LATTICE FILTER WITH ADJUSTABLE GAINS AND ITS APPLICATION IN OPTICAL SIGNAL PROCESSING

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ABSTRACT

In this paper, we consider a class of *N*-th order lattice filter with gains as an extension to the traditional lattice filter structures and develop analysis and synthesis of such filters. In the analysis-synthesis parts, we present recursive methods to derive the input-output transfer function of the filter in terms of parameters of the lattice structure such as the time delay, gains, transmission and reflection coefficients and vice versa. Stability of the filter is analyzed and an algorithm to test the stability is proposed. This class of MIMO lattice filters with adjustable gains models integrated photonic devices under development in our labs for optical communication and high speed signal processing applications. Our signal processing approach to characterizing this type of lattice structures can also be used in filter realization by VLSI, FPGA, or programmable processors for acoustic or speech applications.

Keywords: Lattice filters with adjustable gains, Optical communication and networks, high speed signal processing, stability conditions, Recursive analysis and synthesis.

1. INTRODUCTION

One of the current trends in photonics fields is to build optical devices using semiconductor fabrication techniques (integrated photonics) [3, 11] for optical communication and high speed signal processing applications. Such devices comprise many micro/nano-scale components interconnected by waveguides. The inspiration behind building such devices is the wide range of high speed signal processing operations that conventional electronic integrated circuits can perform. These integrated photonic devices can carry out common signal processing operations like switching, multiplexing, demultiplexing, spectral shaping, equalization, filtering, and generation of desired signals. They can especially be useful to high speed signal processing in the areas of communication, medical, military, and commercial applications. The lattice structure discussed in this paper models the transmission, reflection, and delay of signal paths through optical waveguide and mirror in the presence of gains which is introduced by injecting electric current into the signal propagation paths. The model may be used for other physical systems where signal energy is propagated through mediums with different physical characteristics or impedances and where external signal amplification or attenuation source is available. Communication, acoustic, radar, or seismic channels may utilize such lattice structures with distributed delays and gains. In this paper we develop an analysis-synthesis method complete with stability criteria to an innovative integrated adaptive photonic architecture. Fabrication of current controlled optical gain elements on the same photonic substrate leads to tunable or programmable devices and filter structures [3]. A novel approach to achieve this was introduced and has been under investigation by our team. Controlling the gains by current injection enables adjusting the underlying filter structure, and thus adjusting the filter's transfer function. Propagation of light in optical waveguides and scattering of light at interfaces have been modeled by a class of discrete-time lattice filter structures [1, 2, 4, 5, 7, and 8]. Additional delay and introduction of adjustable gains in the structure leads to the lattice filters with tunable gains that are different from the traditional lattice filters [1, 4, 9, 10]. The traditional lattice structure fails to sufficiently parameterize and model the underlying integrated photonic device we use for optical signal processing. In our model, the only parameters that can be tweaked so as to adapt the transfer function are the gains since the transmission and reflection coefficients are assumed invariant once the device is built. The gains appear in a multiplicative nonlinear form in the coefficients of the filter's transfer functions. Their values define locations of zeros and poles and thus the transfer function and stability of the filter. As such, analysis, synthesis, and deriving stability conditions for such filter of arbitrary order pose a new nontrivial problem to solve. Application of lattice filters with gains is not limited to integrated photonic devices. They can be realized by

VLSI, FPGA, or any programmable signal processor for other non-optical signal processing applications.

In this paper, we consider a class of N-order lattice filters with gains as an extension to the traditional lattice filter structures and develop analysis and synthesis of such filters. In the analysis part, we present a recursive method to derive the input-output transfer function of the filter in terms of the physical parameters of the structure such as the time delay, gains, transmission, and reflection coefficients. Stability of the filter is analyzed and an algorithm to test the stability is proposed. In the synthesis part, we consider how a given transfer function can be realized by the optical lattice filter structure. To that end, we propose a recursive algorithm retrieving parameters of each 2-input, 2-output stage or building block of the structure from the coefficients of the given transfer function.

The contribution of this paper can be summarized as follows. The techniques proposed in this paper provide powerful tools for analysis and design of stable lattice filters with gains. A novel algorithm to test the stability is introduced. In the case of optical filter structure where some of the input-output direct paths are missing, the proposed recursive algorithm can also specify the restrictions imposed on the numerator coefficients of realizable transfer functions.

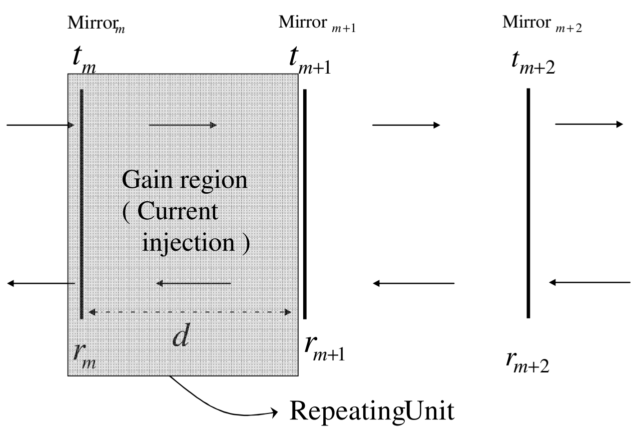

Fig. 1. Multi-mirror Etalon [1]. Each mirror interface reflects a fraction of the light and transmits the other fraction. Spatial delay d defines the temporal delay.

2. SYSTEM MODEL

A 2-input, 2-output N-order optical lattice filter structure was discussed in [1, 2]. Figure 1 shows two stages of the actual physical device that is modeled as a discrete time filter. The device is a series of mirrors with spatial distance between the mirror surfaces providing the temporal delay, and the optical gain. The filter is modular with each m- th module characterized by a gain G_m and a transmission coefficient $t_m = \cos\theta_m$ and a reflection coefficient $r_m = \sin\theta_m$. These two coefficients satisfy the equation $t_m{}^2 + r_m{}^2 = 1$ obeying energy conservation principle assumed for the optical wave propagation in the

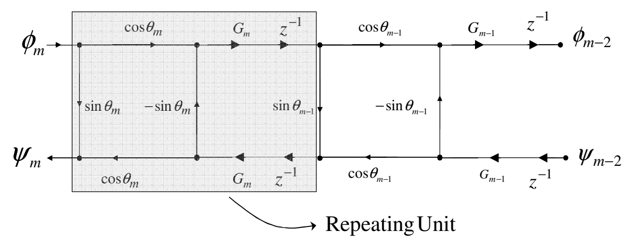

Fig. 2. Signal Flow Graph model of the optical interface. Each mirror interface reflects a fraction of the light $(\sin\theta_m)$ and transmits the other fraction $(\cos\theta_m)$. The spatial delay is modeled by the two unit delays per stage.

system. The discrete-time signal flow graph of the device in Figure 1 is shown in Figure 2. The set of θ_m's alone can realize any stable all-pole configuration. The gains play an important role in the transfer function and the only means to adjust or tune the filter response once the device is manufactured, i.e. once θ_ms are selected. Hence the effect of gains on the zeros, poles, and the stability of the filter must be characterized.

A generalized form of the single-input, single-output lattice filter is shown in Figure 3. The input–output transfer function is defined as the reflection transfer function. The generalization is the introduction of the input-output forward paths $\beta_1, \beta_2, , , , \beta_{N+2}$ in order to realize any arbitrary transfer function. $\theta_m(z)$, m=1,2,…, N, is the transfer matrix of the m-th 2-input, 2-output stage. The system is defined completely by the set of parameters G_m, θ_m, and β_m. A single stage of the lattice filter is shown in Figure 4.

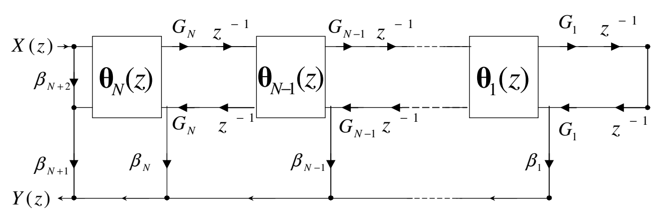

Fig. 3. The generalized structure extended to N stages.

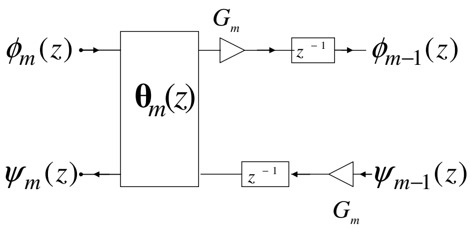

Fig. 4. A single stage or building block as a 2-input, 2-output system characterized by θ_m **,** G_m .

3. ANALYSIS

Using a given set of parameters for N stages, the transfer function of the filter can be obtained by the following recursive equation.

3.1. Analysis Equations

For $m = 1$ to N and given parameters θ_m, G_m, and β_m :

$$\begin{pmatrix} \phi_m(z) \\ \psi_m(z) \end{pmatrix} = \frac{G_m}{\cos\theta_m} \begin{pmatrix} \dfrac{1}{G_m^{\,2}} & \sin\theta_m \\ \dfrac{\sin\theta_m}{G_m^{\,2}} & 1 \end{pmatrix} \begin{pmatrix} z\,\phi_{m-1}(z) \\ z^{-1}\psi_{m-1}(z) \end{pmatrix} \quad (1)$$

with the initial values $\psi_0(z) = \phi_0(z) = 1$.

The transfer function $H_N(z) = \dfrac{Y(z)}{X(z)}$ is then given by

$$H_N(z) = \frac{\sum_{i=0}^{N-1} \beta_{i+1} G_{i+1} z^{-1} \psi_i(z) + \beta_{N+1}\psi_N(z) + \beta_{N+2}\phi_N(z)}{\phi_N(z)} \quad (2)$$

The above equation is derived from Figure 3 directly. For example , for N =3 we have

$$\phi_3(z) = 1 + (r_1 G_1^{\,2} + r_1 r_2 G_2^{\,2} + r_2 r_3 G_3^{\,2}) z^{-1} + (r_2 G_1^{\,2} G_2^{\,2} + r_1 r_2 r_3 G_1^{\,2} G_3^{\,2} + r_1 r_3 G_2^{\,2} G_3^{\,2}) z^{-2} + r_3 G_1^{\,2} G_2^{\,2} G_3^{\,2} z^{-3}$$

4. SYNTHESIS

The Analysis equations of (1) can be inverted to obtain the synthesis equations. This leads to a set of equations called step-down recursions. Notice that the determinant of the matrix in (1) in non-zero as long as the transmission coefficients are non-zero, i.e. as long as $|r_m| = |\sin\theta_m| < 1$.

Let the stable transfer function to be realized be

$$H(z) = \frac{Y(z)}{X(z)} = \frac{N_N(z)}{D_N(z)} = \frac{\sum_{i=0}^{N} b_i z^i}{\sum_{i=0}^{N} a_i z^i} \ . \quad (3)$$

To initialize, we let

$$\phi_N(z) = D_N(z) \ , \psi_N(z) = N_N(z) \ , \ \ \sin\theta_N = \frac{\phi_N(0)}{\psi_N(0)} \quad (4)$$

The recursive synthesis equation (obtained by inverting (1)) is

$$\begin{pmatrix} z\phi_{m-1}(z) \\ z^{-1}\psi_{m-1}(z) \end{pmatrix} = \frac{G_m}{\cos\theta_m} \begin{pmatrix} 1 & -\sin\theta_m \\ -\dfrac{\sin\theta_m}{G_m^{\,2}} & \dfrac{1}{G_m^{\,2}} \end{pmatrix} \begin{pmatrix} \phi_m(z) \\ \psi_m(z) \end{pmatrix} \quad (5)$$

The key step in the order recursive synthesis is the following step-down recursions obtained from (15)

$$\phi_{i-1}(z) = \frac{z^{-1} G_i \left(\phi_i(z) - \sin\theta_i \cdot \psi_i(z)\right)}{\cos\theta_i} \quad (6)$$

$$\psi_{i-1}(z) = \frac{z\left(\psi_i(z) - \sin\theta_i \ \phi_i(z)\right)}{G_i \cos\theta_i} \quad (7)$$

$$\text{where } \sin\theta_i = \left.\frac{\phi_i(z)}{\psi_i(z)}\right|_{z\to 0} \quad (8)$$

Choose G_i such that $|\sin\theta_i| = \left|\dfrac{\phi_i(z)}{\psi_i(z)}\right| < 1$

The result of the step-down algorithm is the generation of the set of polynomials $\psi_N(z), \psi_{N-1}(z), \ldots, \psi_0(z)$ and $\phi_N(z), \phi_{N-1}(z), \ldots, \phi_0(z)$. The β 's are then obtained using

$$N_N(z) = \sum_{i=0}^{N-1} \beta_{i+1} G_{i+1} z^{-1} \psi_i(z) + \beta_{N+1}\psi_N(z) + \beta_{N+2}\phi_N(z) \quad (9)$$

5. STABILITY ANALYSIS

Without loss of generality the denominator polynomial $\phi_N(z)$ can be expressed as

$$\phi_N(z) = \sum_{m=0}^{N} a_m z^m \quad (10)$$

where a_m 's are found recursively , using (1).

The necessary and sufficient conditions for stability of $\phi_N(z)$ can be obtained by the Schur-Cohn stability test [9]. However applying Schur-Cohn conditions lead to a_m^i 's with $i \geq 1$. The degree of non-linearity increases as the order N goes up which makes programming and testing of the conditions very complex and involved. We reduce the complexity by proposing a set of sufficient conditions for which $\phi_N(z)$ is stable. The proposed conditions provide simple inequalities involving linear combination of a_m 's and not a_m^2 's, a_m^3 etc.

We make the substitution $z = \frac{1+s}{1-s}$. This bilinear transformation results in a polynomial in s. Thus the region $|z| \geq 1$ is mapped to the right-half plane in the s domain. $\phi_N(z)$ will be stable iff $\phi_N(s)$ given by

$$\phi_N(s) = \sum_{m=0}^{N} b_m s^m \tag{11}$$

has no roots on the right-half plane.

Let $\mathbf{b} = [b_0\, b_1 \ldots b_N]^T$ and $\mathbf{a} = [a_0\, a_1 \ldots a_N]^T$. $\mathbf{b}$ can be found from $\mathbf{a}$ using a linear transformation $\mathbf{T}_B$.

$$\mathbf{b} = \mathbf{T}_B \mathbf{a} \tag{12}$$

The form of $\mathbf{T}_B$ is given in [12].

Since $\mathbf{a}$ is a function of r_m 's and G_m^2 's, $\mathbf{b}$ will also be a function of r_m 's and G_m^2 's with the same degree of complexity due to the linear transformation of (12)

Let us consider

$$\begin{aligned} Q_N(s) &= \phi_N(s) + \phi_N^*(s) \\ &= \sum_{m=0}^{N} b_m s^m + \sum_{m=0}^{N} b_m \left(s^*\right)^m = \sum_{m=0}^{N} b_m \left(s^m + s^{*m}\right) \end{aligned} \tag{13}$$

(the * sign denotes complex conjugation)

Now every root of $\phi_N(s)$ is also a root of $Q_N(s)$. This means that for stability there should not be any positive real root of $Q_N(s)$.

Let $s = \rho e^{j\Omega}$.

Then

$$\begin{aligned} Q_N(\rho, \Omega) &= \sum_{m=0}^{N} b_m \left(2\rho^m \cos(m\Omega)\right) \\ &= 2\sum_{m=0}^{N} b_m \cos(m\Omega)\,\rho^m \end{aligned} \tag{14}$$

Let

$$Q_N(\rho) = \sum_{m=0}^{N} f_m \rho^m \tag{15}$$

where

$$f_m = 2b_m \cos(m\Omega) \text{ and } 0 \leq \cos\Omega \leq 1 \;\;,\;\; 0 \leq \rho < \infty \tag{16}$$

Define,

$$\begin{aligned} \mathbf{f} &= [f_0\, f_1 \cdots f_N]^T \\ \boldsymbol{\sigma} &= [1 \;\; \cos\Omega \;\; \cos 2\Omega \;...\; \cos N\Omega]^T \\ \mathbf{c} &= [1 \;\; \cos\Omega \;\; \cos^2\Omega \;\; \cos^3\Omega \;.\;.\;.\; \cos^N\Omega \;]^T \\ \boldsymbol{\rho} &= [1 \;\rho\; \rho^2 \;.\;.\;.\; \rho^N\,]^T \\ \mathbf{B} &= diag(b_0, b_1, b_2, \ldots, b_N) \end{aligned} \tag{17-21}$$

The vector $\boldsymbol{\sigma}$ can be expressed in terms of $\mathbf{c}$ using the well known Chebyshev transformation

$$\boldsymbol{\sigma} = \mathbf{T}_C \mathbf{c} \tag{22}$$

The form of $\mathbf{T}_C$ can be found in [12]

Using (17) through (21) and (14) we can express $Q_N(\rho, \Omega)$ in matrix form as follows

$$Q_N(\rho, \Omega) = 2\boldsymbol{\sigma}^T \mathbf{B}\boldsymbol{\rho} \tag{23}$$

Using (22) we can obtain $Q_N(\rho, \Omega) = 2\mathbf{c}^T \mathbf{T}_C^T \mathbf{B}\boldsymbol{\rho}$

As a result the coefficient vector $\mathbf{f}$ of (15) is given by

$$\mathbf{f} = 2\mathbf{c}^T \mathbf{T}_C^T \mathbf{B} \tag{24}$$

The polynomial $Q_N(\rho)$ will not have any positive real root if its coefficients have the same sign. More specifically if

$$f_m \geq 0 \;\text{ for }\; m = 1, 2, \ldots, N-1 \tag{25}$$

$$f_0 > 0 \;\text{ and }\; f_N > 0 \tag{26}$$

$$|r_m| < 1 \;\;,\;\; {G_m}^2 > 0 \;\;,\;\; 0 < \cos\Omega < 1 \tag{27}$$

for a set of parameters r_m 's , G_m^2 's and $\cos\Omega$

This provides the sufficient condition for the stability of $\phi_N(z)$.

To expand on the conditions given by (25),(26), and (27) and to determine whether the solution space for the parameters r_m 's , G_m^2 's and $\cos\Omega$ is empty , and find the parameters if the solution space is non-empty , we can proceed as follows.

Let $Q_N(\rho) = Q_{k,N}(\rho), k = 1$

Step 1: If no set of parameters satisfy (26) for any $Q_{k,N}(\rho)$, then the solution space is empty. Otherwise go to step 2 .

Step 2: If no set of parameters can be found to satisfy (25) for any $Q_{k,N}(\rho)$ then transform $Q_{k,N}(\rho)$ into two new auxiliary polynomials using $\rho = 1 + \rho$ and $\rho = \frac{1}{1+\rho}$ respectively.

Step 3: Apply equation (25),(26), and (27) to each auxiliary polynomials $Q_{k,N}(\rho)$.

Step 4: If every auxiliary polynomial satisfies (25),(26), and (27) then the solution space is non-empty and a set of parameters can be found. Otherwise go to Step 1.

The above algorithm has been shown to converge exponentially, based on Vincent's theorem, in [12].

6. EXAMPLES

We illustrate the proposed method with a simple two-stage example.

6.1. Analysis method

Let us consider a two-stage filter. Let $\theta_1 = \frac{\pi}{6}, \theta_2 = \frac{\pi}{3}$ then by Equation (3) we have

$$G_1^2 < \frac{1}{\sin\theta_1} \Rightarrow G_1^2 < \frac{1}{\sin\frac{\pi}{6}} = 2$$

Let us choose G_1 =1. From (3) we also must have

$$G_1^2 G_2^2 < \frac{1}{\sin\theta_2} \Rightarrow G_2^2 < \frac{1}{\sin\theta_2} = 1.1547 .$$

We now choose $G_2 = 0.75$. The transfer function is then computed by Equations (1) and (2) resulting in

$$H_2(z) = \frac{0.866z^2 + 0.7143 + 0.5625z^{-2}}{z^2 + 0.7436 + 0.4871z^{-2}}$$

which is stable (note that $\beta_1 = \beta_2 = \beta_4 = 0, \beta_3 = 1$)

6.2. Synthesis Method

To illustrate the synthesis procedure we derive the parameters $\theta_1, \theta_2, G_1, G_2$ back from $H_2(z)$ .We consider the same two-stage filter for the sake of simplicity, where

$$\phi_2(z) = z^2 + 0.7436 + 0.4871z^{-2},$$
$$\psi_2(z) = 0.866z^2 + 0.7143 + 0.5625z^{-2}.$$

Then from Equation (8) we get

$\sin\theta_2 = \left.\frac{\phi_2(z)}{\psi_2(z)}\right|_{z\to 0} = 0.866 \;\Rightarrow\; \theta_2 = \frac{\pi}{3}$. Using (6) we get $\phi_1(z) = \frac{z^{-1}G_2(\phi_2(z) - \sin\theta_2\psi_2(z))}{\cos\theta_2}$.

Letting $G_2 = 0.75$ we then get

$\phi_1(z) = 0.3750z + 0.1875z^{-1}$, and

$\psi_1(z) = 0.1875z + 0.3750z^{-1}$ which lead to

$\sin\theta_1 = \left.\frac{\phi_1(z)}{\psi_1(z)}\right|_{z\to 0} = 0.5 \Rightarrow \theta_1 = \frac{\pi}{6}$. This will result in G_1 =1. Thus we get back the parameters that we initially started with. It should be noted that if we had started with a different value of G_2 satisfying the filter stability conditions, we would have got different θ_1, θ_2 for the same transfer function. This is due to the non-linear relationships of the gains in the filter's transfer function.

7. CONCLUSION

A novel *N*-order lattice filter structure with adjustable gains was considered modeling a physical device for optical communication and high speed signal processing applications. The gains were introduced and tuned by electronically injecting and controlling current into the integrated photonic device. The distributed delays in the feed-forward and feed-back paths and presence of the gains separates the lattice structure discussed in this paper from the traditional lattice-ladder structures. The gains appear in multiplicative non-linear form in the filter's transfer function. Recursive methods were introduced to analyze the structure and to synthesize an arbitrary transfer function in terms of the system's parameters. We introduced a convergent algorithm to test the system stability. The lattice structure discussed in this paper can also be used to appropriately model signal propagation channels in other non-optical applications such as radar, acoustic, communication, or seismic signal processing. The analysis-synthesis methods developed for the gain-tunable lattice structure provides necessary tools for actual realization of the desired filters not only by the optical devices but also by VLSI, FPGA, or programmable signal processors for other high speed signal processing applications.

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