ESTIMATION OF THE NUMBER OF SOURCES WITH DOA PRIORS

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ABSTRACT

In this paper, we present a technique for estimating the number of sources based on the eigenvectors of the covariance matrix of received signal at an antenna array when directions of arrival priors are obtained from the received signal, e.g. via a spectral estimation method such as MVDR. The proposed technique is designated the eigenvector order estimation technique (EOET). EOET outperforms AIC, MDL and GDE techniques in practice and provides comparable detection sensitivity as AIC, MDL, and GDE in simulation.

1. INTRODUCTION

Estimation of the number of signals impinging on an antenna array is of significant importance in high resolution direction of arrival estimation methods such as MUSIC and ESPRIT, or in other applications that require the estimation of the signal subspace [1].

Consider an antenna array of M elements and suppose that it receives d signals from directions $\theta_1, \cdots, \theta_d$, e.g.

$$X(n) = AS(n) + W(n)$$

where, A, $M \times d$, is a full-column rank matrix ($M > d$) of array responses to signals received from directions $\theta_1, \cdots, \theta_d$. The n^{th} snapshot source signal vector $S(n)$, $d \times 1$, is modeled as a stationary stochastic process with covariance matrix R_s. The noise vector, $W(n)$, $M \times 1$, is a zeros mean stationary complex Gaussian vector with covariance matrix R_w. Furthermore, $S(n) = [s_1(n), \cdots, s_d(n)]^T$ and $W(n) = [w_1(n), \cdots, w_M(n)]^T$ are uncorrelated. Under these assumptions, the covariance matrix of the received signal at the antenna array can be written as

$$R = \mathcal{E}\{X(n)X^H(n)\} = AR_sA^H + R_w$$

where, superscript ^{H} denotes Hermitian transpose. The subspace spanned by columns of A are called signal subspace, and its orthogonal complement is called noise subspace.

The problem of interest is to determine the dimension of the signal subspace, namely d. Most of the existing methods for estimating d are formulated based on the eigenvalues of the sample covariance matrix. Under the ideal spatially white noise model of $R_w = \sigma^2 I_M$, where σ^2 is a constant representing noise power at each antenna element, and I_M denotes the identity matrix of dimension M, it can be seen that the $M - d$ smallest eigenvalues of the covariance matrix, R, are equal. Therefore, the determination of the number of sources is equivalent to determining the multiplicity of the smallest eigenvalue of R [10] [11].

Akaike information criterion (AIC) and minimum description length (MDL) [2] are two criteria for estimating d that are derived based on the information theoretic criterion proposed by Akaike [3] and by Rissanen and Schwartz [4] [5]. These two criteria are derived based on the assumption that $s_1(.), \cdots, s_k(.)$ are complex, stationary, and ergodic Gaussian random processes and $R_w = \sigma^2 I_M$.

When the noise statistics are unknown, other methods such as Gerschgorin Disk Estimator (GDE) [12] that is based on clustering the Gershgoring radii of the transformed covariance matrix have been proposed.

2. EOET TECHNIQUE

In this section, we present a technique for estimating the number of sources based on the eigenvectors of the covariance matrix of received signal at the antenna array when the set of directions of arrival,$\Theta = \{\theta_1, \cdots, \theta_d\}$, is a subset of a known set E. The proposed technique is designated the eigenvector order estimation technique (EOET). EOET is an improvement to the methods presented in [8], [14]. Information about set E can be inferred e.g. by using a spectral estimation method such as MVDR [13], which does not require knowledge of the number of sources. If no information can be obtained about the directions of arrival, set E can be considered to be $E = [-90°, 90°]$.

Let $\Gamma D \Gamma^H$ be the the spectral decomposition of R, e.g. $R = \Gamma D \Gamma^H$, where $\Gamma = [\gamma_1, \cdots, \gamma_M]$ is an orthogonal matrix whose columns, $\gamma_1, \cdots, \gamma_M$, are the eigenvectors of R, and D a diagonal matrix, whose diagonal elements are the eigenvalues of R. Define $P_m = [\gamma_1, \cdots, \gamma_m]$, $Q_m = I - P_m {P_m}^H$ and $f_m(\theta) = Q_m \mathbf{a}(\theta)$. Where $\mathbf{a}(\theta)$ is the array response to direction θ. For example, for a uniform linear array the array response to direction θ can be written as,

$$\mathbf{a}(\theta) = [1, e^{\frac{-j2\pi\zeta\sin(\theta)}{\lambda}}, \cdots, e^{\frac{-j2\pi(M-1)\zeta\sin(\theta)}{\lambda}}]^T$$

where ζ is the interelement spacing of the antenna array, θ is the direction of arrival of received signal, and λ is the carrier wavelength and superscript ^{T} denotes transpose. Since Span$\{a(\theta_1), \cdots, a(\theta_d)\}$=Span$\{\gamma_1, \cdots, \gamma_d\}$, for $i > d$, we have $f_{i-1}(\theta_j) = f_i(\theta_j) = 0$ for $j = 1, \cdots, d$. EOET is based on a measure of equality between $f_{i-1}(\theta_j)$ and $f_i(\theta_j)$ for $j = 1, \cdots, d$.

One such measure is given by ρ_i, $i = 1, \cdots, M-1$,

$$\rho_i = \frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ||f_i(\theta) + \epsilon\mathbf{1}|| \, ||f_{i+1}(\theta) + \epsilon\mathbf{1}|| g(\theta) d\theta}{\sqrt{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ||f_i(\theta) + \epsilon\mathbf{1}||^2 g(\theta) d\theta \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ||f_{i+1}(\theta) + \epsilon\mathbf{1}||^2 g(\theta) d\theta}}$$

where $||.||$ denotes the norm of $''.''$, $\mathbf{1}$, $M \times 1$, denotes a vector of ones, ϵ is an arbitrary small constant, which is used to avoid the undetermined ratio of $0/0$ and $g(\theta)$ is defined to be,

$$g(\theta) = \sum_{l=1}^{d} \delta(\theta - \theta_l)$$

where, $\delta(.)$ is the Dirac's delta function and $\theta_1, \cdots, \theta_d$ are the directions of arrival of received signals. Then ρ_i is less than one for $i = 1, \cdots, d-1$ and equal to one for $i = d, \cdots, M-1$. Given that the directions of arrival of received signals are not known, we define a related measure μ_i. To do so, let $\{E_1, \cdots, E_r\}$ be the minimal family of open disjoint intervals such that $\Theta \subset \bigcup_{i=1}^r E_i =: E$ and $|E_i| = \Delta_i$, where $|.|$ denotes the length of the interval $''.''$. Define μ_i, $i = 1, \cdots, M-1$, to be,

$$\mu_i = \frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ||f_i(\theta) + \epsilon\mathbf{1}|| \, ||f_{i+1}(\theta) + \epsilon\mathbf{1}|| h(\theta) d\theta}{\sqrt{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ||f_i(\theta) + \epsilon\mathbf{1}||^2 h(\theta) d\theta \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ||f_{i+1}(\theta) + \epsilon\mathbf{1}||^2 h(\theta) d\theta}} \quad (1)$$

where $h(\theta)$ is a simple function [16] defined as,

$$h(\theta) = \sum_{i=1}^{r} \frac{\chi_{E_i}(\theta)}{\Delta_i}$$

where, $\chi_F(.)$ is the indicator function of set F, which is defined as,

$$\chi_F(x) = \begin{cases} 1 & \text{x} \in \text{F} \\ 0 & \text{x} \notin \text{F} \end{cases}$$

Since $f_i(\theta), i = 1, \cdots, M$ are continuous, we expect that $\mu_i \approx 1$ for $i = d, \cdots, M-1$ and $\mu_i < 1$ for $i = 1, \cdots, d-1$. In fact, as $\Delta = \max\{\Delta_1, \cdots, \Delta_r\} \to 0$, $\mu_i \to 1$ for $i = d, \cdots, M-1$ and $\mu_i \to c_i < 1$ for $i = 1, \cdots, d-1$. Further, to amplify the approximate equality of μ_d and μ_{d+1}, we propose the estimator of the signal subspace dimension, $\hat{d}$, to be,

$$\hat{d} = argmax\{\hat{\nu}_i = \left|\frac{\hat{\mu}_i - \hat{\mu}_{i-1}}{\hat{\mu}_i - \hat{\mu}_{i+1}}\right|\}_{i=1,\cdots,M-2}$$

where $\hat{\mu}_0$ is given by,

$$\hat{\mu}_0 = \frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ||\hat{f}_1(\theta) + \epsilon\mathbf{1}|| d\theta}{\sqrt{\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ||\hat{f}_1(\theta) + \epsilon\mathbf{1}||^2 d\theta}}$$

where, $(\hat{.})$ represents the sample estimate of $(.)$.

It should be mentioned that when $d \geq 2$ and set E contains θ_1 and θ_2, i.e. $\theta_1 \in E$ and $\theta_2 \in E$, even if $\{\theta_i\}_{i=3}^d \notin E$, EOET can estimate the number of sources correctly.

In practice, the integral in (1) will be replaced with sum over the available discrete angles and the modeled or calibrated array response vector is utilized.

3. SIMULATION RESULTS

In this section, we investigate the order estimation performance of AIC, MDL, GDE and EOET using computer-based generated data. We consider a uniform linear array with 8 elements, and assume two independent sources with directions of arrival of -1 and

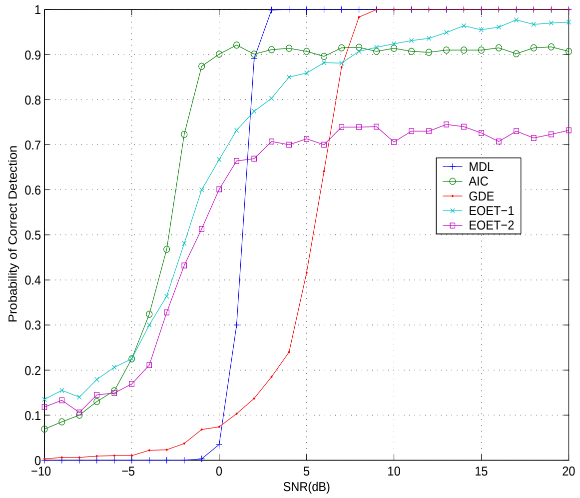

Fig. 1. Probability of correct detection versus SNR for two equi-power sources from -1 and 1 degree

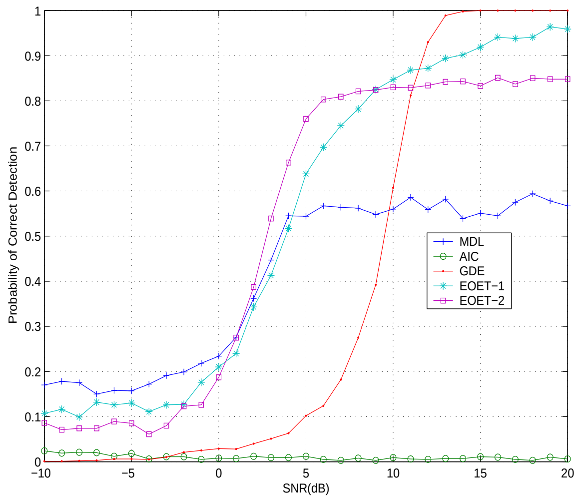

Fig. 2. Probability of correct detection versus SNR for two equi-power sources from -1 and 1 degrees

1 degree. 1000 samples are used to estimate the covariance matrix of received signal. For GDE, $D(N)$ is a nondecreasing parameter in $(0, 1)$ chosen to be 0.01 [12] and for EOET-1 the set E is chosen to be $E = [-4°, 4°]$ and for EOET-2 the set E is chosen to be $E = [-90°, 90°]$. We generate our data based on two models for the covariance matrix of noise. In model A, we generate the data based on $R_w = I$ and in a model B, based on $R_w = I + \xi diag[\kappa_1, \cdots, \kappa_M]$, where ξ is a positive constant and $\kappa_1, \cdots, \kappa_M$ are independent random variables distributed uniformly in the interval $(0, 1)$. In this model SNR denotes the signal to noise ratio (SNR) calculated when $\xi = 0$. We have considered model B to study the effectiveness of AIC, MDL, GDE and EOET when the statistical structure of the data mildly deviates from the ideal model.

Figure 1 depicts the probability of correct detection of the number of received signals versus SNR for the model A.

Figure 2 depicts the probability of correct detection of the number of received signals versus SNR for model B when ξ=1.

These examples show that EOET exhibits comparable performance to AIC and MDL in and ideal model, e.g. model A, and outperforms AIC and MDL when the statistical structure of data mildly deviates from the ideal model, e.g. model B. Further, EOET has a lower detection threshold in comparison to GDE. The results using EOET technique shows that a minimal information about DOA's results a robust number of source estimation.

4. EXPERIMENTAL RESULTS

In this section, we investigate the order estimation performance of AIC, MDL, GDE and EOET using experimental data. In the first subsection, we consider the experimental data obtained in an anechoic chamber and in the second subsection, we consider the experimental data obtained in a multipath environment.

4.1. An Anechoic Chamber

The experiment was carried out at the University of Wyoming. The receiver is equipped with a 6-element uniform linear array with 2.1×wavelength element spacing. This sensor spacing limits the field of view for a spatially non-aliased sector from $-13.5°$ to $13.5°$. The narrowband signal transmitted by transmitters has a wavelength of $\lambda = 8.275mm$. Details of the experimental system can be found in [9]. In the experiment, two equi-power sources are present and SNR is 6dB. One source is stationary at an angle of -3.8 degrees and one source is moving at a constant velocity of 0.0147 degrees per snapshots starting at -10.9 degrees. The covariance matrix of received signal is estimated by

$$\hat{R} = \frac{1}{N}\sum_{n=1}^{N} X(n)X^H(n)$$

where $N = 30$ is the number of snapshots and $X(n)$ is the n^{th} snapshot.

Table I tabulates mean, standard deviation and detection rate (DR) of the estimated number of sources using AIC, MDL, GDE and EOET. These statistics are computed using 150 segments of data.

Figure 3 shows the MVDR spectra of the first and last time segments of the experimental data.

The following is the estimated eigenvalues of the normalized covariance matrix of the first and second segments of experimental data,

$\hat{\lambda}_1 = 0.64, \hat{\lambda}_2 = 0.28, \hat{\lambda}_3 = 0.035, \hat{\lambda}_4 = 0.0151, \hat{\lambda}_5 = 0.01, \hat{\lambda}_6 = 0.007$

$\hat{\lambda}_1 = 0.63, \hat{\lambda}_2 = 0.26\hat{\lambda}_3 = 0.05, \hat{\lambda}_4 = 0.02, \hat{\lambda}_5 = 0.008, \hat{\lambda}_6 = 0.006$

It is evident that the first two eigenvalues are much larger than others. However, the relatively large variation of the four smaller eigenvalues causes AIC and MDL, which contain a test statistic for the equality of the smaller eigenvalues as a part of their formulation, to fail.

Considering the results obtained using EOET, we can conclude that even when no information is available about the directions of arrival, EOET outperforms the other methods. Further, the performance of EOET can be improved with minimal information that can be obtained using a spectral estimation method such as MVDR. In GDE formulation, $D(N)$ is a nondecreasing parameter in $(0, 1)$ which cannot be chosen systematically and our investigation using both computer-based generated data and experimental data shows that GDE's performance highly depends on the choice of D(N) and this makes it difficult to use. Further, we have not been able to find a $D(N)$ such that GDE works in multipath environments which is discussed in the next subsection.

4.2. A Multipath Environment

The experiment was carried out in the city of Sapporo, Japan. The receiver is equipped with a 10-element uniform linear array with half-wavelength element spacing. The transmitter has a single element antennas and is placed in a Line of Sight (LOS) position.

The following is the eigenvalues of the normalized forward-backward spatially smoothed covariance matrix with a subarray of size 8,

$\hat{\lambda}_1 = 0.95, \hat{\lambda}_2 = 0.02, \hat{\lambda}_3 = 0.01, \hat{\lambda}_4 = 0.004, \hat{\lambda}_5 = 0.003, \hat{\lambda}_6 = .000, \hat{\lambda}_7 = .000, \hat{\lambda}_8 = .000,$

Figure 4 depicts the the first four eigenbeams corresponding to the spatially smoothed data at the received antenna array. It can be seen that the first eigenbeam clearly shows a multipath signal that is received from direction 3° and the second eigenbeam shows that a signal is received from direction -6° and the third eigenbeam shows that a signal received from direction 27° and the fourth eigenbeam has a noise-like eigenbeam pattern [15]. Therefore, we expect that the signal subspace is of dimension 3.

Table 2 tabulates mean, standard deviation and detection rate of the estimated number of received signals at the antenna array using AIC, EOET, GDE and MDL. These statistics are computed using 100 segments of data each with 250 snapshots.

Considering the obtained results, it can be seen that both AIC and MDL estimate the number of received signals to be $M - 1$, where M is the number of antenna elements, a result which was observed in other experimental studies as well [8]. Further, GDE also fails to estimate the number of received multipath signals. Considering the results obtained by using EOET, it can be seen that even though one of the the eigenvalues of the estimated covariance matrix is dominant, EOET can still provides good performance with minimal information about the directions of arrival. Further as the measure of set E decreases the performance of EOET improves and also even if the set E may not contain one of the direction, EOET can still perform reasonably well.

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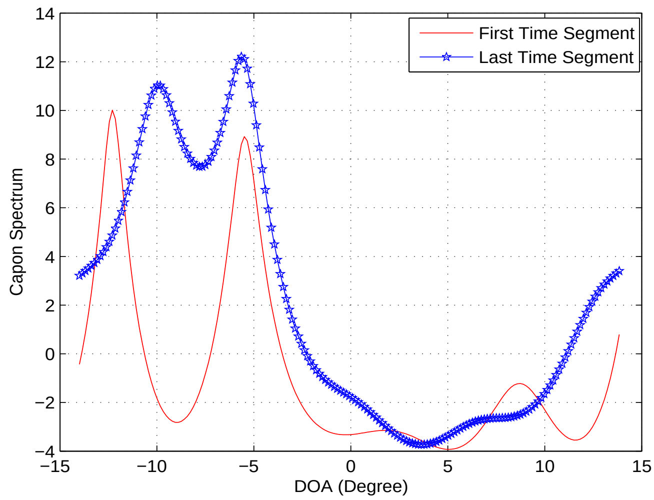

Fig. 3. MVDR spectra of first and last segments of experimental data in the anechoic chamber.

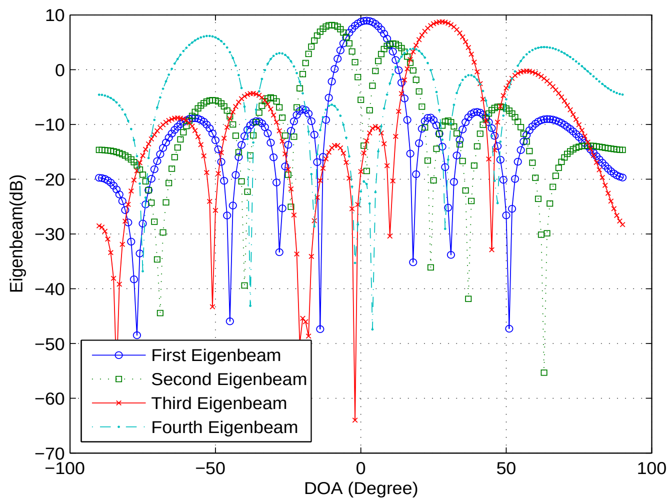

Fig. 4. Eigenbeams of forward-backward spatially smoothed data.

Table 1. Performance of AIC, MDL, GDE and EOET using data of anechoic chamber experiment

	DR	Mean	STD	
AIC	0	4.6	0.53	
MDL	12%	3.8 4	0.95	
GDE	72%	2.2	0.49	D(N)=0.1
GDE	0%	4.	0.0	D(N)=0
GDE	42%	1.4	0.49	D(N)=1
GDE	88%	2.0	0.34	D(N)=$\frac{1}{\log(N=30)}$
GDE	17%	3.3	0.76	D(N)=$\frac{1}{N=30}$
EOET	80%	2.0	0.45	E=$[-13.5, 13.5]$
EOET	80%	2.1	0.43	E=$[-12, 0]$
EOET	85%	2.0	0.38	E=$[-12, -8] \bigcup [-6, -2]$

Table 2. Performance of AIC, MDL, GDE and EOET using experimental data in a multipath environment

	DR	Mean	STD	
AIC	0	7	0.	
MDL	0%	7	0.	
GDE	0%	6	0	D(N)=0.0
GDE	0%	2.	0.	D(N)=0.1
GDE	0%	6	0	D(N)=$\frac{1}{N}$
GDE	0%	2.	0	D(N)=$\frac{1}{\log(N)}$
GDE	0%	1	0.76	D(N)=1.0
EOET	100%	3	0	E=$\{-6°\} \bigcup \{-3°\} \bigcup \{27°\}$
EOET	0%	1	0.0	E=$[-10°, -2] \bigcup [-1°, 7°] \bigcup [23°, 31°]$
EOET	39%	1.7	0.97	E=$[-9°, -3°] \bigcup [0°, 6°] \bigcup [24°, 30°]$
EOET	60%	2.6	1.22	E=$[-8°, -4°] \bigcup [1°, 5°] \bigcup [25°, 29°]$
EOET	100%	3	0.	E=$[-7°, -5°] \bigcup [2°, 4°] \bigcup [26°, 28°]$
EOET	46%	3.0	1.0	E=$[2°, 4°] \bigcup [26°, 28°]$
EOET	100%	3	0.0	E=$[-7°, -5°] \bigcup [26°, 28°]$
EOET	64%	3.7	0.96	E=$[-7°, -5°] \bigcup [2°, 4°]$
EOET	0%	4	0	E=$[-90°, 90°]$