# ROBUST QUADRATIC TIME-FREQUENCY DISTRIBUTIONS FOR THE ANALYSIS OF MULTICOMPONENT FM SIGNALS IN HEAVY-TAILED NOISE

*Mohamed Sahmoudi*[1] *and Karim Abed-Meraim*[2]

[1] LSS, CNRS-SUPELEC-UPS, 91192 Gif-sur-Yvette, France
[2] TSI Departement, Telecom-Paris, 75634 Paris, France

**ABSTRACT**

We consider the problem of instantaneous frequency estimation of multicomponent frequency-modulated signals corrupted by additive heavy-tailed noise. For that, a new time-frequency distribution, named the robust modified B-distribution (R-MBD), is developed as a generalization of the robust minimax M-estimates to handle such signals. We show that this representation outperforms the robust polynomial Wigner-Ville distribution (r-PWVD) in term of high resolution for this class of non-stationary signals. The proposed approach is compared to the higher-order ambiguity function (HAF) algorithm, for the instantaneous frequency estimation of a multicomponent signal. Computer simulations show the superiority of the proposed algorithm over the HAF.

## 1. INTRODUCTION-PROBLEM STATEMENT

This paper is concerned with the analysis of multicomponent frequency modulated (FM) signals, corrupted by additive heavy-tailed noise. Multicomponent signal mean a signal whose time-frequency representation presents multiple ridges in the time-frequency plane.

• **Signal model**: Analytically, the noisy signal considered in this paper may be defined as,

$$x(t) = s(t) + z(t) = \sum_{i=1}^{M} s_i(t) + z(t) \tag{1}$$

where each component $s_i(t)$, of the form $s_i(t) = a_i(t)\ e^{j\phi_i(t)}$, is assumed to have only one ridge, or one continuous curve, in the time-frequency plane. $a_i(t)$ is the amplitude and $\phi_i(t)$ denotes the phase of the $i$th component of the signal. The probability density function (pdf) of the random *impulsive* noise $z(t)$ is modeled as a *heavy-tailed* distribution[1]. Examples of this kind of distributions include $\alpha$-stable with $\alpha < 2$ and generalized Gaussian laws.

• **Symmetric $\alpha$-stable process** $(S\alpha S)$: $S\alpha S$ have proved to be effective in modeling many real-life engineering problems such as outliers and impulse signals [10]. The pdf of $S\alpha S$ processes, denoted as $f_\alpha$, do not have closed form except for the cases $\alpha = 1$ (Cauchy distribution), $\alpha = 1/2$ (Levy distribution) and $\alpha = 2$ (Gaussian distribution). The $S\alpha S$ distribution is defined by means of its characteristic function $\psi(t) = \exp\{j\mu t - \gamma|t|^\alpha\}$, where $\alpha$ $(0 < \alpha \leq 2)$ is the characteristic exponent, controlling the heaviness of the pdf tail, $\gamma(\gamma > 0)$ is the dispersion, which plays an analogous role to the variance, and $\mu$ $(\mu \in \mathbb{R})$ is the location parameter, the symmetry axis of the pdf. Due to their heavy tails, stable distributions do not have finite second or higher-order moments, except for the limiting case of $\alpha = 2$.

• **Generalized Gaussian (GG) pdf**: Another way to model impulsive noise processes is through the generalized Gaussian pdf given by $f_\alpha(x) = A_\alpha \exp\left(-b|x|^\alpha\right)$ where $0 < \alpha \leq 2$, $A_\alpha$ and $b$ are two real constant. For $\alpha = 2$ we have the Gaussian distribution and for $\alpha = 1$ we have the Laplacian distribution which is known to be a good model for impulsive noise.

• **Time-frequency analysis**: Our primary interest, in this work, is to estimate the instantaneous frequency (IF) of each FM signal $s_i(t)$ of (1), defined as

$$IF_i(t) \triangleq \frac{1}{2\pi}\frac{d\phi_i(t)}{dt} \tag{2}$$

Time-frequency analysis techniques are used as they reveal the multicomponent nature of such signals. Ideally, for a given FM signal, the time-frequency distribution (TFD) is represented as a row of delta functions around the signals instantaneous frequency. This property makes the peak of the TFD a very powerful tool as an IF estimator. However, quadratic TFD of multi-component signals suffer from the presence of cross-terms, which can obscure the real features of interest in the signal. The properties of a quadratic TFD are completely determined by its kernel. This kernel should have the shape of a two-dimensional (2-D) low-pass filter to attenuate the cross-terms that exist away from the origin in the ambiguity domain and preserve the auto-terms that concentrate around the origin of this domain [3]. Considerable efforts have been made to define TFD that reduce the effect of cross terms while improving the time-frequency resolution (e.g., [3, 1]). This led to the so-called reduced interference distributions that include the modified B-distribution (MBD), and the signal-dependent optimal time-frequency representation. In this work, we have used the MBD [3] given by:

$$T(t,f) = \int\int_{-\infty}^{+\infty} G_{MB}^{\sigma}(t')[x(t-t'+\frac{\tau}{2})x^*(t-t'-\frac{\tau}{2})]e^{-j2\pi f\tau}dt'd\tau \tag{3}$$

where $G_{MB}^{\sigma}(t') = \frac{k_\sigma}{\cosh(t')^{2\sigma}}$, $0 \leq \sigma \leq 1$ is a real parameter that controls the tradeoff between component's resolution and cross-terms suppression and $k_\sigma = \Gamma(2\sigma)/(2^{2\sigma-1})\Gamma^2(\sigma)$ is the normalizing factor. The choice of the MBD, stems from the fact that it presents a good performance in terms of resolution and cross-terms suppression [3]. The effect of noise on the TFD is another consideration that has direct influence on IF estimation. Study of

This work was in part supported by the CNRS within a MathSTIC project

[1]For complex-valued noise signal, we simply consider that $z(t) = z_r(t) + jz_i(t)$ where $z_r(t)$ and $z_i(t)$ represent two independent heavy-tailed processes with a same pdf function.

additive Gaussian noise influence on TFD-based IF estimation is an important issue [1, 5, 3, 11]. However, in many practical applications, especially in communications, signals are disturbed by impulsive noise due to the propagation environment or to large errors in collecting and recording the data. These noise processes are commonly modeled by heavy-tailed distribution [10]. Since outliers or impulsive noise have an unusually great influence on standard IF estimators, robust procedures attempt to modify those schemes. Only a limited literature was dedicated to the analysis of multi-component FM signals in impulsive noise. In [12], authors propose a class of parametric robust methods to handle linear FM signals. In the same paper [12] a TFD-based technique have been proposed using a pre-processing stage to mitigate the impulsive noise effect. The other alternative, which is the focus of this paper, is to apply the M-estimation principle in order to design a robust TFD with respect to impulsive noise. In [9] and [2], the authors proposed the robust spectrogram and the robust polynomial Wigner-Ville distribution (PWVD), respectively. However, it is known that the spectrogram suffers from low resolution in time-frequency domain, while the PWVD suffers from cross-terms for multi-component signals. In this paper, we use the modified B-distribution [3] and the M-estimation theory to design a new robust TFD, which is referred to as the *robust modified B-distribution (R-MBD)*, for the analysis of multi-component FM signals in heavy-tailed noise. We show that the proposed approach can solve problems that existing time-frequency distributions cannot.

## 2. MINIMAX ROBUST ESTIMATION

The robust minimax approach is an alternative to the conventional maximum likelihood (ML) that overcomes the ML estimates sensitivity and improves the efficiency in an environment with unknown heavy-tailed distribution [7].

• **M-Estimation**: Consider the general signal in noise model given in (1):

$$x(t) = s(t, \theta) + z(t)$$

where $z(t)$ is i.i.d. noise and the signal, $s(t)$, is parameterized by $\theta = (\theta_1, \cdots, \theta_M)^T$, $(.)^T$ denoting transposition. The aim is to estimate $\theta$ from $N$ observations $x(t),\ \ t = 1, \cdots, N$. Given the noise density, $f(.)$, one obtains the ML solution as

$$\hat{\theta} = \arg\min_{\theta} \sum_{t=1}^{N} \rho\{x(t) - s(t, \theta)\} \tag{4}$$

where $\rho(x) = -\log f(x)$. Alternatively, one can solve the $M$ coupled equations

$$\sum_{t=1}^{N} \psi\{x(t) - s(t, \theta)\} \frac{\partial s(t, \theta)}{\partial \theta} = 0 \tag{5}$$

where $\psi(x) = -f'(x)/f(x)$ is the location score function of $f(x)$. It is clear that without a priori knowledge of $f(x)$ estimation of $\theta$ cannot be optimal. Huber considered estimation in the presence of outliers or impulsive noise and proposed the concept of M-estimation [7]. In an M-estimation framework $-\log f(x)$ is replaced with a similarly behaved function, $\rho(x)$, chosen to confer robustness on the estimator under deviations from a nominal density. Thus, a M-Estimate for $\theta$ can be obtained as a solution of the optimization problem given in equation (4) or by solving the $M$ coupled equations

$$\sum_{t=1}^{N} \varphi\{x(t) - s(t, \theta)\} \frac{\partial s(t, \theta)}{\partial \theta} = 0 \tag{6}$$

where $\varphi(x) = \rho'(x)$. When $f(x)$ is unknown one is unsure of how close $\varphi(x)$ is to $\psi(x)$.

• **Theoretical performances**: Let $F$ be the distribution of the noise and $F_n$ its empirical counterpart from a sample of size $n$. Then an estimate of $\theta$ can be defined in terms of a functional $T$ operating on $F_n, T(F_n)$, while the true parameters are obtained as $T(F)$. Using the influence function concept, it is proved in [14] that the asymptotic covariance of estimation errors of $\theta$ has the form

$$Cov\,\{T(F_n), T(F)\} = \frac{E[\varphi^2(x)]}{E[\varphi'(x)]^2} \quad \sum_{n=1}^{N} \Lambda_n \Lambda_n^T \Bigg)^{-1} \tag{7}$$

where $\varphi(x) = \rho'(x)$ and $\Lambda_n$ is the gradient of $s_n(\theta)$. Then the only degree of freedom at our disposal for minimizing the asymptotic covariance is through appropriate choice of $\varphi(x)$. It is then critical to choose $\varphi(x)$ such that the asymptotic relative efficiency (ARE) with respect to Fisher information

$$ARE(\varphi|\psi) = \frac{E[\varphi'(x)]^2}{E[\varphi^2(x)]E[\psi^2(x)]} \tag{8}$$

is maximized.

• **Optimal loss function**: Let the noise distribution $f$ be known incompletely; what is known is only that it belongs to a certain class $\mathcal{P}$. Applying to our M-estimator the Cramer-Rao inequality, under certain regularity assumptions, gives

$$Cov\,\{T(F_n), T(F)\} \geq A(\Lambda_n) I(f)^{-1} \tag{9}$$

where $I(f)$ is the Fisher information and $A(\Lambda_n)$ is a matrix depending only on $\Lambda_n$. The worst distribution is naturally the one for which the right-hand part in (9) is maximal, or $I(f)$ is minimal. In other words, the robust Huber's minimax estimator over $\mathcal{P}$ is defined as in the ML method by equation (4) with the loss function

$$\rho^*(z) = -\ln(f^*(z)) \tag{10}$$

where $f^*(z)$ is selected in $\mathcal{P}$ such that the information on the parameter contained therein is minimal, i.e. a solution of the problem

$$f^*(z) = arg \min_{f \in \mathcal{P}} I(f) \tag{11}$$

where $I(f) = \int (f'(z))^2/f(z))dz$ denotes the Fisher information. We call the M-estimator robust if the loss function $\rho$ is given according to (11) and (10). This approach consist to consider the worst case (among $\mathcal{P}$) corresponding to the pdf giving the minimum Fisher information value. Solving the worst case would insure robustness (good estimation performance) if the considered signal pdf belongs to $\mathcal{P}$. It is emphasized that the robustness property of the estimator depends on how the class $\mathcal{P}$ is defined. Thus, in order to obtain the robust minimax estimator, first, an appropriate class $\mathcal{P}$ should be defined, and after that, the loss function $\rho$ is given by (10) and (11) [7].

• **$L_p$-norm criterion**: The analogue of variance for an $\alpha$-stable variable is the dispersion, $\gamma$ . Therefore, in analogy to the minimum mean squared error (MMSE) criterion we can define a similar criterion, namely the minimum dispersion (MD) criterion which aims at minimizing the dispersion of the estimation errors rather than the variance [13]. Recall that the *fractional lower order moments (FLOMs)* of an alpha-stable random variable with zero location parameter and dispersion $\gamma$ are given by

$$E|X|^p = C(p,\alpha)\gamma^{\frac{p}{\alpha}} \text{ for } 0 < p < \alpha \tag{12}$$

where $C(p,\alpha) = 2^{p+1}\frac{\Gamma(\frac{p+1}{2})\Gamma(\frac{-p}{\alpha})}{\alpha\sqrt{\pi}\Gamma(\frac{-p}{2})}$ is a constant depending only on $p$ and $\alpha$. This tells us that the $p$th order moment of an $\alpha$-stable random variable and its dispersion are related through only a constant. Therefore, the MD criterion is equivalent to the least $L_p$-norm estimation where $0 < p < \alpha$ and the estimates of a parameter $\theta$ can be obtained from equation (4) using the $L_p$-norm loss function

$$\rho_p(x) = |x|^p \text{ where } 1 \leq p \leq 2 \tag{13}$$

Equivalently, we could define it from equation (6) through the nonlinear function $\varphi_p(x) = \mathrm{sign}(x)p|x|^{p-1}$, where $\mathrm{sign}(x) \triangleq x/|x|$, as a tool of the robust estimation which appears originally as a heuristic idea supported latter by theoretical and experimental studies. In particular, for $p = 1$, $L_1$-norm criterion referred to as "modulus function" in [2, 9] was used to define the robust periodogram and robust PWV distributions. It should be emphasized that the least $L_p$-norm estimates are not only optimal in a MD sense for $\alpha$-stable data, but also optimal in the ML sense for the family of generalized Gaussian distribution. Indeed, for ML estimator, the noise pdf is assumed to be known and for $L_p$-norm criterion, $p$ chosen as the same value of the index $\alpha$ in the generalized Gaussian pdf. In addition, applying equations (10) and (11) over the class of generalized Gaussian pdf, we can easily show that the least $L_p$-norm estimate is optimal also in the robust minimax sense if we choose $p$ as the smallest value in the considered set of $\alpha$ values. In addition, the numerical evaluations of the score functions of $\alpha$-stable laws, for diferent values of $1 \leq \alpha < 2$ show that these functions are all odd and bounded. Since it can be shown that these score functions behave similarly to all the nonlinearity functions $\varphi_p$ which are also odd and bounded, we propose them as a natural choice of the nonlinearity $\varphi$.

## 3. ROBUST TIME-FREQUENCY BASED IF ESTIMATION

•**Combining M-estimation and time-frequency analysis**: It is well known that the conventional TFD are quite sensitive with respect to non-Gaussian noise. In particular, in presence of impulsive noise they produce poor estimation results. In order to get a good estimation performance in this context, we use the robust statistics theory to define a new robust quadratic time-frequency distribution. Let consider the noisy signal (1) in discrete-time $x(kT) = s(kT) + z(kT)$ where $T$ is a sampling period. A standard time-frequancy distribution, at a point $(kT, f)$, is shown to be a solution of the optimization problem [9]

$$\hat{\mathcal{B}} = arg\min_{\mathcal{B}} \mathcal{J}\left(kT, f, \mathcal{B}\right) \tag{14}$$

where

$$\mathcal{J}(kT, f, \mathcal{B}) = \sum_{n=-N/2}^{N/2} w(nT)\rho[e(k, f, n)], \tag{15}$$

$$e(k, f, n) = \mathcal{G}_x(kT, nT)e^{-j2\pi f nT} - \mathcal{B}$$

where $w(nT)$ is a window function, $\mathcal{G}_x(kT, nT)$ being the kernel of the considered quadratic time-frequency distribution of the FM signal $x(kT)$ and $\mathcal{B}$ is an estimate of the expectation of the sample average of the quantity $\mathcal{G}(kT, nT)e^{-j2\pi f nT}$. If we choose the loss function $\rho(e) = |e|^2$, we can show by solving for $\mathcal{B}$ the expression

$$\frac{d\mathcal{J}(kT, f, \mathcal{B})}{d\mathcal{B}^*} = 0$$

that the optimal solution corresponds to the standard TFD

$$B_x^s(kT, f) = \sum_{n=-N/2}^{N/2} \frac{w(nT)}{\sum_{n=-N/2}^{N/2} w(nT)} \mathcal{G}_x(kT, nT)e^{-j\pi f nT} \tag{16}$$

Thus, for a weighted window, the standard TFD can be treated as an estimate of the mean, calculated over a set of complex-valued observations $G = \{\mathcal{G}_x(kT, nT)e^{-j\pi f nT}; n \in [-N/2, N/2]\}$. It has been shown that the optimal loss function $\rho$ derived in the minimax Huber's estimation theory (see section 2) could be applied to the design of a new class of robust time-frequency distributions, inheriting properties of strong resistance to impulsive noise. In particular, some robust TFDs have been derived by using the absolute error loss function $\rho(e) = |e|$ in (15) [9]. In this work, we propose to choose the loss function $\rho$ in the criterion (15) as the $L_p$-norm criterion $\rho(e) = |e|^p$ with $1 \leq p \leq 2$. The parameter $p$ control the exponential loss function degree. In the $\alpha$-stable noise case, we choose $p < \alpha$, then we must choose $p \in [1; \alpha[$. The optimal choice of this criterion is well detailed in section 2.

•**Robust time-frequency distribution**: In this work, we use the MBD to handle multi-component non-stationary FM signals given by model (1). However, similarly to the standard spectrogram, WVD and PWVD, the standard MB-distribution is not an adequate analysis tool in presence of heavy-tailed noise. To mitigate this problem, we use the MB-distribution kernel given in Equation (3) and the $L_p$ norm loss function in the design of the proposed robust MBD to analyze FM signals affected by impulsive noise. In this case, we find the optimal solution, labelled the robust modified B-distribution (R-MBD), to be the solution of

$$\frac{\partial}{\partial\mathcal{B}^*}\left\{\sum_{n=-N/2}^{N/2} w(nT)|\mathcal{G}_x(kT, nT)e^{-j\pi f nT} - \mathcal{B}|^p\right\} = 0$$

$$\Longleftrightarrow$$

$$\sum_{n=-N/2}^{N/2} w(nT)(\mathcal{G}_x(kT, nT)e^{-j\pi f nT} - \mathcal{B})|\mathcal{G}_x(kT, nT)e^{-j\pi f nT} - \mathcal{B}|^{p-2} = 0$$

$$\Longleftrightarrow$$

$$\mathcal{B}_x^r(kT, f) = \sum_{n=-N/2}^{N/2} \frac{d(k, f, n)}{D_0(kT, f)} \mathcal{G}_x(kT, nT)e^{-j\pi f nT}, \tag{17}$$

$$d(k, f, n) = w(nT)|\mathcal{G}_x(kT, nT)e^{-j\pi f nT} - \mathcal{B}_x^r(kT, f)|^{p-2},$$

$$D_0(kT, f) = \sum_{n=-N/2}^{N/2} d(k, f, n)$$

Since, the quantity $\mathcal{B}_x^r(kT, f)$ appears on the right as well as on the left hand side of equation (17), an iterative procedure is necessary in order to obtain the R-MBD. The robust-MBD algorithm can be summarized as follows:

**Robust-MBD algorithm**

**Step 1.** *Evaluate the standard MBD using equation (16)*
**Step 2.** *For initialization purposes, set the iteration index $i = 0$ and*

$$\mathcal{B}_x^{r\,0}(kT, f) = \mathcal{B}_x^{s}(kT, f)$$

**Step 3.** *Sweep. Set $i = i + 1$ and do*

- *Compute $d(k, f, n)$ and $D_0(kT, f)$ using equations (3) and (3) respectively.*
- *Compute the robust MBD, for iteration $i$, $\mathcal{B}_x^{r\,i}(kT, f)$ using Equation (17).*

**Step 4.** *If the relative absolute difference between two iterations is smaller than a fixed threshold $\epsilon$, i.e.*

$$|\mathcal{B}_x^{r\,i}(kT, f) - \mathcal{B}_x^{r\,i-1}(kT, f)|/|\mathcal{B}_x^{r\,i}(kT, f)| \leq \epsilon$$

then stop the algorithm. Otherwise go to Step 3.

It was shown in [8] that the above iterative algorithm will converge to a single (global) minimum under a good choice of the initial value. In our case, the choice of $\mathcal{B}_x^{r\,0}(kT, f) = \mathcal{B}_x^{s}(kT, f)$ satisfies the necessary condition of the convergence.

•**Component Separation & IF Estimation**: In [1], an algorithm that separates the signal components and estimate their respective IF laws from the signal TFD in the Gaussian noise case, has been presented. In this work, we apply this algorithm, using the proposed robust MB-distribution of the noisy signal, to estimate the IF of each component $s_i(t)$ of the multi-component signal given by model (1).

## 4. PERFORMANCE EVALUATION & COMPARISON

•**Experiment 1: Clear TFD representation**

To check the validity and effectiveness of the proposed algorithm, we consider the time-frequency representation of a three-component FM signal corrupted by an impulsive noise modeled as a generalized Gaussian distribution with $\alpha = 1.5$ . The standard MBD, displayed in Fig. 2. yields a poor representation; while, the R-MBD (computed using $p = 1$) displayed in Fig. 1. reveals clearly the features of the noisy signal. The superiority of the R-MBD over the standard MBD is obvious.

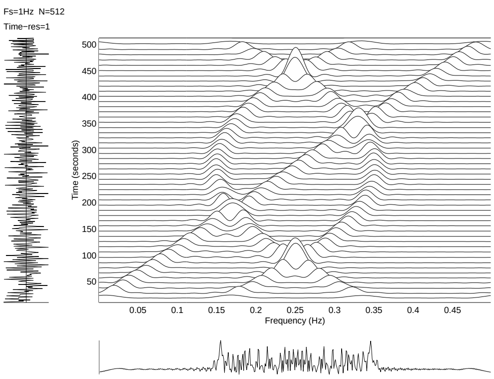


**Fig. 1**. The Robust-MBD of the multi-component signal test.

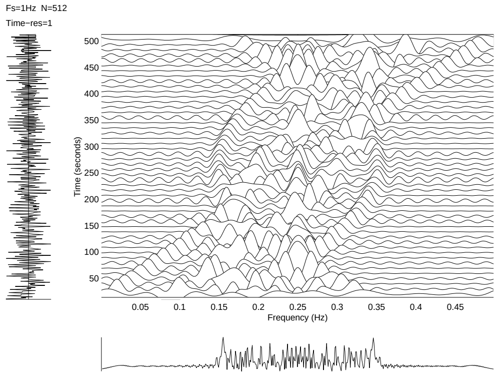


**Fig. 2**. The standard MBD of the multi-component signal test.

•**Experiment 2: Sensitivity to the noise power**

Here, we assess the statistical performance of the R-MBD based IF estimator of multicomponent FM signals. For that let us consider a two linear FM components signal embedded in additive impulsive $\alpha$-stable noise $z(t)$ modeled as $x(t) = s_1(t)+s_2(t)+z(t)$ where $s_1(t) = \exp\{j2\pi(a_1t + b_1t^2)\}$ and $s_2(t) = \exp\{j2\pi(a_2t + b_2t^2)\}$. The noise $z(t)$ is chosen with zero location parameter, characteristic exponent $\alpha = 1$ and dispersion $\gamma$. The signals IF coefficients are given by $a_1 = 0.2$, $b_1 = 0.1 * 10^{-3}$, $a_2 = 0.45$ and $b_2 = -1.5 * 10^{-3}$. To validate the proposed method and to compare it with some existing methods, we implement the following procedure:

1. Compute the TFD of the noisy signal $x(t)$ using r-PWVD [2] and the proposed R-MBD. For that, we choose $\sigma = 0.01$ for the MBD kernel and $p = \alpha/3$ for the fractional $L_p$-norm loss function to design the R-MBD. In the experiments, we fix the signal length equal to $N = 501$ and the window length, used in the r-PWVD implementation, equal to 101 samples.
2. Put the computed TFD matrix through the component separation algorithm [1] in order to extract the two respective components. The peaks of the extracted components (in the time-frequency domain) are, then, used to estimate the IFs of the chirps [1].
3. Put the same noisy signal through the HAF algorithm to estimate the four chirp parameters $a_1, b_1, a_2$ and $b_2$.
4. For the HAF algorithm, use a simple polynomial fit to obtain estimates of $IF_1(t)$ from $(a_1, b_1)$ and estimates of $IF_2(t)$ from $(a_2, b_2)$.

The estimation performance is measured by the normalized MSE defined by

$$NMSE = \frac{1}{N_r} \sum_{r=1}^{N_r} \frac{\|\hat{\theta}_r - \theta\|^2}{\|\theta\|^2}$$

where $\theta$ is the considered parameter, $\hat{\theta}_r$ is the estimate of $\theta$ at the $r$th experiment and $N_r$ is the number of Monte-Carlo runs chosen here equal to 500. In Fig. 3., we display the NMSE of the IF estimate versus the inverse noise dispersion $1/\gamma$ in dB for HAF, r-PWVD and R-MBD. The accuracy and superiority of the R-MBD over both algorithms r-PWVD [2] and HAF [11] is evident.

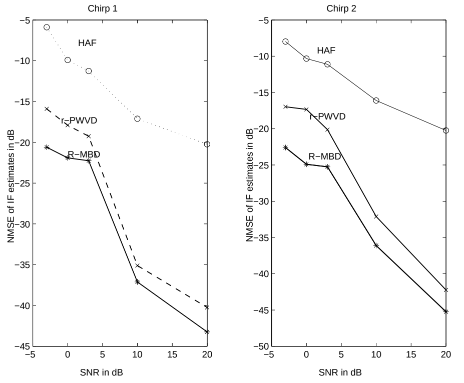


**Fig. 3**. NMSE of IF estimates, corresponding to the HAF, r-PWVD and the R-MBD for a noisy two-component chirp signal versus the inverse dispersion noise in dB.

**•Experiment 3: Sensitivity to $\alpha$**

To check the sensitivity to deviations in the value of $\alpha$ from our model, we have computed the information loss due to the replacement of the score function $\psi_\alpha = -f'_\alpha/f_\alpha$ in the estimating equations by the Cauchy score function proposed in [6], i.e., $\phi_c = 2x/(1+x^2)$ and by $L_p$ norm nonlinearity with different $p$ values. We measure the information loss by the asymptotic relative efficiency (ARE) defined in equation (8). The Fisher information has been evaluated using the well known (e.g. [6]) good approximation $I(f_\alpha) \approx \alpha/2$. Using the formula (12), we easily obtain the ARE of the nonlinearities $\varphi_p$ as

$$ARE(\varphi_p|\psi_\alpha) = I(f_\alpha)^{-1}\gamma^{\frac{-2}{\alpha}}(p-1)^2\frac{C(p-2,\alpha)}{C(2(p-1),\alpha)}$$

On the other hand, we evaluated numerically the ARE of the Cauchy score function using the following expression,

$$ARE(\phi_c|\psi_\alpha) = I(f_\alpha)^{-1}E[\phi_c^2(x)] \approx \frac{2}{\alpha}\int \phi_c^2(x)f_\alpha(x)dx$$

This expression has been easily obtained from (8) using the score function property $E[\phi_c^2] = E[\phi'_c]$. Fig. 4 shows a numerical eval-

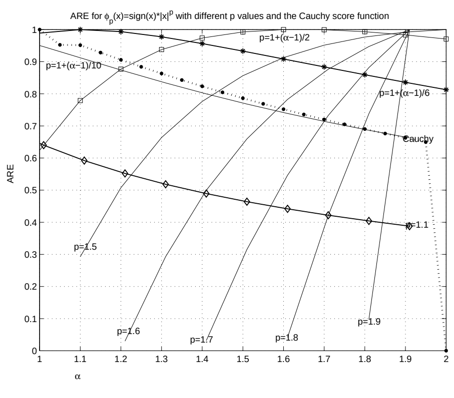


**Fig. 4**. Information loss in different $\alpha$'s with dispersion $\gamma = 1$.

uation of the information loss ARE when the noise is $\alpha$-stable distributed with different $\alpha$'s and with dispersion $\gamma = 1$. For "good" $p$ values, the information loss is relatively small for any $\alpha$. In other words, the use of the $L_p$-norm lost function is robust to deviation in the characteristic exponent $\alpha$ in our model. We can see also that for small deviations from Gaussianity ($1.8 \leq \alpha \leq 2$), the performance loss is small for any reasonable choice of $p$ ($1.4 \leq p \leq 2$). However, for any $p$, there is a level where the noise is more impulsive, and the information loss increases as $\alpha$ decreases. The $L_p$ norm estimator is superior to the Cauchy estimator over some good $p$ values, its efficiency being up to 15% higher.

## 5. CONCLUSION

In this paper, we proposed a new approach to analyze multicomponent non-stationary FM signals corrupted by additive heavy-tailed noise. The fractional $L_p$-norm ($1 \leq p \leq 2$) loss function has been used in the M-estimation framework to design a new robust TFD referred to as R-MBD. Computer simulations confirm the effectiveness of the proposed algorithm and that the best results in terms of estimation accuracy are obtained by the R-MBD based algorithm which is, on the other hand, the most expensive one followed by the r-PWVD based method.

## 6. REFERENCES


[1] B. Barkat and K. Abed-Meraim, "Algorithms for blind components separation and extraction from the time-frequency distribution of their mixture", *to appear in Journal of App. Sig. Proc.*, 2005.

[2] B. Barkat and LJ. Stankovic, "Analysis of polynomial FM signals corrupted by heavy-tailed noise", *Signal Processing*, 84, pp: 69-75, 2004.

[3] B. Boashash, V. Sucic, "Resolution measure criteria for the objective assessment of the performance of quadratic time-frequency distributions", *IEEE Trans. On Signal Processing*, Vol. 51, No. 5, pp: 1253 - 1263, May 2003.

[4] P. Flandrin, "Some features of time-frequency representations of multicomponent signals", *Proc. of IEEE Inter. Conference ICASSP '84*, Vol. 9, pp: 266 - 269, 1984.

[5] P. Duvaut, A. Doucet, C. Veaux, and P. Flandrin, "Instantaneous frequency estimation: Bayesian approaches versus reassignment-application to gravitational waves" *Proc. of IEEE Inter. Conference ICASSP'96*, vol. 5, pp: 2968 - 2971, 1996.

[6] J. Friedmann, H. Messer and J.-F. Cardoso, "Robust parameter estimation of a deterministic signal in impulsive noise", *IEEE Trans. On Signal Processing*, vol. 48, No. 4, April 2000.

[7] P. J. Huber, *Robust Statistics*. New York: Wiley, 1981.

[8] S. Kaluri, G.R. Arce, "Fast algorithms for weighted myriad algorithm computation by fixed point search", *IEEE Trans. on Signal Processing*, April 2000.

[9] V. Katkovnik, I. Djurovic, LJ. Stankovic, "Robust time-frequency representation", in B. Boashash (Ed), Time-frequency signal analysis and processing, Elsevier, 2003.

[10] C.L. Nikias and M. Shao, "Signal processing with alpha-stable distributions and applications", *John Wiley* , 1995.

[11] S. Peleg and B. Friedlander. "Multicomponent signal analysis using the polynomial-phase transform", *IEEE Trans. On AES*, Jan. 1996.

[12] M. Sahmoudi, K. Abed-Meraim and B. Barkat, "IF estimation of multicomponent chirp signal in impulsive $\alpha$-stable noise environment using parametric and non-parametric approaches" *In Proc. of EUSIPCO'2004*, Austria, Sept., 2004.

[13] M. Sahmoudi, K. Abed-Meraim and M. Benidir, "Blind separation of impulsive alpha-stable sources using minimum dispersion criterion", *IEEE Signal Processing Letters*, Vol. 12, April 2005.

[14] R. F. Brcich and A. M. Zoubir, "Robust estimation with parametric score function estimation", *In Proc. of the IEEE Conference ICASSP 2002.*, vol.2, pp: 1149-1152, 2002.