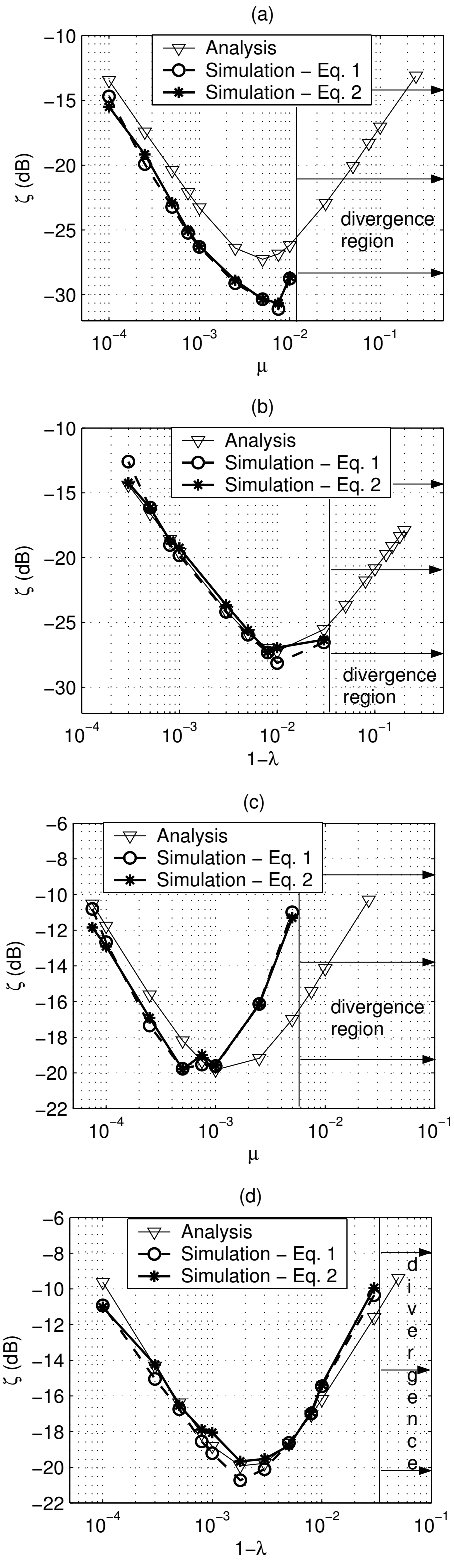

Fig. 2. Analysis and simulation MSE of (a) MU-CMA (4-QAM), (b) MU-SWA (4-QAM), (c) MU-CMA (16-QAM), and (d) MU-SWA (16-QAM); $\lambda_r = 0.999$; $\alpha = 4$; $\sigma_q = 10^{-3}$; average over 100 experiments.

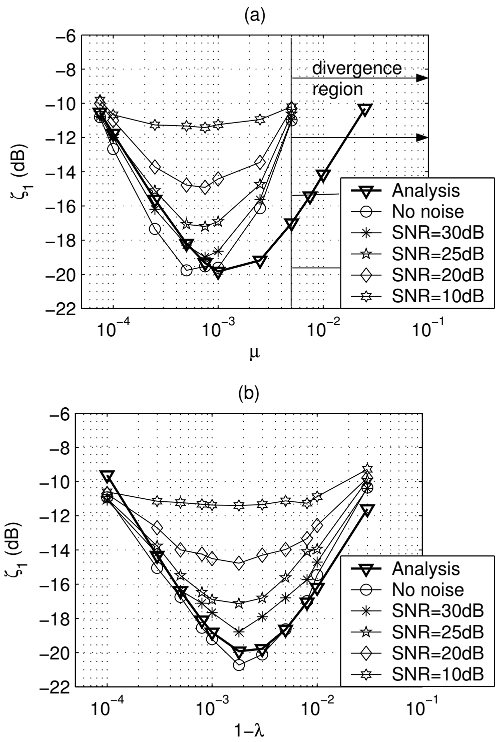

Fig. 3. Analysis and simulation MSE of (a) MU-CMA, (b) MU-SWA; 16-QAM; $\lambda_r = 0.999$; $\alpha = 4$; $\sigma_q = 10^{-3}$; average over 50 experiments.

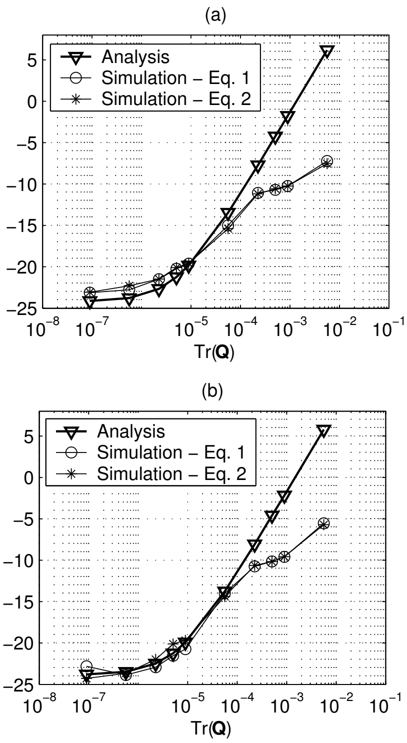

Fig. 4. Analysis and simulation MSE as function of $\mathrm{Tr}\mathbf{Q}$; (a) MU-CMA; (b) MU-SWA; 16-QAM; $\mu = 10^{-3}$; $\lambda = 0.9982$; $\lambda_r = 0.999$; $\alpha = 4$; average over 50 experiments.

6. CONCLUSION

Under certain conditions, the considered algorithms can reach the same steady-state MSE. The ratio between the minimum MSE for MU-SWA and MU-CMA is the same obtained between the RLS and LMS algorithms; this allows a direct extension of the comparison of these algorithms to blind and multiuser context. Through simulations, we verify that theoretical analysis can predict experimental steady-state results. This analysis is validated in an environment with a small degree of nonstationarity and high signal-to-noise ratio. Although there is a large gap between analysis and simulation results for low signal-to-noise ratios, the optimal adaptation factors can be predicted by the obtained expressions.

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