COMPUTATIONALLY EFFICIENT METHOD FOR ESTIMATION OF THE NUMBER OF SIGNALS IN MULTIPATH ENVIRONMENT

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ABSTRACT

This paper proposes a nonparametric detection method for the coherent narrowband signals impinging on a uniform linear array (ULA). By exploiting the array geometry and its shift invariance property, the coherency of incident signal is decorrelated through subarray averaging, and the number of incident signals is equal to the rank of a matrix formed from the cross-correlations between some sensor data, where the effect of additive noise is also eliminated. Then a new criterion is formulated in the terms of the row elements of the QR upper-triangular factor of the cross product of formed correlation matrix, and the number of signals is determined as the value for which this QR-based criterion is maximized. The proposed method has remarkable insensitivity to the correlation of incident signals and flexibility to the spatially correlated noise. Simulation results show that the proposed method is superior in detecting closely-spaced signals with small number of snapshots and at low signal-to-noise ratio (SNR).

1. INTRODUCTION

Estimation of the number of incident signals from the noisy array data is an indispensable prerequisite for high-resolution direction-of-arrival (DOA) estimation methods in array processing. Although many eigenvalue-based nonparametric detection methods such as the Akaike information criterion (AIC) and minimum description length (MDL) criterion [1] were proposed, these nonparametric methods suffer serious degradation in performance when the incident signals are coherent (fully correlated) such as in multipath propagation environments, where the rank of the signal covariance matrix is smaller than the number of incident signals. Some parametric detection methods are generally the optimal approaches for the coherent signals by solving the detection of the number of signals and the estimation of their directions simultaneously (e.g., [2], [3]), but they are computationally intensive. Even though the AIC and MDL criterion can be modified to combat the deleterious effect of coherency between the incident signals by using decorrelation techniques such as (forward-backward (FB)) spatial smoothing (SS) [4], [5], [11], they perform poorly in difficult scenarios with closely-spaced signals, low signal-to-noise ratio (SNR), and small number of snapshots. Furthermore, most of the aforementioned nonparametric and parametric methods require the eigendecomposition of the (smoothed) correlation matrix, and thus their applications are limited in some applications, because the eigendecomposition process is not suitable for real-time implementation due to its computational burdensomeness and time-consuming (e.g., [6]-[9]). Additionally a QR-based detection method [8] was presented for coherent signals without eigendecomposition, where the number of signals is determined from the row elements of the QR upper-triangular factor of the smoothed subarray covariance matrix, but *a priori* knowledge of the true noise variance and a subjective judgment are needed, and further its performance generally degrades in difficult scenarios.

Therefore in this paper, we propose a computationally simple and efficient nonparametric detection method for the coherent narrowband signals impinging on a uniform linear array (ULA). By exploiting the array geometry and its shift invariance property, the coherency of incident signal is decorrelated through subarray averaging, and the number of incident signals is equal to the rank of a correlation matrix formed from the cross-correlations between some sensor data regardless of the signal coherency. Since the effect of additive noise is eliminated is the resultant correlation matrix, the robustness to the noise can be improved. Then when the finite array data is available, a new criterion is formulated in the terms of the row elements of the QR upper-triangular factor of the cross product of formed correlation matrix, and the number of signals is determined as the value for which this QR-based criterion is maximized. The proposed method is computationally efficient and suitable for real-time implementation because the computationally cumbersome eigendecomposition and the evaluation of all correlations of the array data are not needed, and it has remarkable insensitivity to the correlation of incident signals and flexibility to the spatially correlated noise. The detection perforamnce of the proposed method is verified through numerical examples.

2. DATA MODEL AND BASIC ASSUMPTIONS

We consider a ULA of M sensors with spacing d and suppose that p $(p < M/2)$ narrowband signals $\{s_k(n)\}$ with the carrier frequency f_0 are far away and impinge on the array from distinct directions $\{\theta_k\}$. The received signal $y_i(n)$

at the ith sensor can be expressed [3], [7]

$$y_i(n) = \sum_{k=1}^{p} s_k(n)\, e^{j\omega_0 (i-1)\tau(\theta_k)} + w_i(n) \tag{1}$$

where $w_i(n)$ is the additive noise, $\omega_0 = 2\pi f_0$, $\tau(\theta_k) = (d/c)\sin\theta_k$, and c is the propagation speed frequency. Then the received signals can be rewritten in a compact form as

$$\boldsymbol{y}(n) = \boldsymbol{A}\boldsymbol{s}(n) + \boldsymbol{w}(n) \tag{2}$$

where $\boldsymbol{y}(n)$, $\boldsymbol{s}(n)$, and $\boldsymbol{w}(n)$ are the vectors of the received signals, incident signals, and additive noise, and $\boldsymbol{A}$ is the array response matrix given by $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \cdots, \boldsymbol{a}(\theta_p)]$, with $\boldsymbol{a}(\theta_k) = [1, e^{j\omega_0\tau(\theta_k)}, \cdots, e^{j\omega_0(M-1)\tau(\theta_k)}]^T$.

Here we assume that the array is calibrated and the matrix $\boldsymbol{A}$ is unambiguous. Without loss of generality, the signals $\{s_k(n)\}$ are assumed to be coherent under the flat-fading multipath propagation and given by [4], [7], [11]

$$s_k(n) = \beta_k s_1(n), \qquad \text{for} \quad i = 1, 2, \cdots, p \tag{3}$$

where $\{\beta_k\}$ are the complex attenuation coefficients with $\beta_k \neq 0$ and $\beta_1 = 1$. The incident signals and additive noise are assumed to be independent and complex circularly Guassian noise with zero-mean and variance as $E\{s_1(n) \cdot s_1^*(t)\} = r_s\delta_{n,t}$, $E\{s_1(n)s_1(t)\} = 0$, $E\{\boldsymbol{w}(n)\boldsymbol{w}^H(t)\} = \sigma^2\boldsymbol{I}_M\delta_{n,t}$, and $E\{\boldsymbol{w}(n)\boldsymbol{w}^T(t)\} = \boldsymbol{O}_{M\times M}\ \forall n,t$, where $E\{\cdot\}$, $(\cdot)^H$, $\delta_{n,t}$, $\boldsymbol{I}_m$, and $\boldsymbol{O}_{m\times m}$ denote the expectation, Hermitian transpose, Kronecker delta, and $m\times m$ identity and null matrices.

3. ESTIMATION OF THE NUMBER OF SIGNALS WITHOUT EIGENDECOMPOSITION

3.1. Decorrelation with Subarray Averaging

Under the assumptions of data model, we get the array covariance matrix $\boldsymbol{R}$ as

$$\boldsymbol{R} = E\{\boldsymbol{y}(n)\boldsymbol{y}^H(n)\} = \boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^H + \sigma^2\boldsymbol{I}_M \tag{4}$$

where $\boldsymbol{R}_s$ is the signal covariance matrix given by $\boldsymbol{R}_s = E\{\boldsymbol{s}(n)\boldsymbol{s}^H(n)\}$. By defining the correlation r_{ik} between the noisy signals $y_i(n)$ and $y_k(n)$ as $r_{ik} = E\{y_i(n)y_k^*(n)\}$, where $r_{ik} = r_{ki}^*$, we can find that the diagonal elements $\{r_{kk}\}$ of $\boldsymbol{R}$ in (4) are affected by the additive noise. Additionally since the incident signals are coherent, the matrix $\boldsymbol{R}_s$ is singular (i.e., $\text{rank}(\boldsymbol{R}_s) < p$, when $p > 1$), so the number of incident signals cannot be estimated directly from the multiplicity of the eigenvalues of the array covariance matrix $\boldsymbol{R}$ in (4). To circumvent this crucial rank deficit problem, the spatial smoothing based on subarray [4] is a well-known preprocessing to decorrelate the signal coherency (e.g., [5]), where the eigendecomposition of the spatially smoothed covariance matrix of subarrays is required. In this paper, we deal with the estimation of the number of signals in an efficient and effective way by using subarray averaging.

First for simplicity, the noisy received signal $y_i(n)$ in (1) can be reexpressed by

$$y_i(n) = \boldsymbol{b}_i^T\boldsymbol{s}(n) + w_i(n) \tag{5}$$

where

$$\boldsymbol{b}_i = [e^{j\omega_0(i-1)\tau(\theta_1)}, e^{j\omega_0(i-1)\tau(\theta_2)}, \cdots, e^{j\omega_0(i-1)\tau(\theta_p)}]^T.$$

Now we can divide the full array into L overlapping subarrays with $\bar{p}$ ($\bar{p} \geq p$) sensors in the forward and backward directions [4], [7], [11], and the lth forward or backward subarray comprises $\{l, l+1, \cdots, l+\bar{p}-1\}$ or $\{M-l+1, M-l, \cdots, L-l+1\}$ sensors, respectively, where $l = 1, 2, \cdots, L$, and $L = M - \bar{p} + 1$, the signal vectors of the lth forward and backward subarrays are given by

$$\begin{aligned}\boldsymbol{y}_{fl}(n) &= [y_l(n), y_{l+1}(n), \cdots, y_{l+\bar{p}-1}(n)]^T \\ &= \bar{\boldsymbol{A}}_1\boldsymbol{D}^{l-1}\boldsymbol{s}(n) + \boldsymbol{w}_{fl}(n) \end{aligned}\tag{6}$$

$$\begin{aligned}\boldsymbol{y}_{bl}(n) &= [y_{M-l+1}(n), y_{M-l}(n), \cdots, y_{L-l+1}(n)]^H \\ &= \bar{\boldsymbol{A}}_1\boldsymbol{D}^{-(M-l)}\boldsymbol{s}^*(n) + \boldsymbol{w}_{bl}(n) \end{aligned}\tag{7}$$

where $\boldsymbol{w}_{fl}(n) = [\, w_l(n),\ w_{l+1}(n),\ \cdots,\ w_{l+\bar{p}-1}(n)\,]^T$, $\boldsymbol{w}_{bl}(n) = [w_{M-l+1}(n), w_{M-l}(n), \cdots, w_{L-l+1}(n)]^H$, $\bar{\boldsymbol{A}}_1$ is the $\bar{p}\times p$ submatrix of $\boldsymbol{A}$ in (2) consisting of the first $\bar{p}$ rows with the column $\bar{\boldsymbol{a}}_1(\theta_k) = [\,1,\ e^{j\omega_0\tau(\theta_k)},\ \cdots,\ e^{j\omega_0(\bar{p}-1)\tau(\theta_k)}]^T$, and $\boldsymbol{D} = \text{diag}(e^{j\omega_0\tau(\theta_1)}, e^{j\omega_0\tau(\theta_2)}, \cdots, e^{j\omega_0\tau(\theta_p)})$. Then by defining the correlation vectors $\boldsymbol{\varphi}_{fl}$, $\bar{\boldsymbol{\varphi}}_{fl}$, $\boldsymbol{\varphi}_{bl}$, and $\bar{\boldsymbol{\varphi}}_{bl}$ between the signal vectors $\boldsymbol{y}_{fl}(n)$ and $\boldsymbol{y}_{bl}(n)$ in (6) and (7) and the signals $y_1(n)$ and $y_M(n)$ in (1) $\boldsymbol{\varphi}_{fl} = E\{\boldsymbol{y}_{fl}(n)y_M^*(n)\}$, $\bar{\boldsymbol{\varphi}}_{fl} = E\{\boldsymbol{y}_{fl}(n)y_1^*(n)\}$, $\boldsymbol{\varphi}_{bl} = E\{y_1(n)\boldsymbol{y}_{bl}(n)\}$, and $\bar{\boldsymbol{\varphi}}_{bl} = E\{y_M(n)\boldsymbol{y}_{bl}(n)\}$, after some algebraic manipulations, we can obtain four Hankel correlation matrices [7]

$$\boldsymbol{\Phi}_f = [\boldsymbol{\varphi}_{f1}, \boldsymbol{\varphi}_{f2}, \cdots, \boldsymbol{\varphi}_{f,L-1}]^T = \bar{\rho}_M r_s\bar{\boldsymbol{A}}\boldsymbol{B}\bar{\boldsymbol{A}}_1^T \tag{8}$$

$$\bar{\boldsymbol{\Phi}}_f = [\bar{\boldsymbol{\varphi}}_{f2}, \bar{\boldsymbol{\varphi}}_{f3}, \cdots, \bar{\boldsymbol{\varphi}}_{fL}]^T = \bar{\rho}_1 r_s\bar{\boldsymbol{A}}\boldsymbol{B}\boldsymbol{D}\bar{\boldsymbol{A}}_1^T \tag{9}$$

$$\begin{aligned}\boldsymbol{\Phi}_b &= [\boldsymbol{\varphi}_{b1}, \boldsymbol{\varphi}_{b2}, \cdots, \boldsymbol{\varphi}_{b,L-1}]^T \\ &= \bar{\rho}_1^* r_s\bar{\boldsymbol{A}}\boldsymbol{B}^*\boldsymbol{D}^{-(M-1)}\bar{\boldsymbol{A}}_1^T \end{aligned}\tag{10}$$

$$\begin{aligned}\bar{\boldsymbol{\Phi}}_b &= [\bar{\boldsymbol{\varphi}}_{b2}, \bar{\boldsymbol{\varphi}}_{b3}, \cdots, \bar{\boldsymbol{\varphi}}_{bL}]^T \\ &= \bar{\rho}_M^* r_s\bar{\boldsymbol{A}}\boldsymbol{B}^*\boldsymbol{D}^{-(M-2)}\bar{\boldsymbol{A}}_1^T \end{aligned}\tag{11}$$

where $\bar{\boldsymbol{A}}$ is the $(M-\bar{p})\times p$ submatrix of the matrix $\boldsymbol{A}$ in (2) consisting of its first $M-\bar{p}$ rows with the column $\tilde{\boldsymbol{a}}(\theta_k) = [1, e^{j\omega_0\tau(\theta_k)}, \cdots, e^{j\omega_0(L-1)\tau(\theta_k)}]^T$, $\boldsymbol{B} = \text{diag}(\beta_1, \beta_2, \cdots, \beta_p)$, $\bar{\rho}_i = \boldsymbol{\beta}^H\boldsymbol{b}_i^*(\theta)$, and $\boldsymbol{\beta} = [\beta_1, \beta_2, \cdots, \beta_p]^T$.

Clearly the correlation matrices $\boldsymbol{\Phi}_f$, $\bar{\boldsymbol{\Phi}}_f$, $\boldsymbol{\Phi}_b$, and $\bar{\boldsymbol{\Phi}}_b$ in (8)–(11) are not affected by the additive noise, and $\boldsymbol{\Phi}_b = \boldsymbol{J}_{M-\bar{p}}\bar{\boldsymbol{\Phi}}_f^*\boldsymbol{J}_{\bar{p}}$, and $\bar{\boldsymbol{\Phi}}_b = \boldsymbol{J}_{M-\bar{p}}\boldsymbol{\Phi}_f^*\boldsymbol{J}_{\bar{p}}$, where $\boldsymbol{J}_m$ is an $m\times m$ counteridentity matrix. Moreover these Hankel matrices in (4)-(7) can be formed simply from the elements $\{r_{i1}\}$ and $\{r_{iM}\}$ in the Mth and first columns $\boldsymbol{\varphi}$ and $\bar{\boldsymbol{\varphi}}$

of array covariance matrix $\boldsymbol{R}$ in (4) except for the auto-correlations r_{11} and r_{MM}, which contain the noise variance σ^2, where $\boldsymbol{\varphi} = E\{\boldsymbol{y}(n)y_M^*(n)\}$, and $\bar{\boldsymbol{\varphi}} = E\{\boldsymbol{y}(n)y_1^*(n)\}$. Furthermore under the assumptions of data mode, we easily find that the ranks of these $(M-\bar{p}) \times p$ matrices in (8)–(11) equal the number of incident signals p iff $M - \bar{p} \geq p$ and $\bar{p} \geq p$ (see [7], [10] for reference), i.e., the dimension of signal subspace of the matrices is restored to the number of incident signals without spatial smoothing.

Remark 1: Although the incident signals are assumed to be all coherent, the proposed method can be extended to the case of partly correlated or incoherent signals and can accommodate a more general noise model of the spatially correlated noise if we choose appropriate subarrays (i.e., cross-correlations of the array data) (see [7] for reference).

3.2. QR-Based Approach to Detection of Number of Signals

From (8)–(11), we can get [7], [10]

$$\boldsymbol{\Phi} = [\boldsymbol{\Phi}_f, \bar{\boldsymbol{\Phi}}_f, \boldsymbol{\Phi}_b, \bar{\boldsymbol{\Phi}}_b] = \bar{\rho}_M r_s \bar{\boldsymbol{A}}\boldsymbol{B}\boldsymbol{C} \tag{12}$$

where

$$\begin{aligned}\boldsymbol{C} = [&\bar{\boldsymbol{A}}_1^T, (\bar{\rho_1}/\bar{\rho_M})\boldsymbol{D}\bar{\boldsymbol{A}}_1^T, (\bar{\rho}_1^*/\bar{\rho_M})\bar{\boldsymbol{B}}\boldsymbol{D}^{-(M-1)}\bar{\boldsymbol{A}}_1^T,\\ &(\bar{\rho}_M^*/\bar{\rho_M})\bar{\boldsymbol{B}}\boldsymbol{D}^{-(M-2)}\bar{\boldsymbol{A}}_1^T]\end{aligned}$$

and $\bar{\boldsymbol{B}} = \mathrm{diag}(\beta_1^*/\beta_1, \beta_2^*/\beta_2, \cdots, \beta_p^*/\beta_p)$. From the above analysis, evidently the rank of this $(M-\bar{p}) \times 4\bar{p}$ correlation matrix $\boldsymbol{\Phi}$ equals the number of incident signals p iff $M - \bar{p} \geq p$ and $\bar{p} \geq p$. Then we have the following relationship for the estimation of number of signals without the eigendecomposition process.

Theorem 1 *Let $\boldsymbol{\Psi} = \boldsymbol{\Phi}\boldsymbol{\Phi}^H$. The number of incident signals equals the rank of the QR upper-triangular factor $\bar{\boldsymbol{R}}$ of the $(M-\bar{p}) \times (M-\bar{p})$ matrix $\boldsymbol{\Psi}$ when the identifiability condition that $p \leq \bar{p} < M - p$ is satisfied, where the Householder QR decomposition of the matrix $\boldsymbol{\Psi}$ is given by*

$$\begin{aligned}\boldsymbol{\Psi} &= \bar{\boldsymbol{Q}}\bar{\boldsymbol{R}}\\ &= [\underbrace{\bar{\boldsymbol{Q}}_1}_{p}, \underbrace{\bar{\boldsymbol{Q}}_2}_{M-\bar{p}-p}] \begin{bmatrix} \bar{\boldsymbol{R}}_{11}, & \bar{\boldsymbol{R}}_{12} \\ \multicolumn{2}{c}{\boldsymbol{O}_{(M-\bar{p}-p)\times(M-\bar{p})}} \end{bmatrix} \begin{matrix} \}p \\ \}M-\bar{p}-p \end{matrix}\end{aligned} \tag{13}$$

in which $\bar{\boldsymbol{Q}}$ is the $(M-\bar{p}) \times (M-\bar{p})$ unitary matrix, $\bar{\boldsymbol{R}}_{11}$ is the $p \times p$ upper-triangular and nonsingular matrix, and $\bar{\boldsymbol{R}}_{12}$ is the $p \times (M-\bar{p}-p)$ matrix with non-zero elements.

Proof: Omitted.

Apparently the elements $\{\bar{r}_{ik}\}$ of the $(M-\bar{p})\times(M-\bar{p})$ matrix $\bar{\boldsymbol{R}}$ in (13) are given by $\bar{r}_{ik} \neq 0$ for $i \leq k$ with $i = 1, 2, \cdots, p$ and $k = i, i+1, \cdots, M-\bar{p}$ whereas $\bar{r}_{ik} = 0$ for others, and the number of incident signals p is revealed in the rank of the QR factor $\bar{\boldsymbol{R}}$ of matrix $\boldsymbol{\Psi}$. Thus the number of signals can be determined as the number of nonzero rows of $\bar{\boldsymbol{R}}$ without the need of eigendecomposition when the double inequality $p \leq \bar{p} < M - p$ is satisfied. This is the basic principle for detecting the number of signals. Furthermore the signal detection capability can be improved, because the affection of the additive noise does not appear in the cross product $\boldsymbol{\Psi}$ of the correlation matrix $\boldsymbol{\Phi}$ as well.

3.3. Estimation of Number of signals with Accessible Noisy Data

In practice, only a finite and noisy array data is available, and the correlation matrices $\boldsymbol{\Phi}_f$, $\bar{\boldsymbol{\Phi}}_f$, $\boldsymbol{\Phi}_b$, and $\bar{\boldsymbol{\Phi}}_b$ in (8)–(11) (and $\boldsymbol{\Psi}$) should be replaced with their estimates. From the finite array data $\{\boldsymbol{y}(n)\}_{n=1}^N$, the sample estimates of the correlation vectors $\hat{\boldsymbol{\varphi}}$ and $\hat{\bar{\boldsymbol{\varphi}}}$ are obtained as

$$\hat{\boldsymbol{\varphi}} = [\hat{r}_{1M}, \hat{r}_{2M}, \cdots, \hat{r}_{MM}]^T = \frac{1}{N}\sum_{n=1}^N \boldsymbol{y}(n)y_M^*(n) \tag{14}$$

$$\hat{\bar{\boldsymbol{\varphi}}} = [\hat{r}_{11}, \hat{r}_{21}, \cdots, \hat{r}_{M1}]^T = \frac{1}{N}\sum_{n=1}^N \boldsymbol{y}(n)y_1^*(n) \tag{15}$$

Then the estimated Hankel correlation matrices $\hat{\boldsymbol{\Phi}}_f$, $\hat{\bar{\boldsymbol{\Phi}}}_f$, $\hat{\boldsymbol{\Phi}}_b$, and $\hat{\bar{\boldsymbol{\Phi}}}_b$ and hence the estimated matrix $\hat{\boldsymbol{\Phi}}$ can be formed from $\hat{\boldsymbol{\varphi}}$ and $\hat{\bar{\boldsymbol{\varphi}}}$ as

$$\hat{\boldsymbol{\Phi}}_f = \mathrm{Hank}\{\boldsymbol{h}_c, \boldsymbol{h}_r\}, \quad \hat{\bar{\boldsymbol{\Phi}}}_f = \mathrm{Hank}\{\bar{\boldsymbol{h}}_c, \bar{\boldsymbol{h}}_r\} \tag{16}$$

$$\hat{\boldsymbol{\Phi}}_b = \boldsymbol{J}_{M-p}\hat{\bar{\boldsymbol{\Phi}}}_f^*\boldsymbol{J}_p, \quad \hat{\bar{\boldsymbol{\Phi}}}_b = \boldsymbol{J}_{M-p}\hat{\boldsymbol{\Phi}}_f^*\boldsymbol{J}_p \tag{17}$$

$$\hat{\boldsymbol{\Phi}} = [\hat{\boldsymbol{\Phi}}_f, \hat{\bar{\boldsymbol{\Phi}}}_f, \hat{\boldsymbol{\Phi}}_b, \hat{\bar{\boldsymbol{\Phi}}}_b] \tag{18}$$

where $\boldsymbol{h}_c = [\hat{r}_{1M}, \hat{r}_{2M}, \cdots, \hat{r}_{M-p,M}]^T$, $\boldsymbol{h}_r = [\hat{r}_{M-p,M}, \hat{r}_{M-p+1,M}, \cdots, \hat{r}_{M-1,M}]^T$, $\bar{\boldsymbol{h}}_c = [\hat{r}_{21}, \hat{r}_{31}, \cdots, \hat{r}_{L1}]^T$, $\bar{\boldsymbol{h}}_r = [\hat{r}_{L1}, \hat{r}_{L+1,1}, \cdots, \hat{r}_{M1}]^T$, and $\mathrm{Hank}\{\cdot\}$ denotes the Hankel operation.

When the number of snapshots N is not sufficiently large, the QR factor $\hat{\bar{\boldsymbol{R}}}$ of the cross product $\hat{\boldsymbol{\Psi}} = \hat{\boldsymbol{\Phi}}\hat{\boldsymbol{\Phi}}^H$ may become an upper-triangular and nonsingular matrix with full-rank due to the effect of estimation error (i.e., $\hat{\bar{r}}_{ik} \neq 0$ for $i = p+1, \cdots, M-\bar{p}$ and $k = i, i+1, \cdots, M-\bar{p}$), and the number of incident signals could not be determined directly by comparing magnitude relation between the elements $|\hat{\bar{r}}_{pp}|$ and $|\hat{\bar{r}}_{p+1,p+1}|$ of $\hat{\bar{\boldsymbol{R}}}$. Here we suggest a criterion to estimate the number of signals (i.e., the effective rank of $\hat{\boldsymbol{\Psi}}$) by using the QR decomposition.

Now we consider the QR decomposition with column pivoting of matrix $\hat{\boldsymbol{\Psi}}$ given by (e.g., [12])

$$\hat{\boldsymbol{\Psi}}\boldsymbol{\Pi} = \hat{\bar{\boldsymbol{Q}}}\hat{\bar{\boldsymbol{R}}} = \hat{\bar{\boldsymbol{Q}}}\begin{bmatrix} \hat{\bar{\boldsymbol{R}}}_{11}, & \hat{\bar{\boldsymbol{R}}}_{12} \\ \boldsymbol{O}_{(M-\bar{p}-p)\times p}, & \hat{\bar{\boldsymbol{R}}}_{22} \end{bmatrix} \begin{matrix} \}p \\ \}M-\bar{p}-p \end{matrix} \tag{19}$$

where $\boldsymbol{\Pi}$ is a $(M-\bar{p}) \times (M-\bar{p})$ permutation matrix, which is used to represent different methods of QR decomposition with column interchanges, and $\hat{\bar{r}}_{ik} = 0$ for $i > k$ with $i = 2, 3, \cdots, M-\bar{p}$ and $k = 1, 2, \cdots, i-1$. By introducing an auxiliary quantity $\zeta(i)$ in terms of the ith row elements of QR factor $\hat{\bar{\boldsymbol{R}}}$ in (19) as

$$\zeta(i) = \sum_{k=i}^{M-\bar{p}} |\hat{\bar{r}}_{ik}| \tag{20}$$

we define a ratio criterion $\xi(i)$ as

$$\xi(i) = \frac{\zeta(i)}{\zeta(i+1)} \tag{21}$$

for $i = 1, 2, \cdots, M - \bar{p} - 1$. Then the number of incident signals is determined as the value of i for which the criterion $\xi(i)$ is maximized, i.e.,

$$\hat{p} = \arg\max_i \, \xi(i). \tag{22}$$

Remark 2: Although this paper considers the case of estimating the number of signals in multipath environment, where $p \geq 2$, the proposed method is applicable to the case of a single incident signal (i.e. $p = 1$). Furthermore the proposed method can be modified for the case of that there is no signal (i.e. $p = 0$) (yet it is beyond the scope of this paper).

3.4. Choice of Subarray Size

In the proposed method, the subarray size $\bar{p}$ is crucial to detecting the number of signals and should be chosen appropriately, because the information on the number of signals is unavailable and the subarray size usually affects the detection performance when the number of snapshots is small or the SNR is low.

From the identifiability condition $p \leq \bar{p} < M - p$ described in Theorem 1, we can find that the maximum detectable number of signals is $p < M/2$ (i.e., $p_{\max} = \lceil M/2 \rceil - 1$), which is similar to the identifiability condition that guarantees the uniqueness of direction estimation for the SUMWE (subspace-based method without eigendecomposition) with the known number of signals (see [7] for reference), where $\lceil x \rceil$ denotes the smallest integer not less than x. Therefore we can choose a conservative value of the subarray size as $\bar{p} = \lfloor M/2 \rfloor$, which satisfies the inequality condition that $p_{\max} \leq \bar{p} = \lfloor M/2 \rfloor < M - p_{\max}$, where $\lfloor x \rfloor$ denotes the largest integer not greater than x.

4. NUMERICAL EXAMPLES

The effectiveness of the proposed method in estimating the number of signals is evaluated through numerical examples. The ULA with M sensors is separated by a half-wavelength, and two coherent signals with equal power come from angles θ_1 and θ_2. The SNR is defined as the ratio of the power of the source signals to that of the additive noise at each sensor. For comparing the detection performance of the proposed method, the SS- and FBSS-based AIC and MDL criteria [1], [4], [5], [11] and the QR-based method [8] are carried out. Further the QR-based method [8] is modified in a similar way to the proposed method shown in (20)-(22) for avoiding the needs of the true noise variance and a subjective judgment (referred as modified Reilly herein). The simulation results shown below are obtained by the ensemble-averaging over 1000 independent trials.

Example 1—Performance versus SNR: The incident directions of two coherent signals are $\theta_1 = 5^{\rm o}$ and $\theta_2 = 12^{\rm o}$, and their SNR is varied from -10 to 25dB. The number of sensors is $M = 10$, and the number of snapshots is $N = 64$. The subarray size is set at $\bar{p} = \lfloor M/2 \rfloor = 5$ for the proposed method, while the subarray size is set at $m = 5$ for the SS-based AIC and MDL criteria [1], [4], [5], [11] and the QR-based method [8]. Further the QR-based method [8] is modified in a similar way to the proposed method shown in (20)-(22) for avoiding the needs of the true noise variance and a subjective judgment (referred as modified Reilly herein). The proposed method is carried out with three different permutation matrices such as i) $\boldsymbol{\Pi} = \boldsymbol{I}_{M-\bar{p}}$ (referred as QR), ii) $\boldsymbol{\Pi} = [\boldsymbol{e}_1, \boldsymbol{e}_5, \boldsymbol{e}_3, \boldsymbol{e}_2, \boldsymbol{e}_4]$ according to a column index maximum-difference bisection rule [13], [14] (referred as QRPP) and iii) a data-dependent $\boldsymbol{\Pi}$ which makes the absolute values of diagonal elements $\{\hat{\bar{r}}_{ii}\}$ in decreasing order [12] (referred as QRP), where $\boldsymbol{e}_i$ is an $(M - \bar{p}) \times 1$ unit vector with unity element at the ith location and zeros elsewhere.

When $\mathrm{SNR} = 2.5\mathrm{dB}$, the averaged criteria $\{\xi(i)\}$ in (21) for the proposed method with three permutation matrix are plotted in Fig. 1 for 1000 trials. Obviously the number of incident signals can be estimated as the value of i for which the criterion $\xi(i)$ in (21) has the maximum. Then probability of correct detection of the proposed method in terms of the SNR are shown in Fig. 2. It is found that the proposed detection method generally outperforms the more common SS- and FBSS-based AIC and MDL criteria with eigendecomposition and the modified QR-based method of [8] as low SNR.

Example 2—Performance versus Snapshot Number: The simulation conditions are similar to those in Example 1, except that the SNR is set at 2.5dB, and the number of snapshots is varied from $N = 1$ to $N = 1000$. From Fig. 3, it can be seen that the proposed method is superior to the other methods such as SS- and FBSS-based AIC and MDL criteria and modified Reilly even for a small number of snapshots.

Example 3—Performance versus Angular Separation: The simulation conditions are similar to those in Example 1, except that the SNR is set at 2.5dB, and two coherent signals impinge on the array along $\theta_1 = 5^{\rm o}$ and $\theta_2 = \theta_1 + \Delta\theta$, where $\Delta\theta$ is varified from $\Delta\theta = 1^{\rm o}$ to $\Delta\theta = 10^{\rm o}$. Fig. 4 showes that the proposed method has good performance in detecting the closely-spaced signals with insignificantly degraded performances as compared to the FBSS-based MDL and AIC methods with time-consuming EVD when the number of snapshots is small and/or the SNR is low.

Example 4—Performance versus Subarray Size: The sim-

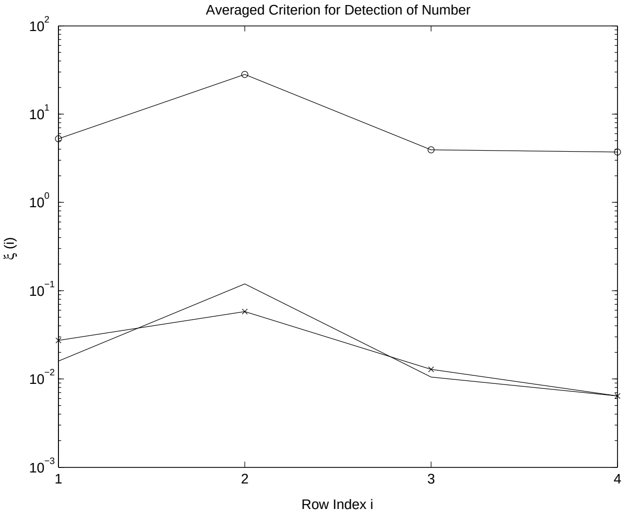

Fig. 1. Averaged criterion for the detection of number of signals (solid line with "o": proposed method (QR); solid line with "x": proposed method (QRP); and solid line: proposed method (QRPP)) for Example 1 ($\mathrm{SNR} = 2.5$d, $N = 64$, and $M = 10$).

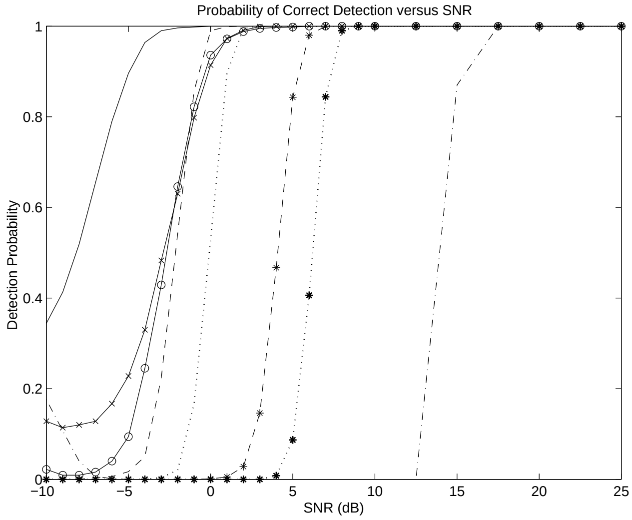

Fig. 2. Probability of correct detection in terms of the SNR (solid line with "o": proposed method (QR); solid line with "x": proposed method (QRP); solid line: proposed method (QRPP); dashed line with "*": SS-AIC; dashed line: FBSS-AIC; dotted line with "*": SS-MDL; dotted line: FBSS-MDL; and dash-dot line: modified Reilly method) for Example 1 ($N = 64$ and $M = 10$).

ulation conditions are similar to those in Example 1, except that the SNR is set at 2.5dB, and the subarray size $\bar{p}$ for the proposed method is varied from $\bar{p} = 2$ to $\bar{p} = 7$ so that the condition $p \leq \bar{p} < M - p$ is satisfied, whereas the subarray size m for the SS-based methods is varied $m = 3$ to $m = 10$. Because the dimension of matrix $\hat{\mathbf{\Psi}}$ is $(M - \bar{p}) \times (M - \bar{p})$, which varies with the subarray size $\bar{p}$, and the decorrelation of signal coherency is also related with the subarray size, the detection performance is affected by the subarray size when the number of snapshots is small and/or the SNR is low. From the results shown in Fig. 5, we can find that high probability of correct detection can be achieved for the proposed method by choosing the subarray size as $\bar{p} = \lfloor M/2 \rfloor$.

5. CONCLUSION

In this paper, a computationally simple and efficient non-parametric method was proposed for estimating the number of coherent narrowband signals impinging on a ULA. The proposed method does not require the computationally cumbersome eigendecomposition and the evaluation of all correlations of the array data, and the effect of additive noise is eliminated, hence it is suitable for real-time implementation. Furthermore the proposed method has remarkable insensitivity to the correlation of incident signals and flexibility to the spatially correlated noise. The effectiveness of the proposed method was verified through numerical examples, and simulation results showed that the proposed method is superior in detecting closely-spaced signals with small number of snapshots and/or at relatively low SNR

6. REFERENCES

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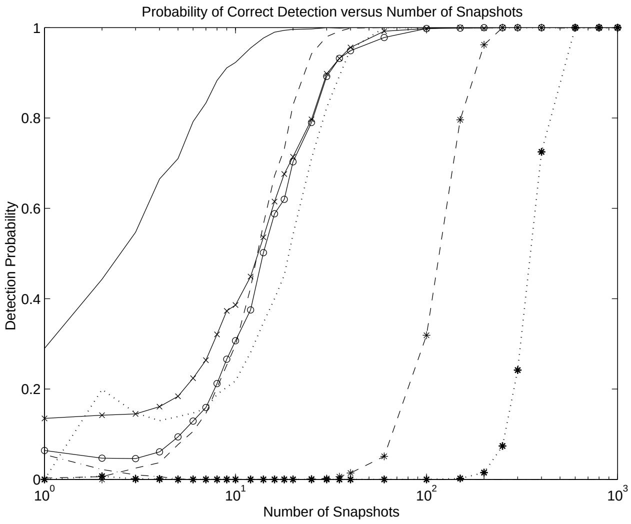

Fig. 3. Probability of correct detection in terms of the number of snapshots (solid line with "o": proposed method (QR); solid line with "x": proposed method (QRP); solid line: proposed method (QRPP); dashed line with "*": SS-AIC; dashed line: FBSS-AIC; dotted line with "*": SS-MDL; dotted line: FBSS-MDL; and dash-dot line: modified Reilly method) for Example 2 ($\mathrm{SNR} = 2.5\mathrm{dB}$ and $M = 10$).

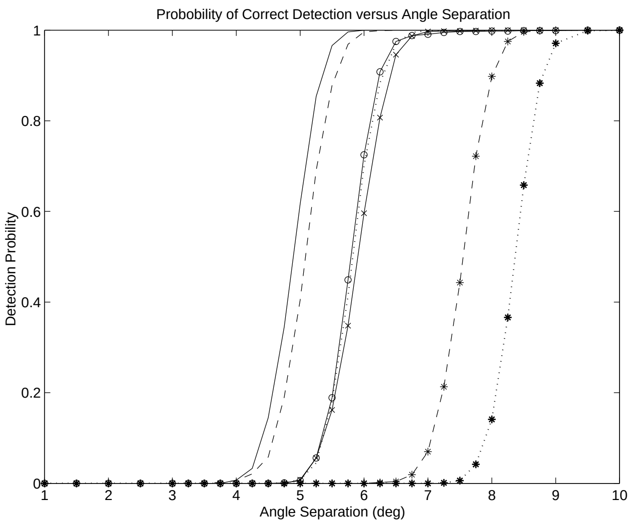

Fig. 4. Probability of correct detection in terms of the angular separation (solid line with "o": proposed method (QR); solid line with "x": proposed method (QRP); solid line: proposed method (QRPP); dashed line with "*": SS-AIC; dashed line: FBSS-AIC; dotted line with "*": SS-MDL; dotted line: FBSS-MDL; and dash-dot line: modified Reilly method) for Example 3 ($\mathrm{SNR} = 2.5\mathrm{dB}$, $N = 64$, and $M = 10$).

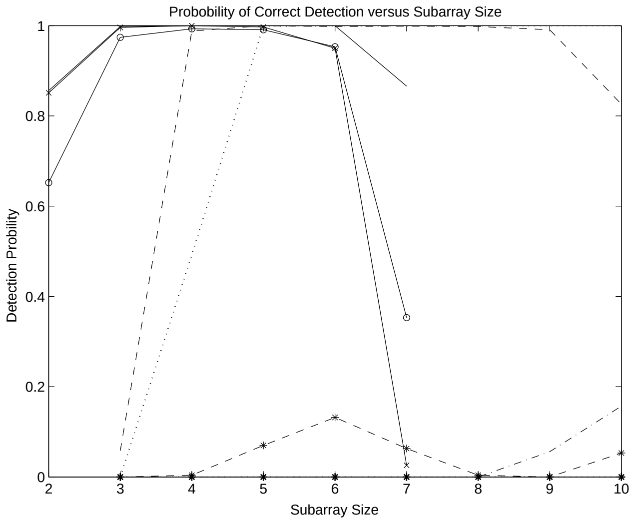

Fig. 5. Probability of correct detection in terms of the subarray size (solid line with "o": proposed method (QR); solid line with "x": proposed method (QRP); solid line: proposed method (QRPP); dashed line with "*": SS-AIC; dashed line: FBSS-AIC; dotted line with "*": SS-MDL; dotted line: FBSS-MDL; and dash-dot line: modified Reilly method) for Example 4 ($\mathrm{SNR} = 2.5\mathrm{dB}$, $N = 64$, and $M = 10$).

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