PARALLEL DEFLATION WITH ALPHABET-BASED CRITERIA FOR BLIND SOURCE EXTRACTION

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ABSTRACT

Blind source extraction aims at estimating the source signals which appear mixed at the output of a sensor array. A novel approach to blind source extraction is presented in this contribution, which exploits the discrete character (finite alphabet property) of digital modulations in the case where sources with different alphabet exist. An alphabet polynomial fitting (APF) criterion matched to the specific signal constellation is employed to extract, through deflation, the sources with the same modulation. Using the appropriate APF criteria, the sources with different modulations can be extracted in parallel. This new concept, referred to as parallel deflation, presents the potential of reducing both the signal estimation errors that typically accumulate in the conventional deflationary approach and the spatio-temporal diversity required for a satisfactory source extraction. In addition, APF criteria can be optimized through a cost-effective optimal step-size technique that can escape local extrema.

Keywords : blind equalization, deflation, finite alphabet, MIMO, parallel processing, underdetermined mixtures.

1. INTRODUCTION

Channel equalization aims to reconstruct the transmitted signals that have distorted by the propagation medium. Blind equalization has been the subject of intense research interest since the pioneering work of Sato [1] and Godard [2]. The main advantage of blind techniques is arguably that training sequences are not required, so that the effective transmission rate, and thus the spectral efficiency, are increased. In multiple-input multiple-output (MIMO) scenarios, the spatial mixing of several transmitted sources adds to the intersymbol interference introduced by the time dispersive channel. Blind signal extraction can be accomplished through a deflation approach, where the input signals are estimated one after another [3, 4]. The major limitation of classical deflation is that estimation errors accumulate along successive extraction stages. Also, sufficient diversity must be available in general; i.e., for a satisfactory equalization, the number of sensors needs to be higher than the number of sources.

The present contribution addresses the problem of blind extraction of discrete signals, particularly in the underdetermined case where there are less sensors than sources. The originality of this work lies in the use of a polynomial criterion named *alphabet polynomial fitting (APF)*, which exploits the knowledge of the modulation alphabet in order to accomplish the source extraction [5, 6]. In contrast to to traditional source-distribution independent principles such as constant modulus [2] or kurtosis maximization (KM) [7], the APF criterion targets a specific modulation. This feature leads to the novel concept of parallel deflation: a polynomial criterion can be used in a deflationary process to extract the signals of each modulation. Parallel deflation can thus reduce the diversity required for the extraction of all sources from a mixture while extracting different modulations simultaneously. As a result, this new approach can increase the extraction performance while reducing the computational cost compared to classical deflation.

Moreover, APF criteria can be optimized by efficient gradient- or Newton-descent procedures based on an optimal step size computed algebraically at each iteration. The optimal step-size strategy is able to avoid local extrema at an affordable computational cost.

2. BLIND SOURCE EXTRACTION

2.1. Problem and Signal model

We consider a time-dispersive MIMO linear time-invariant (LTI) system with the input-output relationship

$$\boldsymbol{w}(n) = \sum_{k=0}^{L_c} \mathbf{C}_k \boldsymbol{s}(n-k) + \boldsymbol{b}(n), \quad n \in \mathbb{N}$$

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where

$\boldsymbol{s}(n) \in \mathbb{C}^N$	source signal vector,
$\boldsymbol{w}(n) \in \mathbb{C}^P$	channel output signal vector,
$\boldsymbol{b}(n) \in \mathbb{C}^P$	noise vector,
$\mathbf{C}_k \in \mathbb{C}^{P\times N}$	channel impulse response.

The sequence $\mathbf{C}_k$, $k = 0, \ldots, L_c$ corresponds to the impulse response matrix taps of the finite impulse response (FIR) MIMO channel. An equalizer described by the impulse response matrix taps $\mathbf{H}_k \in \mathbb{C}^{N\times P}$, $k = 0, \ldots, L_h$, processes the channel output signals and aims at extracting the sources. The output signal vector is thus given by

$$\hat{\boldsymbol{s}}(n) = \sum_{k=0}^{L_h} \mathbf{H}_k \boldsymbol{w}(n-k), \quad n \in \mathbb{N}.$$

The extraction of the pth output component $\hat{s}_p(n)$ can alternatively be expressed as:

$$\hat{s}_p(n) = \boldsymbol{h}_p{}^{\mathsf{T}} \tilde{\boldsymbol{w}}(n) \tag{1}$$

where $\tilde{\boldsymbol{w}}(n) = [\boldsymbol{w}(n)^{\mathsf{T}}, \boldsymbol{w}(n-1)^{\mathsf{T}}, \ldots, \boldsymbol{w}(n-L_h)^{\mathsf{T}}]^{\mathsf{T}} \in \mathbb{C}^{P(L_h+1)}$ (symbol $^{\mathsf{T}}$ stands for transposition) and $\boldsymbol{h}_p = [(\boldsymbol{H}_0)_{(p,:)}, (\boldsymbol{H}_1)_{(p,:)}, \ldots, (\boldsymbol{H}_{L_h})_{(p,:)}]^{\mathsf{T}} \in \mathbb{C}^{P(L_h+1)}$, notation $(\boldsymbol{H}_j)_{(p,:)}$ denoting the pth row of the equalizer matrix tap $\boldsymbol{H}_j$.

2.2. Classical deflation

Classical deflation aims at extracting one by one the N source signals mixed at the output of P sensors. This scheme can be employed with a source-distribution independent criterion such as the CM or KM principles; for instance, the KM cost function [7] is used in the original paper [3]. Thus, a unique criterion is applied to extract each source from the observations. In order to avoid extracting the same signal twice, the contribution of the extracted source has to be estimated (e.g., via correlation techniques) and subtracted from the sensors. This procedure is repeated until the N sources are extracted. The required diversity for the N-source extraction is limited by a number of sensors $P \approx N$. Moreover, estimation errors accumulate with the number of extractions, so that the extraction quality gradually decreases. Classical deflation is illustrated in Fig. 1.

3. ALPHABET-BASED SOURCE EXTRACTION

3.1. Alphabet-based criteria

In the sequel, $N = \sum_i K_i$ denotes the total number of emitted signals, where K_i is the number of signals having the same alphabet $\mathcal{A}_i$. This corresponds to the following additional hypothesis about the input signals:

S1. Sources $\mathrm{s}^{(i)} = [s_1^{(i)}, \ldots, s_{K_i}^{(i)}]^{\mathsf{T}}$ belong to a finite alphabet $\mathcal{A}_i$, characterized by d_i complex distinct roots of the polynomial $Q_i(s(n)) = 0$, where d_i corresponds to the total number of possible symbols in the constellation.

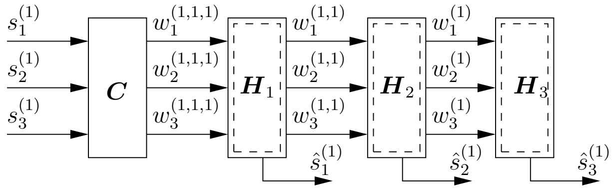

Fig. 1. Classical deflation. Extraction of 3 signals $\{s_p^{(1)}\}_{p=1}^3$, typically (but not necessarily) having the same modulation $\mathcal{A}_1$. Conventional deflation estimates the input signals one by one.

Modulation	$\mathcal{A}$	$Q(s)$
BPSK	$\{-1, +1\}$	$s^2 - 1$
q-PSK	$\{e^{\jmath 2k\pi/q}\}\lvert_{k\in 0,\ldots,q-1}$	$s^q - 1$
QAM-16	$\{\pm1, \pm3\} + \{\pm\jmath, \pm3\jmath\}$	$\sum_{k=0}^4 \alpha_k s^{4k}$

$\alpha_0 = 50625/256, \alpha_1 = 12529/16, \alpha_2 = -221/8,$
$\alpha_3 = 17, \alpha_4 = 1.$

Table 1. Alphabets and associated polynomials of some discrete modulations.

This hypothesis is essential to alphabet-based criteria. For instance, a q-PSK modulated signal s is characterized by the roots of polynomial $Q(s) = s^q - 1$. Thus, each discrete modulation can be associated with an APF criterion, as illustrated by the examples in Table 1.

Considering hypothesis **S1** on the discrete inputs of a MIMO channel, it is possible to perform source extraction by minimizing the following polynomial criterion [5]:

Theorem 1 *: Consider $\mathcal{S}_i$ the set of processes taking their values in alphabet $\mathcal{A}_i$, and $\mathcal{H}_i$ the set of FIR filters. Criterion:*

$$\mathcal{J}_{APF}^{(i)}(\boldsymbol{H}_i, \hat{\mathrm{s}}^{(i)}) = \sum_{n=1}^{K_i} \sum_m \left| Q_i\big(\hat{s}_n^{(i)}(m)\big)\right|^2 \tag{2}$$

*is a contrast function under hypothesis **S1**.*

An APF criterion can be used for classical deflation when the emitted signals have all the same alphabet, i.e., $N = K_1$ and $K_i = 0, \forall i > 1$. However, novel extraction approaches are enabled by the discriminating character of APF criteria, which is stronger than that of traditional principles such as CM and KM. The new approaches consist of extracting the sources with different alphabets in parallel, thus the terms of parallel extraction and parallel deflation, which are explained next.

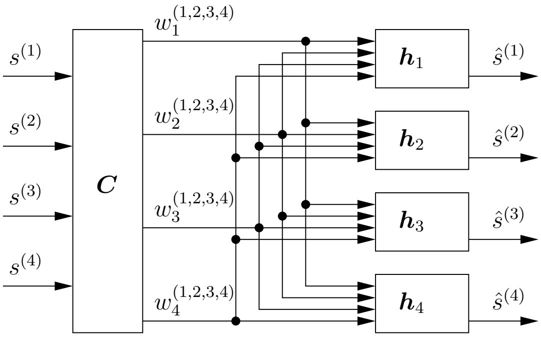

Fig. 2. Parallel Extraction. From the observed sensor output, parallel extraction allows the simultaneous separation of source signals having different modulations.

3.2. Parallel extraction

Parallel extraction can take place when the N emitted signals all have different modulations, i.e., $K_i = 1, \forall i$. Each equalizer is computed from an APF criterion corresponding to one alphabet. Thus, the equalizers for each modulation can be determined in parallel from the observed sensor output. Fig. 2 shows an example of parallel extraction of signals $\{s^{(i)}\}_{i=1}^4$ with alphabets $\{\mathcal{A}_i\}_{i=1}^4$, respectively. Parallel extraction can be considered as a particular case of the more general parallel deflation.

3.3. Parallel deflation

In the general case, the sensor output observes mixtures of M groups of sources where the ith group is composed of K_i signals having the same modulation. Thus we have $N = \sum_{i=1}^M K_i$. Then, it is possible to extract the sources of the same group by means of a deflation approach operating on a criterion matched with the corresponding modulation. This process can be carried out in parallel for other groups having a different modulation and hence their own APF criterion. Consequently, the discriminating property of APF criteria is able to decouple a separating problem of N signals into M extraction problems of K_i sources, $i = 1, \ldots, M$. Contrary to classical deflation, the required diversity for parallel deflation is reduced to $P \approx \max(K_i)$. This diversity improvement offers further advantages in terms of performance (e.g., less error accumulation), computational complexity and cost. Parallel deflation reduces to parallel extraction when $M = N$, so that deflation is no longer required.

4. OPTIMIZATION OF APF CRITERIA

In order to estimate a source with alphabet $\mathcal{A}_i$, contrast function (2) must be minimized with respect to the equalizer

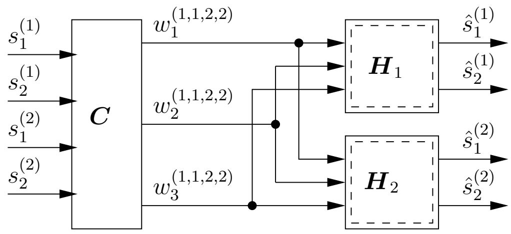

Fig. 3. Parallel deflation in an underdetermined case. The extraction of more sources than sensors is possible with parallel deflation, provided that enough diversity is available for extracting the sources of each alphabet.

tap vector $\boldsymbol{h}$, which is used to extract a single component as in eqn. (1). After a suitable initialization (e.g., via the conventional center-tap filter), the equalizer vector is iteratively updated in the descent direction $\boldsymbol{g}$:

$$\boldsymbol{h}' = \boldsymbol{h} - \mu \boldsymbol{g}$$

In a gradient-based algorithm, we have $\boldsymbol{g} = \nabla \mathcal{J}_{APF}^{(i)}(\boldsymbol{h})$, whereas a Newton-based algorithm would involve the Hessian of $\mathcal{J}_{APF}^{(i)}$ as well.

The interesting feature of APF criteria is that $\mathcal{J}_{APF}^{(i)}(\boldsymbol{h}')$ is a $2q$th-degree polynomial in the step size μ, for constellations composed of q symbols. This feature is not exclusive of APF contrasts, but it is also shared by other equalization criteria such as CM and KM [5]. As a result, steepest descent minimization of contrast (2) can be carried out by finding the optimal step size

$$\mu_{\text{opt}} = \min_{\mu} \arg \mathcal{J}_{APF}^{(i)}(\boldsymbol{h} - \mu \boldsymbol{g})$$

among the roots of the $(2q-1)$th-degree polynomial $\partial \mathcal{J}_{APF}^{(i)}(\boldsymbol{h} - \mu \boldsymbol{g}) / \partial \mu$. In some cases, this root finding can be accomplished algebraically: the APF criterion matched to BPSK signals and the CM criterion are associated with respective 3rd-degree polynomials, solved by Cardano's formula; the normalized KM criterion involves a 4th-degree polynomial whose roots are obtained by Ferrari's formula. The coefficients of these polynomials are simple polynomial functions of the observed data vectors and the current equalizer and gradient vectors [6, 8]. Consequently, the incorporation of the optimal step-size technique only entails a moderate increase in computational complexity. In return, since μ_{opt} yields the global minimum of $\mathcal{J}_{APF}^{(i)}$ along direction $\boldsymbol{g}$, the optimal step-size technique shows an improved robustness against local extrema relative to conventional gradient-descent minimization [9].

After convergence of the equalizer vector, the contribution of the estimated source signal to the observations is

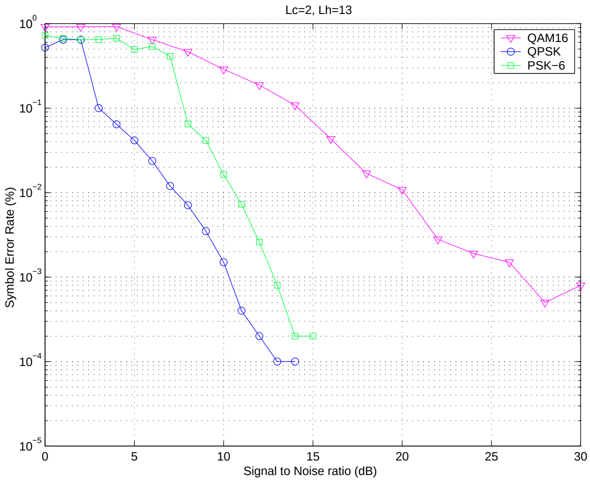

Fig. 4. Parallel extraction of 3 different sources for various SNRs.

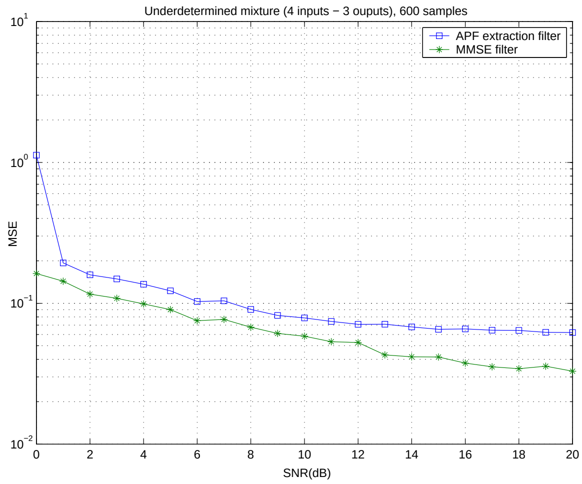

Fig. 5. APF extraction of a QPSK signal from an underdetermined mixture.

calculated and subtracted from the sensor output, to prevent extracting the same source twice. This contribution is easily obtained as the cross-correlation between the estimated source signal and the sensor output vector. To extract the next source, the APF criterion needs to be minimized again, but using the sensor output data without the contribution from the source previously extracted. This process is repeated until all sources with the same modulation have been obtained In parallel deflation, the deflation processes of the different APF criteria can be executed in parallel.

5. PRELIMINARY EXPERIMENTAL RESULTS

5.1. Parallel extraction

In this experiment, $N = 3$ sources with different modulations (QPSK, QAM-16, PSK-6) are mixed by a length-3 equalizer ($L_c = 2$). $P = 3$ noisy observations are processed by a parallel extraction algorithm made up of the APF criteria associated with each modulation. The channel coefficients are randomly drawn from a Gaussian distribution, and so is the noise added to the observations. Fig. 4 summarizes the parallel extraction performance for different signal-to-noise ratios (SNRs).

5.2. Parallel deflation

The second experiment tests a channel spanning two baud periods ($L_c = 1$) and mixing $N = 4$ source signals (2 QPSK and 2 QAM16, i.e., $M = 2$) at the output of only $P = 3$ sensors:

$$C(z) = C_0 + C_1 z^{-1}$$

with

$$C_0 = \begin{bmatrix} -0.66 & -0.19 & 0.65 & 0.92 \\ 0.22 & -0.96 & 0.43 & -0.85 \\ -0.30 & -0.76 & 0.95 & 0.85 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0.75 & -0.98 & -0.75 & -0.38 \\ -0.97 & 0.27 & 0.90 & 0.53 \\ 0.95 & 0.65 & 0.30 & -0.52 \end{bmatrix}$$

Hence, this situation describes the underdetermined mixture context. The extraction of one of the QPSK signals is illustrated in Fig. 5. Note that, despite the hardness of the underdetermined scenario, the APF extraction performance lies very close to the MMSE bound.

6. CONCLUSIONS

The use of contrast functions matched to the signal modulation enables the definition of a novel approach to blind source extraction whereby sources with different constellations can be extracted in parallel, provided that no alphabet be a subset of another. Parallel deflation may prove useful when different modulations coexist in the same transmission environment. Such a scenario is likely in future-generation wireless communication networks, where signal constellations will be dynamically allocated according to the service required and the channel conditions, analogously to the bit-loading schemes used in multicarrier communications [10]. The preliminary experiments reported in this paper are encouraging. More detailed experimental results illustrating the performance of the parallel deflation approach will be presented at the conference.

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