QUASI-PARABOLIC IONOSPHERE MODELING TO TRACK WITH OVER-THE-HORIZON RADAR

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ABSTRACT

As Over-The-Horizon Radars use ionosphere reflection to detect and track targets, they are faced with a possible appearance of multipaths and difficulties to estimate the target ground location. To solve these problems we propose to use a ionosphere model called Multi-Quasi-Parabolic (MQP) model. Introducing the coefficients of the MQP model into a target tracking algorithm leads to nonlinear evolution and measurement equations. Furthermore we only have estimation, through ionosphere measurements, of these parameters values. To take into account MQP parameters in a target tracking algorithm, we propose two different approaches built on the same algorithm: the Unscented Kalman Filter. In the first approach, we use a Joint Unscented Kalman Filter and in the second one we use an Unscented Particle Filter applied on both target state space and MQP parameters space (we could call this a Joint Unscented Particle Filter). We compare these two approaches and present our conclusions illustrating them by numerical simulations on the French Over-The-Horizon Radar Nostradamus.

1. INTRODUCTION

Over-The-Horizon (OTH) radar perform long distance survey for wide areas using ionospheric reflections of electromagnetic waves. The French OTH radar NOSTRADAMUS offers two particularities: a monostatic implementation and elevation angle measurement.

The aim of OTH Radar Nostradamus is to estimate the target's ground coordinates (ground range, ground range rate, azimuth and elevation angle). We investigate the interest of using ionospheric knowledge to achieve this. Thus, we can define two different approaches using either a simplified propagation model or a full ionospheric model describing the propagation.

The first approach has already been studied, particularly in [1]. The second one is generally less attractive because of the elevation angle measurement lack. As OTHR Nostradamus measures this angle, we studied the opportunity of using the ionospheric model called Multi-Quasi-Parabolic model (MQP) [2]. This model involves nonlinear and hard to compute equations. One way to integrate it into tracking algorithm is to use a tracking algorithm derivative-free and able to deal with highly nonlinear equations. We can find these two characteristics in the Unscented Kalman Filter [3]. To take into account uncertainties on MQP parameter values we used this UKF in a joint estimation problem approach, which consists in a simultaneous estimation of the target state and the model parameters, from the data.

In section 2 we briefly describe the UKF principles. The joint estimation problem is introduced in section 3. Section 4 gives a description of the two algorithms used to handle the joint estimation problem. Some results are given in section 5.

2. UNSCENTED KALMAN FILTER

MQP model equations are clearly nonlinear and their too high nonlinearity prevents us from using linearization as a solution, thus Extended Kalman filter cannot be used. Besides Jacobians or Hessians based on these equations would be difficult to implement. These remarks justify the use of an Unscented Kalman Filter (UKF). The UKF built on a Kalman filter uses an Unscented Transformation (UT), to deal with nonlinear problems. This transformation consists in propagating mean and covariance information through nonlinear transformations using a set of weighted ”sigma points”. Besides its derivative freeness (and so its implementation easiness) and the fact that it is able to handle nonlinear problems, it proves to be more accurate than an EKF. Different methods can be chosen to calculate the ”sigma points” and their weights, depending on the aim of the filter (for more details we can refer to [3]).

2.1. UKF equations

Let consider a dynamical target state model

$$\begin{aligned} x_{k+1} &= F(x_k) + v_k \\ y_k &= H(x_k) + w_k \end{aligned}$$

where x_k is the state vector at time k (dimension d_x), v_k a Gaussian process noise with known covariance matrix Q_k, y_k the observed measurement signal, and w_k is the Gaussian observation noise with known covariance R_k. We set $p = 2d_x$.

Initialization:

$$\begin{aligned} \hat{x}_0 &= E(x_0) \\ P_0 &= E\left[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\right] \end{aligned}$$

for $k = 1...\infty$,
Calculate sigma points:
weights ($\forall k$):

$$\begin{aligned} W_0 = &\quad \tfrac{\lambda}{d_x+\lambda} \\ W_i = &\quad \tfrac{1}{2(d_x+\lambda)} \quad i = 1...p \end{aligned}$$

vectors:

$$\begin{aligned}
\alpha &= \sqrt{(d_x+\lambda)} \\
X_{0,k-1} &= \hat{x}_{k-1} \\
X_{i,k-1} &= \hat{x}_{k-1} + \alpha\left(\sqrt{P_{k-1}}\right)_i && i = 1..d_x \\
X_{i,k-1} &= \hat{x}_{k-1} - \alpha\left(\sqrt{P_{k-1}}\right)_{i-d_x} && i = d_x+1..2d_x
\end{aligned}$$

λ is a scaling parameter. $\left(\sqrt{P_{k-1}}\right)_i$ is the i^{th} column of the matrix square root (e.g. lower triangular Cholesky factorization).

Time update:

$$\begin{aligned}
X_{i,k|k-1} &= F(X_{i,k-1}) && i = 0..2d_x \\
\hat{x}_{k|k-1} &= \textstyle\sum_{i=0}^{2d_x} W_i X_{i,k|k-1} \\
Y_{i,k|k-1} &= H(X_{i,k|k-1}) && i = 0..2d_x \\
\hat{y}_{k|k-1} &= \textstyle\sum_{i=0}^{2d_x} W_i Y_{i,k|k-1}
\end{aligned} \tag{1}$$

$$\begin{aligned}
P_{k|k-1} &= \textstyle\sum_{i=0}^{2d_x} W_i (X_{i,k|k-1} - \hat{x}_{k|k-1}) \times ... \\
&\quad (X_{i,k|k-1} - \hat{x}_{k|k-1})^T + Q_k
\end{aligned}$$

Measurement update:

$$\begin{aligned}
P_{y_k} &= \textstyle\sum_{i=0}^{2d_x} W_i (Y_{i,k|k-1} - \hat{y}_{k|k-1}) \times ... \\
&\quad (Y_{i,k|k-1} - \hat{y}_{k|k-1})^T + R_k \\
P_{x_k y_y} &= \textstyle\sum_{i=0}^{2d_x} W_i (X_{i,k|k-1} - \hat{x}_{k|k-1}) \times ... \\
&\quad (Y_{i,k|k-1} - \hat{y}_{k|k-1})^T
\end{aligned}$$

$$\begin{aligned}
K_k &= P_{x_k y_k} P_{y_k}^{-1} \\
\hat{x}_k &= \hat{x}_{k|k-1} + K_k (y_k - \hat{y}_{k|k-1}) \\
P_k &= P_{k|k-1} - K_k P_{y_k} K_k^T
\end{aligned}$$

3. TARGET STATE AND MODEL PARAMETERS JOINT ESTIMATION

3.1. Vector and function definitions

Let consider a target generating m paths, and flying at constant ground range rate (r_g) and azimuth rate. Suppose we know the true MQP model parameters, and we want to estimate the target state. Then from MQP model equations [4] we necessary have to include all paths' elevation angles in the state vector. Furthermore ground range is useless in the filter equations. So the state vector is

$$x_t = \left[r_G, A, \dot{A}, \theta_1, \dot{\theta}_1, ...\theta_m, \dot{\theta}_m\right]^T$$

where A is the azimuth, and θ_i the i^{th} path elevation angle. From each elevation angle we can calculate a ground range. The final target ground range is obtained by a fusion of these results using [5]. OTHR Nostradamus gives the slant range R_s, the slant range rate r_s, the azimuth, and elevation angles. For the i^{th} path we call $y_{p_i} = [R_{s_i}, r_{s_i}, A_i, \theta_i]^T$. The measurement vector is then $y = [y_{p_1}; y_{p_2}; ...; y_{p_m}]$. Function F is defined as:

$$\begin{aligned}
A_{k+1} &= A_k + \hat{A}_k \Delta_t \\
\theta_{i_{k+1}} &= \theta_{i_k} + \hat{\theta}_{i_k} \Delta_t \\
r_{G_{k+1}} &= r_{G_k} \\
\dot{A}_{k+1} &= \dot{A}_k \\
\dot{\theta}_{i_{k+1}} &= \frac{r_{G_{k+1}}}{\frac{\partial R_G}{\partial \theta}(\theta_{i_{k+1}})}
\end{aligned}$$

Function H is so that for each y_{p_i}:

$$\begin{aligned}
R_{s_i} &= R_s(\theta_i) \\
r_{s_i} &= r_G \cos(\theta_i)
\end{aligned}$$

3.2. Joint estimation

We do not know the real MQP model parameters otherwise a *simple* UKF would be sufficient. Furthermore the logic idea to say that the best these parameters are estimated, the best the results are, leads us to take into account a joint estimation of the parameters and the state, knowing the measurements. The state vector is then extended to

$$x = \begin{bmatrix} x_t \\ x_m \end{bmatrix}$$

x_m is the MQP parameters vector (composed of height of the base, height of the top and critical frequency of each ionosphere layer). In this paper we suppose x_m constant in the considered ionospheric area and during the time the target is tracked. In some cases, this assumption may be wrong but the following algorithms may be adapted to variable ionosphere parameters.

4. ALGORITHMS PRESENTATION

We compare in the following sections two algorithms designed for joint estimation: the Joint UKF, and a Joint Unscented Particle Filter.

4.1. Joint UKF

We can briefly describe it saying it is a Joint Extended Kalman Filter where the extended Kalman filter is replaced by a UKF. Basically it means that the UKF is applied on the extended state vector . Parameters used in functions F and H are those estimated in $\hat{x}_{k|k-1}$. For a more detailed description of the joint UKF we can refer to [6].

4.2. Joint estimation using an Unscented Particle Filter

Particle filters for state-space models with the presence of unknown parameters are envisaged and described in [7]. In the context of static parameters, they are known to have poor results because of an impoverishment problem due to the presence of the static parameters in the state vector. To handle this problem we can add small random disturbance, as proposed in [8]. This artificial evolution of the parameters leads to a loss of information because parameters ARE really static. Another proposed approach uses a kernel smoothing of the parameters [9]. To avoid any loss of information [9] suggests to introduce a shrinkage rule in the kernel location. Finally in [7] we can find a method where artificial evolution is directly connected to kernel smoothing with shrinkage.

Because of our extended vector dimension, and as some radar resolutions are large (especially elevation angle's one), a huge number of particles has to be used. To reduce this number and by the way the computational time, we marginalized the target state so that it would be a Rao-Blackwelized Particle filter. Sampling step is thus used only for the MQP parameters, considered as static. To avoid the impoverishment we use the technique presented above and further described below.

Filter equations

$$E(x_{1:k}|Y_{1:k}) = \int x_{1:k} p(x_{1:k}|Y_{1:k}) dx_{1:k}$$

as $p(x_{1:k}|Y_{1:k}) = p(x_{t,1:k}|Y_{1:k}, x_{m,1:k}) p(x_{m,1:k}|Y_{1:k})$

$$E(x_{1:k}|Y_{1:k}) = \int \left(\int x_{1:k} p(x_{t,1:k}|Y_{1:k}, x_{m,1:k}) dx_t \right) \times ... \\ p(x_{m,1:k}|Y_{1:k} dx_{1:k}) dx_m$$

we can re-write this integral as

$$= \int x_{m,1:k} \left(\int x_{t,1:k} p(x_{t,1:k}|Y_{1:k}, x_{m,1:k}) dx_t \right) \times ... \\ p(x_{m,1:k}|Y_{1:k}) dx_{1:k} dx_m$$

With

$$p(x_{m,1:k}|Y_{1:k}) = \frac{p(Y_{1:k}|x_{m,1:k}) p(x_{m,1:k})}{p(Y_{1:k})}$$

we have

$$E(x_{1:k}|Y_{1:k}) = \int \int x_{t,1:k} p(x_{t,1:k}|Y_{1:k}, x_{m,1:k}) dx_t \\ ... \times x_{m,1:k} p(Y_{1:k}|x_{m,1:k}) p(x_{m,1:k}) dx_{1:k} dx_m \frac{1}{p(Y_{1:k})}$$

which can be approximated by

$$E(x_{1:k}|Y_{1:k}) \approx \frac{1}{N} \sum_{n=1}^{N} \left[x_{t,1:k}; x^n_{m,1:k} \right] \\ ... \times p(x_{t,1:k}|Y_{1:k}, x^n_{m,1:k}) p(Y_{1:k}|x_{m,1:k}) \frac{1}{p(Y_{1:k})}$$

where $x^n_{m,1:k} \sim p(x_{m,1:k}), n = 1...N$.
Knowing the MQP parameters, we can deduce the result of the second integral running a basic UKF:

$$\int x_{t,1:k} p(x_{t,1:k}|Y_{1:k}, x^n_{m,1:k}) dx_t \approx \hat{x}^{UKF}_{t,1:k}(n)$$

The normalization term $p(Y_{1:k})$ can be expressed as

$$\begin{aligned} p(Y_{1:k}) = & \int p(Y_{1:k}|x_{m,1:k}) p(x_{m,1:k}) dx_m \\ \approx & \frac{1}{N} \sum_{n=1}^{N} p(Y_{1:k}|x^n_{m,1:k}) \end{aligned}$$

Thus, with the usual particle filter approximations we finally have:

$$\hat{x}_k \approx \sum_{n=1}^{N} \left[x^{UKF}_{t,k}; x^n_{m,k} \right] w^*_k(n) \qquad (2)$$

where

$$\begin{aligned} x^n_{m,k} = & \quad p(x_{m,k}|x^n_{m,k-1}) \qquad n = 1...N \\ w^*_k(n) = & \quad \frac{w_k(n)}{{}^{N}_{n=1}\, p(Y_{1:k}|x^n_{m,1:k})} \\ w_k(n) = & \quad p(Y_k|x^n_{m,k}, Y_{1:k-1}) w_{k-1}(n) \qquad (3) \end{aligned}$$

$$p(Y_k|x^n_{m,k}, Y_{1:k-1}) = \mathcal{N}(Y_k; \hat{Y}^{UKF}_{k|k-1}(n), P_{yy,k|k-1}(n)) \qquad (4)$$

according to [7]:

$$p(x_{m,k}|x^n_{m,k-1}) \sim \\ \mathcal{N}(x_{m,k}; a x^n_{m,k-1} + (1-a)\hat{x}_{m,k-1}, h^2 P_{x_{m,k-1}}) \qquad (5)$$

where $\hat{x}_{m,k-1}, P_{x_{m,k-1}}$ are the Monte Carlo posterior mean and variance matrix computed from the samples $\left\{x^n_{m,k-1}\right\}^N_{n=1}$ associated to the weights $\{w_{k-1}(n)\}^N_{n=1}$. The controlling smoothing parameter h is so that $h^2 = 1 - a^2$ and $a = \frac{3\delta - 1}{2\delta}$ with the discount factor δ typically around 0.95-0.99.
Note: the same UKF is used to calculate $x^{UKF}_{t,k}(n)$, $\hat{Y}^{UKF}_{k|k-1}(n)$ and $P_{yy,k|k-1}(n)$.

Algorithm
Initialization: $x_{t,0}, P_0$ are initialized using measurements. Non-measured values are set to 0. $x_{m,0}$ is initialized using a network of ionospheric sounder.

- For $k = 1...\infty$ (time)
  - for $n = 1 : N$ (particles)
    - ∗ draw $x^n_{m,k} \sim p(x_{m,k}|x^n_{m,k-1})$ according to (5)
    - ∗ run the n^{th} UKF using MQP parameters $x^n_{m,k}$
    - ∗ assign the particle a weight $w_k(n)$ according to (3)
  - END *for n*
  - calculate the sum $s = \sum_{n=1}^{N} w_k(n)$ and normalize $\left(w^*_k(n) = \frac{w_k(n)}{s} \text{ for each } n \epsilon \{1..N\} \right)$
  - calculate $\hat{x}_k$ (2)
  - calculate the empirical covariance matrix S_k of $\left\{x^n_{m,k}, w_k(n)\right\}^N_{n=1}$
  - resample
- END *for k*

Note: Resampling step may not be used at each iteration. The resampling step is a classical one. All details may be found in [10].

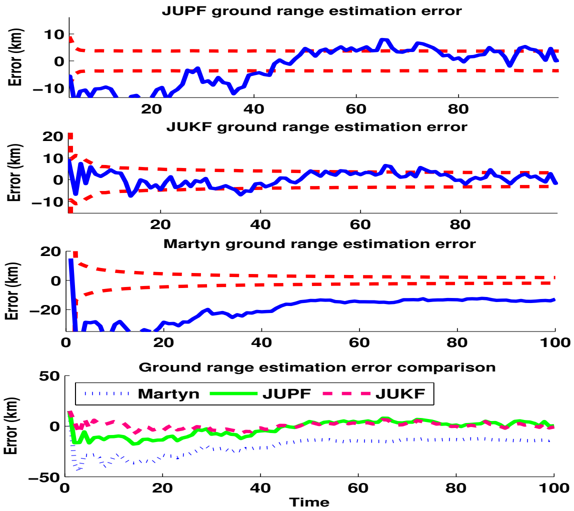

Fig. 1. Convergence example.

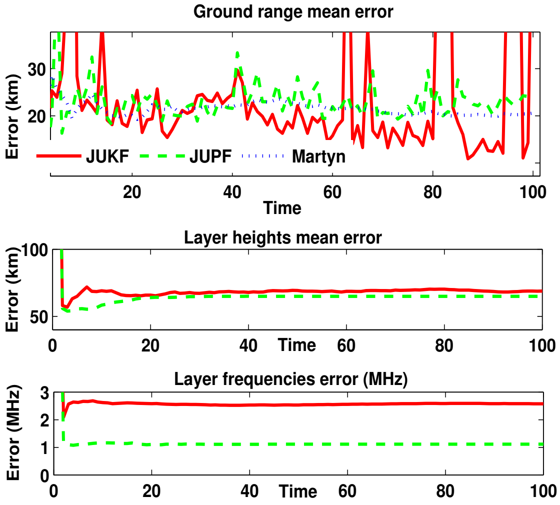

Fig. 2. Ground range and MQP parameters Root Mean Square Error computed over 50 simulations.

5. RESULTS

We give in this section Joint UKF and Joint UPF results (1000 particles are used in the JUPF). All are numerical simulation results based on OTHR Nostradamus characteristics. They are compared with a third method based on an approximation known as Martyn's Theorem [11]: this approximation is independent from ionosphere parameters. In this paragraph we will refer at this method as the Martyn's method. Figure 1 gives a ground range estimation error example for a target settled at $2000km$ from the radar (two paths are generated). On figure 2, we give ground range and parameters root mean square errors computed over 50 independent simulations.

All covariances results are centered on zeros.

5.1. Interpretation

With only 1000 particles, JUPF ground range mean estimations are the same as Martyn's method ones. JUKF is better. Both, JUKF and JUPF estimate better state error covariance matrix than the Martyn's method. This point is interesting for multitarget adaptation. On Figure 2, JUKF peaks are a consequence of the scale parameter λ. When this scale parameter (which determines the spread of the sigma points) is too low numerical problems may occur, but when it is too high and when for some reasons the covariance matrices increase (path disappearance, measurement loss) sigma points spread increases too, and some points may not have physical reality any more. Because of its structure (N particles are used), this problem does not arise for the JUPF.

JUPF parameters estimation is generally better than JUKF one. Even if the JUPF generally improves the knowledge on parameters values (especially on the layer critical frequency values), we can say that parameters estimation is not really efficient. This is due to Nostradamus elevation angle resolution. Under this resolution, we can often find equivalent elevation angle and ionosphere profile that lead to similar slant range. The problem with such equivalent parameters is that they cannot be used to track a different target.

6. CONCLUSIONS

In this paper we have investigated the interest of using MQP model parameters directly in OTHR tracking algorithms. We have adopted two methods based on the use of an Unscented Kalman Filter with a joint estimation approach. The Joint Unscented Kalman Filter we have implemented showed good results though scaling problems may occur. These problems were easier to handle with the JUPF. As both algorithms showed at least equivalent results as an ionosphere independent method (ground range and state error covariance matrix are generally better) we can conclude that even with Nostradamus elevation angle resolution it is worth using MQP parameters directly in tracking algorithms. Joint approach main interest was to enable to take into account parameter uncertainties. Indeed, with Nostradamus radar resolutions it reveals to be difficult to estimate accurately these parameters: algorithms may converge toward an equivalent ionosphere profile. Nevertheless, on this point, the JUPF generally gave better results than the JUKF. Future work will focus on the interest of using these MQP parameters to track in a multitarget context.

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