

# APPLICATION OF THE FRACTIONAL FOURIER TRANSFORM TO ISAR RANGE PROFILE COMPENSATION OF FAST-MOVING TARGET

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## ABSTRACT

The authors consider the problem of range profile compensation (RPC) in the inverse synthetic aperture radar (ISAR) imaging of fast-moving target. A novel RPC technique based on fractional Fourier transform (FrFT) is proposed. In this scheme, the compensation parameters are estimated by the FrFT based on the Shannon entropy minimizing criterion. The following burst error elimination technique and least mean square curve fitting technique are used successively to obtain more accurate and robust estimation. By combing all those techniques, a detailed RPC algorithm is presented and the effectiveness of the proposed algorithm is shown by presenting and discussing some computer simulation results.

## 1. INTRODUCTION

Inverse synthetic aperture radar (ISAR) is a well-established technique for providing high resolution images of moving targets by means of a coherent processing of radar echoes<sup>[1-5]</sup>. The high range resolution is produced by transmitting wide-band signals and the high cross-range resolution is obtained by using the Doppler frequency gradient generated by the rotation of the target relative to radar line-of-sight (RLOS). In ISAR systems, the stretch signal waveform is widely used for ISAR imaging because of the simplicity of its design and implementation. The stretch concept provides high range resolution by transmitting a chirp signal, and then mix (multiplying) the echo signal by an identical frequency-modulated (FM) signal or slightly different FM signal. After mix processing, each sub-component of mixer output signal, which is corresponds to all scatterers in the same range bin, is a sinusoid signal and then the fast Fourier transform (FFT) can be used to generate the ISAR high resolution range profile. Extensive experiment results have shown that the stretch processor can obtain good ISAR range profile for slow-moving target. Hence the range profile of slow-moving target can be directly applied to ISAR translation compensation and cross-range

focus processing to achieve ISAR two-dimensional (2-D) image. But when the radial velocity of target is very high, such as satellite and ballistic missile, the high radial velocity will cause severe range profile aberration (RPA) and it consequently can degrade the ISAR 2-D image. It is obvious that the RPA from radial velocity of target must be effectively compensated and good compensation of RPA is crucial to ISAR imaging of fast-moving target in this case.

The ISAR echo signal model of fast-moving target had been investigated and the impacts of fast-moving character on ISAR 1-D range profile had been analyzed in [4-6]. The results show that the radar echo component corresponding to each scatterer of fast-moving target is approximately a chirp signal and the RPA happened inevitably because of its spectrum expansion, whereas the sub-component is a sinusoid signal and the RPA can be ignored in the case of slow-moving target. In this paper, we have analyzed the effect of RPA on ISAR translation compensation processing and ISAR 2D images by means of computer simulation. Simulation results indicate that the RPA will result in performance deterioration of the translation compensation techniques and the ISAR images blurring. To resolve those problems, a novel range profile compensation (RPC) technique containing two main steps is proposed. The first step is to estimate the chirp rate (denotes as  $K_1$ ) of the stretch mixer output signal. The second step is to compensate RPA by multiplying the output signal with dechirp reference signal constructed using the estimated value of chirp rate. Finally an algorithm using the fractional Fourier transform (FrFT) for RPC is proposed and computer simulation experiment is presented to demonstrate its effectiveness.

The remainder of this paper is organized as follows: Section 2 analyzes the ISAR echo signal model of fast-moving target. The basic theory of chirp signal parameter estimation using FrFT is introduced in Section 3. Section 4 introduces the RPC algorithm using the FrFT in detail. Finally, the computer simulation experiments are provided to demonstrate the performance of the RPC algorithm in Section 5 and the conclusions are made in Section 6.

## 2. ISAR ECHO SIGNAL MODEL OF FAST-MOVING TARGET

In this section, we briefly analyze the ISAR echo signal model of fast-moving target. Suppose the transmitted chirp signal of ISAR is

$$S_T(t) = a(t) \exp[j\pi(2f_0 t + Kt^2)] \quad (1)$$

where its complex amplitude of  $S_T(t)$  has the form

$$a(t) = \begin{cases} 1 & 0 \leq t \leq T_p \\ 0 & T_p < t < T_r \end{cases}$$

$T_p$  is the pulse width,  $T_r$  is the radar pulse repetition interval (PRI),  $f_0$  is the carrier frequency;  $K$  is the chirp rate. Assume the complex reflectivity density of a scatterer is 1, the sub-component received from this scatterer by a radar receiver is

$$S_R(t) = a(t-\tau) \exp[j\pi(2f_0(t-\tau) + K(t-\tau)^2)] \quad (2)$$

where  $\tau$  is the time delay of this scatterer. Suppose the time delay of reference scatterer is  $\tau_1$ , the reference chirp signal of stretch mixer is

$$S_f(t) = a(t-\tau_1) \exp[-j\pi(2f_0 t + K(t-\tau_1)^2)] \quad (3)$$

After simple mathematical development, the output signal of stretch mixer, which mixes the received signal with reference chirp signal, can be determined as

$$S_f(t) = S_R(t) S_f(t) = A(t) \exp[-j\Phi_0] \exp[j2K\pi(\tau_1 - \tau)t] \quad (4)$$

where  $A(t) = a(t-\tau)a(t-\tau_1)$ ,  $\Phi_0 = \pi[2f_0\tau - K(\tau^2 - \tau_1^2)]$ . For ground-to-air ISAR system, the radial velocity of moving target probably affects the ISAR echo signal model. When the radial velocity of moving target is relative slow, the slant range change of each scatterer on moving target during pulse chirp signal operation period can be ignored, and then the time delay of each scatterer is

$$\tau = \tau_0 = 2R_0/C \quad (5)$$

where the  $R_0$  is the slant range between scatterer and the radar while the transmitted chirp signal arrives at the target,  $C$  is the light speed. Let  $\tau = \tau_0$  and  $t = \tau_1 + nT_s$  ( $n=1, 2, \dots, N$ ) where  $N$  is the available sampling values of mixer output signal. Then the equation (4) can be rewritten as

$$S_f(n) = A(n) \exp(-j\Phi_1) \exp[j2K\pi(\tau_1 - \tau_0) nT_s] \quad (6)$$

where  $\Phi_1 = \pi[2f_0\tau_0 - K(\tau_0 - \tau_1)^2]$  and  $T_s$  is the analogy-to-digital converter sampling period.

Letting

$$f_{d0} = K(\tau_1 - \tau_0)$$

Then the discrete version of equation (6) has the form

$$S_f(n) = A(n) \exp(-j\Phi_1) \exp[j2\pi f_{d0} nT_s] \quad (7)$$

The equation (7) indicates that each sub-component of mixer output signal corresponding to scatterer is nearly a sinusoid signal and the its frequency value denoted by  $f_{d0}$  is determined by chirp rate of transmitted chirp signal and the distance between this scatterer and the tracking reference point in the case of slow-moving target. However, the slant range change of each scatterer on fast-moving target during pulse chirp signal operation period cannot be ignored anymore. Suppose the range resolution

of ISAR system is 0.15m, in which the bandwidth and the pulse width of the radar transmitted signal are 1GHz and 50 microseconds respectively. When the radial velocity of moving target is 300m/s, the slant range change during pulse signal operation period is 15mm .i.e. ten percent of range resolution. Hence the slant range change can be ignored. But when the radial velocity is 9000m/s, the slant range change during pulse operation period is 0.45m, which is 3 times of range resolution. In this case, the slant range change must be considered and the time delay of the scatterer is determined as  $\tau = \tau_0 - 2vt/C$ . Letting  $\tau = \tau_0 - 2vt/C$  and  $t = \tau_1 + nT_s$ , the equation (4) yields

$$S_f(n) = A(n) \exp(j\Phi_2) \exp[j\pi(2f_{d1} nT_s + K_1(nT_s)^2)] \quad (8)$$

where

$$\Phi_2 = -2\pi f_0 \tau_0 + 2\pi f_d \tau_1 + K\pi D^2$$

$$f_d = 2v/\lambda$$

$$D = \tau_0 - \tau_1 (1 + 2v/C)$$

$$f_{d1} = f_d - KD (1 + 2v/C)$$

$$K_1 = 4K(v/C + 1)(v/C)$$

$\lambda$  is the wavelength of transmitted signal;  $v$  is the radial velocity of moving target. The equation (8) shows that the echo signal from each scatterer of fast-moving target is approximately a chirp signal and the chirp rate denoted by  $K_1$  is determined by chirp rate of transmitting chirp signal and the radial velocity of moving target. Based on this radar echo signal model, we can easily make conclusion that the ISAR received signal from the moving target, which has many scatterers, is approximately a multi-component chirp (MMC) signal with the same chirp rate. Because the chirp signal occupies certain bandwidth in frequency domain, the echo signal model of fast-moving target probably results in severe one-dimensional (1-D) RPA and 2-D image blurring, the chirp rate  $K_1$  of the echo signal must be compensated effectively. After RPA compensation, the remainder task is to apply, for example, the conventional range-Doppler (R-D) imaging algorithm to obtain the ISAR image, which can be efficiently implemented by using 2-D FFT.

## 3. PARAMETER ESTIMATION OF CHIRP SIGNAL USING THE FRACTIONAL FOURIER TRANSFORM

The FrFT of signal  $s(t)$  can be defined by<sup>[7]</sup>

$$S_p(u) = \begin{cases} \sqrt{\frac{1-j \cot a}{2\pi}} \exp\left(j \frac{u^2}{2} \cot a\right) \cdot \int_{-\infty}^{\infty} s(t) \exp\left(j \frac{t^2}{2} \cot a\right) \exp(jut \csc a) dt & a \neq n\pi \\ s(t) & a = 2n\pi \\ s(-t) & a = (2n+1)\pi \end{cases} \quad (9)$$

where  $n=0, \pm 1, \pm 2, \dots$ . If we choose the transformation kernel  $K_a(t, u)$  as

$$K_a(t,u) = \begin{cases} \sqrt{\frac{1-j\cot a}{2\pi}} \cdot e^{j[0.5(u^2+t^2)\cot a - ut\csc a]} & a \neq n\pi \\ \delta(t-u) & a = 2n\pi \\ \delta(t+u) & a = (2n+1)\pi \end{cases}$$

The FrFT of signal  $s(t)$  is  $S_a(u) = \int_{-\infty}^{+\infty} s(t)K_a(t,u)dt$ .

Suppose that the FrFT operator is designated by  $F^a$ , where  $a$  is the rotation angle. Then it has the following basic properties [7-9].

Linear operator:

$$F^a [c_1 f(t) + c_2 g(t)] = c_1 F^a f(t) + c_2 F^a g(t).$$

Identity operator:  $F^0 [s(t)] = F^{2\pi} [s(t)]$ .

Consistency with Fourier transform:

$$F^{\pi/2} [s(t)] = F^{5\pi/2} [s(t)] = S(f).$$

Additivity of rotation:  $F^{a+\beta} = F^a F^\beta$ .

From those, it can also be concluded that the inverse FrFT with angle  $a$  is the FrFT with angle  $-a$ . Some valuable discussion for discrete FrFT mathematical definition can be seen in [7-9].

To apply FrFT for ISAR RPC of fast-moving target, we would discuss the detection performance of the FrFT for a chirp signal. A chirp signal is a typical non-stationary signal, its instantaneous frequency rises linearly with time. In the time-frequency plane, the chirp distributes along a slant edge. The classical Fourier transform of signal can be regarded as the projection of its time-frequency distribution onto the frequency axis  $f$ . The spectrum of a chirp signal in the Fourier domain will spread, just as shown in Fig.1. If the time-frequency distribution of a chirp signal is projected onto a rotated frequency axis  $v$  with a proper angle  $a$ , i.e. the FrFT, the energy distribution of the chirp may be more highly concentrated in the fractional Fourier domain, as shown in Fig.2. So the FrFT can be applied to chirp signal detection and parameter estimation by the following steps:

*Step1:* Compute the FrFT of signal  $s(t)$  in rotation angle  $a$  from zero to  $2\pi$ .

*Step2:* Find the maximum in 2D FrFT plane by 2D searching.

*Step3:* Estimate the chirp signal parameters based on the angle  $a_0$  and the frequency in fractional Fourier axis corresponding to maximum: chirp rate is  $atan(a_0)$  and initial frequency  $f_{a0} = f_{u0} \csc(a_0)$  where the  $atan(\cdot)$  and  $\csc(\cdot)$  denote the arc tangent and cosecant function respectively.

#### 4. THE PROPOSED ALGORITHM FOR RANGE PROFILE ABERRATION COMPENSATION

The analysis results in Section 3 show that the echo signal of fast-moving target is approximately a MCC signal with the same chirp rate. So the initial frequencies corresponding to near range resolution cells are near in

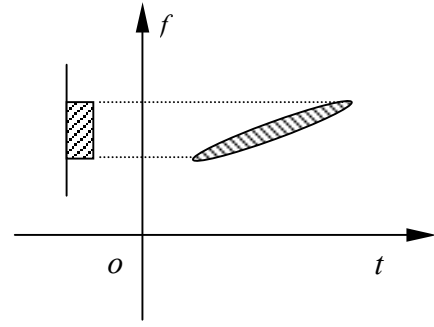


Figure 1: The projection of chirp signal onto Fourier domain

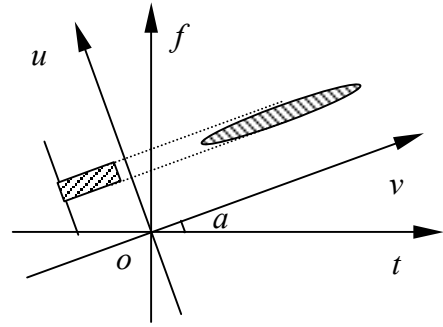


Figure 2: The projection of chirp signal onto fractional Fourier domain

frequency domain and then it probably results in severe spectrum overlap in FrFT spectrum  $S_p(u)$ . Obviously, the height maximum criterion, which is commonly used in parameter estimation, can hardly achieve ideal performance in this case. To obtain better performance, we propose a novel parameter estimation technique for mono-component chirp signal or MCC signals with the same chirp rate by using Shannon entropy minimizing criterion. The basic idea is that when the chirp rate  $k$  corresponding to certain rotation angle in FrFT spectrum nears the chirp rate of the MCC signal denoted by  $K_1$ , the FrFT spectrum of MCC signals in this angle is similar to that of multicomponent sinusoid (MCS) signal in angle zero. Because the sharper envelope waveform means less Shannon entropy, the Shannon entropy criterion can be applied to chirp rate estimation of MCC signal in this case. Let us introduce this technique, which constitutes the computational kernel of the proposed RPC algorithm.

*Chirp Rate Estimation:* Suppose the envelope waveform of FrFT spectrum in angle  $a$  is of the form

$$S_a(u) = |S_1, S_2, \dots, S_N| \quad (10)$$

where  $m$  ( $m=1,2,\dots,L$ ) and  $L$  are the index and the number of angle selected in FrFT spectrum respectively. the angle  $a$  satisfy

$$k = atan(a) \quad (11)$$

Letting  $P_i = |S_i| / \sum_{i=1}^N |S_i|$  ( $i=1, 2, \dots, N$ ) where  $N$  is defined in Section 2. The Shannon entropy of  $S_a(u)$  is defined by

$$H[S_a(u)] = -\sum_{i=1}^N P_i \log P_i \quad (12)$$

Then the chirp rate estimation of the echo signal can be estimated by

$$K_1 = \text{atan}(\arg \min_{k_m} H[S_a(u)]) \quad (13)$$

From the discussion above, it can be seen that the chirp rate of each echo signal denoted by  $K_{1i}$  ( $i=1, 2, \dots, M$ ) where  $M$  is the pulse number for coherent processing can be estimated using the FrFFT based on Shannon entropy minimizing criterion. After chirp rate estimation, the reference signal  $s_{ri}(t) = \exp(-j\pi K_{1i} t^2)$  can be generated and be applied to RPC by multiplying it with the mixer output signal. Then the compensated range profile can be obtained by FFT processing consequently.

In practice, the chirp rate estimation technique above can hardly achieves enough accuracy and probably has burst error. To improve this problem, we propose a burst error elimination technique and apply the least mean square (LMS) curve fitting technique to improve the stability and accuracy of chirp rate estimation. The whole RPC algorithm using FrFFT contains follow steps:

*Step 1:* Calculate the FrFFT spectrum of the  $i$ th ( $i=1, 2, \dots, M$ ) echo signal one by one during certain angle scope.

*Step 2:* Estimate the chirp rate  $K_{1i}$  by (10)-(13).

*Step 3:* Eliminate the burst error of chirp rate estimation.

*Substep (a):* Compute the mean value of  $K_{1i}$  ( $i=1, 2, \dots, M$ ).

*Substep (b):* Compute the differences values between the mean value and each  $K_{1i}$ .

*Substep (c):* Compare the  $i$ th ( $i=1, 2, \dots, M$ ) difference value with a predetermined threshold. If it exceeds this threshold, let the  $K_{1i}$  equals to the mean of all original  $K_{1i}$ ; else the  $K_{1i}$  never be processed.

*Step 4:* Calculate a new group of chirp rate  $K_{1i}$  by using LMS curve fitting technique.

*Step 5:* Obtain all compensation reference signal  $s_{ri}(t) = \exp(-j\pi K_{1i} t^2)$  using  $K_{1i}$  and then multiply them with their corresponding output signal of stretch mixer.

*Step 6:* Produce compensated ISAR range profile of fast-moving target using a sequence of FFTs.

## 5. SIMULATION AND DISCUSSION

In this section, we present simulation examples to illustrate the performance of the proposed RPC algorithm. Some important simulation parameters are as follows. The radar works in X band with a wavelength of 3cm; the bandwidth and the pulse width of transmitting signal are 1GHz and 51.2 microsecond respectively; the sampling

frequency of echo signal is 5MHz; the number of sampling data is 256; the pulse number for coherent image processing is 128; the total rotation angle is 5.19 degree; the pulse repetition frequency is 400Hz. Suppose the simulated target is composed of 88 ( $8 \times 11$ ) ideal scatterer and all the transverse and radial distance among close scatterers are 1 meter. In coherent image processing interval, the target moves along a straight line, which is parallel to X axis, with the invariable velocity of 10km/s in the X-Y coordinate system, as shown in Fig.3. The distance of the target from the radar is about 500km and the height is about 250km. During this CPI, the target only yaws to right for a small angle with time duration about 9.6 seconds.

Before image processing, the range alignment is performed on the collected data within the 9.6s using the sliding window correlation method [1], and the initial phase is calibrated using Doppler centroid tracking method [1]. To reduce the computation load, the raw data are extracted with a ratio of 30:1.

After the translation compensation, the range profile and 2-D ISAR image of fast-moving target, which are obtained by using R-D image algorithm with Hamming windows smoothing, are shown in Fig.4 and Fig.5, respectively. From Fig.4 and Fig.5, we can observe severe RPA and ISAR 2-D image blurring due to the echo signal model variance. The estimated curves of chirp rate using the proposed method are shown in Fig.6. From Fig.6, we can see that the proposed break error elimination technique and LMS curve fitting technique can improve the stability and the accuracy of parameter estimation effectively. After RPC using the proposed algorithm, the compensated range profile of fast-moving target is shown in Fig.7. Comparing the Fig.4 with Fig.7, it is obvious that the RPA can be compensated effectively. The 2-D image of the moving target, in which the compensated range profile is used, is shown in Fig.8. Fig.8 shows that the 2-D image of the fast-moving target using the compensated range profile can achieve better image and the 2-D image blurring can be improved greatly.

## 6. CONCLUSIONS

In this paper, we have investigated the echo signal model of fast-moving target and analyzed its impacts on ISAR images. Based on those analysis results, we can draw several conclusions as follows: 1) The sub-component of the echo signal is approximately a chirp signal; 2) The echo signal is a MCC signal; 3) The commonly used stretch processing will result in severe RPA and ISAR 2-D image blurring. A new RPC scheme using the FrFFT has been proposed and its performance has been demonstrated through simulation experiments. Simulation experiment results have shown that the proposed algorithm can compensate the RPA of fast-moving target effectively.

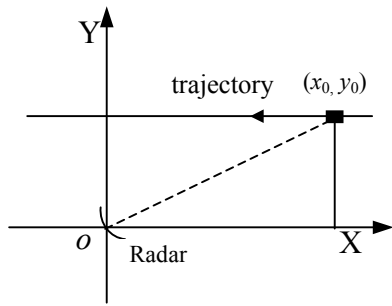


Figure 3: ISAR imaging geometry

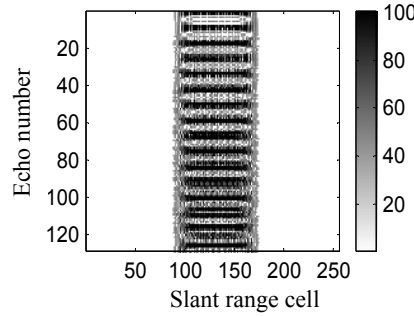


Figure 4: The origin range profile of fast-moving target

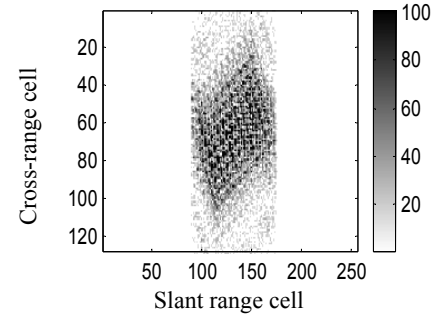


Figure 5: The origin 2-D image of fast-moving target

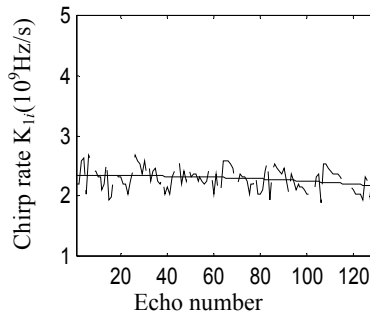


Figure 6: Curves of chirp rate estimation  $K_1$ .  
Dashed line: original estimation; Solid line: estimation after break error elimination and curve fitting

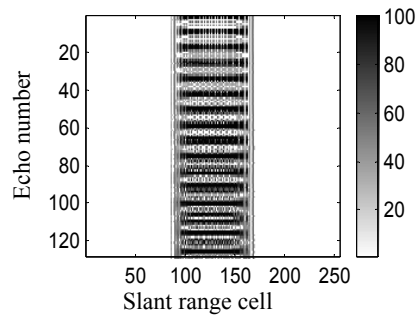


Figure 7: The compensated range profile of fast-moving target

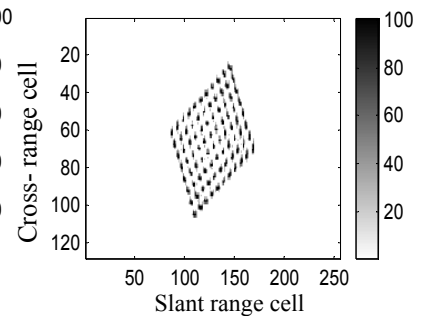


Figure 8: The compensated 2-D image of fast-moving target

Unlike the Wigner-ville distribution, which is widely used for parameter estimation of chirp signal, the FrFT is a kind of linear time-frequency transform and then it would never be impacted by strong and much cross terms. Moreover, the FrFT is computational very efficient and requires only a sequence of FFTs. Hence it can be easily implemented in real-time by using currently available high speed FFT chips. So the proposed algorithm is a very effective and efficient tool for RPA compensation of fast-moving target.

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