RAPID EQUALISATION FOR A HIGH INTEGRITY BLUETOOTH RECEIVER

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ABSTRACT

The Bluetooth standard has a lax specification for parameters such a carrier frequency and modulation index. Research has shown that significant system degradation can occur even when operating within the permitted limits. Hence, a high-integrity Bluetooth receiver will need to compensate for this mismatch. Where multipath propagation exists, it will undermine parameter synchronising algorithms, and taking into account the limited burst lengths of Bluetooth packets, it is essential that equalisation occurs fast enough so as to give algorithms downstream sufficient time to converge, and minimise information loss.

The coloured nature of Bluetooth signals and the possibility of a carrier frequency offsets, limit the choice of equaliser algorithms that can guarantee fast convergence. As a remedy, in this paper we propose the normalised sliding window constant modulus algorithm (NSWCMA), and determine regularisation factors that will optimise speed. We also show that the NSWCMA can outperform the minimum mean square error solution in terms of bit error ratio.

1. INTRODUCTION

This paper extends work carried out in the University of Southampton to build a high-integrity Bluetooth receiver for software defined radio, that is robust to common signal adversities. Already we have proposed a new technique to reduce the complexity of a high-performance matched filter bank (MFB) receiver for Bluetooth signals by up to 90% [1], and derived novel algorithms to correct carrier frequency and modulation index offsets [1, 2], which could otherwise cause serious system degradation even when operating within the limits permitted by the Bluetooth standard [3, 1].

However, algorithms meant to rectify frequency errors in Bluetooth will be undermined if the channel is even moderately dispersive, and hence, they are likely to be deployed downstream from an equaliser. Therefore the equaliser should converge quickly to give sufficient time to other signal processing blocks to complete their tasks, ideally within the time it takes to receive the mutually known 72-bit access code, and thus prevent information loss. This problem is compounded by the fact that a master node may receive alternate packets from different transceivers [4], each having different carrier frequency and modulation index, and experiencing different channel conditions. Therefore fresh adaptive processing begins with each arriving packet, and a slow converging equaliser will impede the progress of other parameter synchronising blocks, and increase the bit error ratio (BER).

As a remedy we propose the normalised sliding window constant modulus algorithm (NSWCMA) [5, 6], which is akin to the affine projection algorithm (APA) [7, 8, 9], but is based on a constant modulus (CM) criterion [10, 11]. The CM criterion is appropriate because it is not susceptible to carrier frequency offsets, which we must assume exist, and which would not have been corrected prior to equalisation. However, we note that implementation of the NSWCMA requires the inversion of the signal correlation matrix, and since Bluetooth signals are coloured, this matrix is ill-conditioned [12]. A method of regularisation can resolve this [13, 14]. Hence, in this paper we offer some insight into important design considerations involved in determining the best regularisation factor, and do this exemplarily for a baseband model of a Bluetooth signal.

We also show through simulation that the BER performance attainable with the NSWCMA, surpasses that provided by the minimum mean square error (MMSE) solution. Hence, for clarity, without making the common assumption of a white transmit signal, the MMSE solution used is derived.

The structure of this paper is as follows: Following this introduction, the transmission system model is developed in Sec. 2, after which the MMSE solution and regularised NSWCMA is derived in Sec. 3. Simulation results are presented and discussed in Sec. 4, before concluding in Sec. 5.

2. TRANSMISSION SYSTEM MODEL

Transmit signal generation explained in this section is depicted in Fig. 1. The modulation scheme specified for Bluetooth is binary Gaussian frequency shift keying (GFSK) [4]. It involves expanding a bipolar data symbol sequence $p[k] \in \{\pm 1\}$ by a factor of N, and passing the result through a Gaussian filter with an impulse response $g[n]$, a bandwidth-time product K_{BT}=0.5, and a support length of $L_g = 3$ symbol periods. The filter output is scaled by $2\pi h$, where h is a modulation index that may lie in the range (0.28,0.35) [4], thus yielding a instantaneous angular frequency signal

$$\hat{\omega}[n] = 2\pi h \sum_{k=-\infty}^{\infty} p[k] g[n - kN] \quad ,$$

where k and n stand for the symbol and chip indices respectively. The phase of the baseband version of the transmitted signal,

$$s[n] = \exp\{j \sum_{\nu=-\infty}^{n} \hat{\omega}[\nu]\} = \prod_{\nu=-\infty}^{n} e^{j\hat{\omega}[\nu]} \quad ,$$

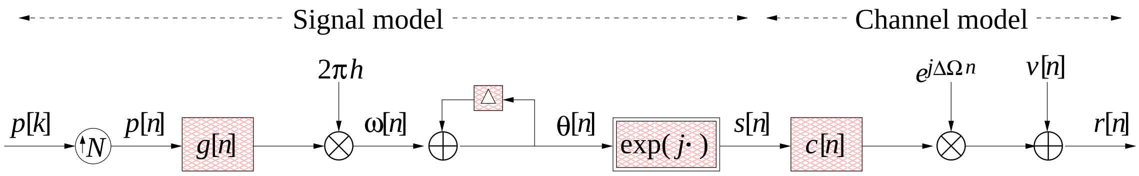

Fig. 1. Transmission system model.

is determined as the cumulative sum over all previous frequency values $\hat{\omega}[n]$.

Furthermore, the signal $s[n]$ is corrupted by additive Gaussian white noise (AWGN) $v[n]$, a stationary dispersive channel impulse response (CIR) $c[n]$, of 300 ns root mean square (RMS) [15, 16], and a normalised angular carrier frequency offset of $\Delta\Omega = \frac{2\pi\Delta f_c}{NR}$, whereby R is the data rate and Δf_c=75 kHz is the maximum carrier frequency offset permitted in Bluetooth [4]. Hence, the received signal can be expressed as

$$r[n] = \sum_{\lambda=0}^{L_c-1} c[\lambda]\, s[n-\lambda]\, e^{j\Delta\Omega n} \;+\; v[n] \quad , \qquad (1)$$

with L_c being the length of the CIR.

3. EQUALISATION

We wish to equalise adverse channel effects in presence of carrier frequency errors, this rules out phase sensitive equalisation algorithms like the popular least mean square (LMS) algorithm, which would misinterpret the resulting phase changes as a rapidly varying channel, and hence, would never converge. The constant modulus algorithm (CMA), is suitable because it is insensitive to signal phase [10, 11].

Unfortunately correlation introduced by the Gaussian filter between signal samples causes unbearably slow convergence, and Sec. 1 highlighted why this would decrease the integrity of a high-performance Bluetooth receiver. Several methods have been proposed to speed up the CMA when faced with correlated inputs [17, 18, 19, 20], but these techniques assume that the transmit signal $s[n]$ is white, and that the intersymbol interference (ISI) is only introduced by the channel, they are therefore not suitable for Bluetooth. The NSWCMA however, is based in the CM criterion, but imposes more constraints than the classical CMA, and as a result achieves faster convergence [5, 6]. It is this method that is proposed here, but regularisation shall be performed to cater for correlated transmit signal $s[n]$. However, prior to formulation of the NSWCMA in Sec. 3.2, the MMSE solution, which serves as a useful performance benchmark is derived in Sec. 3.1.

3.1. Formulation of the MMSE

The MMSE (or Weiner-Hopf) solution refers to the equaliser coefficient vector $\mathbf{w}_{\rm opt}$ for which the mean square error $\xi_{\rm MSE}$ is minimised [21, 22, 23]. Stated mathematically

$$\xi_{\rm MSE} \;=\; \mathcal{E}\left\{(z[n]-s[n-\delta])^2\right\} = \min \quad , \qquad (2)$$

where $z[n]$, or short z_n, is the equaliser output and δ is the channel-equaliser delay. We define the channel and equaliser coefficient vectors as

$$\mathbf{c} = [\, c_0, c_1, \cdots, c_{L_c-1}\,]^H \quad ,$$

and

$$\mathbf{w}[n] = \mathbf{w}_n = [\, \mathrm{w}_0[n], \mathrm{w}_1[n], \cdots, \mathrm{w}_{L_\mathrm{w}-1}[n]\,]^H \quad ,$$

and denote the vector of samples of $s[n]$ and $v[n]$ contributing to the nth equaliser output $z[n]$, as

$$\mathbf{s}[n] = \mathbf{s}_n = [\, \mathrm{s}_0[n], \mathrm{s}_1[n], \cdots, \mathrm{s}_{L_\mathrm{w}+L_c-1}[n]\,]^T \quad ,$$

and

$$\mathbf{v}[n] = \mathbf{v}_n = [\, v_0[n], v_1[n], \cdots, v_{L_\mathrm{w}-1}[n]\,]^T \quad ,$$

with $(\cdot)^T$ and $(\cdot)^H$ symbolising the transpose and hermitian transpose respectively. Hence the channel input signal and AWGN covariance matrices can be defined as

$$\mathbf{S} = \mathcal{E}\left\{\mathbf{s}[n]\mathbf{s}[n]^H\right\} \qquad \in \mathbb{C}^{L_\mathrm{w}+L_c \times L_\mathrm{w}+L_c}$$

and

$$\mathbf{V} = \mathcal{E}\left\{\mathbf{v}[n]\mathbf{v}[n]^H\right\} \qquad \in \mathbb{C}^{L_\mathrm{w} \times L_\mathrm{w}}$$

respectively, while the channel convolutional matrix is given by

$$\mathbf{C} \;=\; \begin{bmatrix} \mathbf{c}^H & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{c}^H \end{bmatrix} \qquad \in \mathbb{C}^{L_\mathrm{w} \times L_\mathrm{w}+L_c} \quad .$$

Now if we designate a pinning vector as

$$\mathbf{\Delta}_i = [\, 0\,,\; \cdots\,,\; 0\,,\; 1\,,\; 0\,,\; \cdots\,,\; 0\,]^T \quad ,$$

with a '1' in the ith position, and assume that $s[n]$ and $v[n]$ are statistically independent random processes, then the following formulation arises from (2)

$$\begin{aligned} \xi_{\rm MSE} &= \mathcal{E}\left\{(z_n - \mathbf{\Delta}_i^H \mathbf{s}_n)^2\right\} \\ &= \mathcal{E}\left\{(\mathbf{w}^H(\mathbf{C}\mathbf{s}_n+\mathbf{v}_n) - \mathbf{\Delta}_i^H\mathbf{s}_n)(\mathbf{w}^H(\mathbf{C}\mathbf{s}_n+\mathbf{v}_n) - \mathbf{\Delta}_i^H\mathbf{s}_n)^H\right\} \\ &= \mathbf{w}^H\mathbf{C}\mathbf{S}\mathbf{C}^H\mathbf{w} + \mathbf{w}^H\mathbf{V}^H\mathbf{w} - \mathbf{w}^H\mathbf{C}\mathbf{S}\mathbf{\Delta}_i - \mathbf{\Delta}_i^H\mathbf{S}\mathbf{C}^H\mathbf{w} \\ &\quad + \mathbf{\Delta}_i^H\mathbf{S}\mathbf{\Delta}_i \quad . \end{aligned}$$

The minimum point of $\xi_{\rm MSE}$ can now be obtained by equating its derivative to zero as follows

$$\frac{\partial \xi_{\rm MSE}}{\partial \mathbf{w}^*} \;=\; \mathbf{C}\mathbf{S}\mathbf{C}^H\mathbf{w} + \mathbf{V}^H\mathbf{w} - \mathbf{C}\mathbf{S}\mathbf{\Delta}_i = \mathbf{0} \quad ,$$

so that

$$\mathbf{w}_{\rm opt} \;=\; \min_i (\mathbf{C}\mathbf{S}\mathbf{\Delta}_i)(\mathbf{C}\mathbf{S}\mathbf{C}^H + \mathbf{V}^H)^{-1} \quad ,$$

and in uncorrelated noise conditions this can be further simplified to

$$\mathbf{w}_{\rm opt} \;=\; \min_i (\mathbf{C}\mathbf{S}\mathbf{\Delta}_i)(\mathbf{C}\mathbf{S}\mathbf{C}^H + \sigma_v^2\mathbf{I}_{L_\mathrm{w}})^{-1} \quad ,$$

where σ_v^2 is the variance of the noise and $\mathbf{I}_{L_\mathrm{w}}$ is an $L_\mathrm{w} \times L_\mathrm{w}$ identity matrix.

3.2. Formulation of the NSWCMA

To simplify the derivation of the NSWCMA we first assume knowledge of the desired equaliser output sequence $d[n]$, or short d_n, and similar to the APA [14] we aim to solve the constrained optimisation (minimisation) problem [24], whereby

$$||\Delta \mathbf{w}_{n+1}|| = ||\mathbf{w}_{n+1} - \mathbf{w}_n|| = \min \quad , \tag{3}$$

is minimised, subject to the constraints

$$\mathbf{r}_n^H \mathbf{w}_{n+1} = d_n^* \quad , \tag{4}$$
$$\mathbf{r}_{n-1}^H \mathbf{w}_{n+1} = d_n^* \quad , \tag{5}$$
$$\vdots$$
$$\mathbf{r}_{n-P+1}^H \mathbf{w}_{n+1} = d_n^* \quad , \tag{6}$$

where P is the window size, $||\mathbf{w}||^2 = \mathbf{w}^H \mathbf{w}$, and

$$\mathbf{r}[n] = \mathbf{r}_n = [\, \mathrm{r}_0[n], \mathrm{r}_1[n], \cdots, \mathrm{r}_{L_\mathrm{w}-1}[n] \,]^T \quad .$$

The system of equations (4)-(6) can conveniently be expressed in matrix notation as

$$\mathbf{R}_n^H \mathbf{w}_{n+1} = \mathbf{d}_n^* \quad ,$$

where

$$\mathbf{R}_n = [\, \mathbf{r}_n, \mathbf{r}_{n-1}, \cdots, \mathbf{r}_{n-P+1} \,] \in \mathbb{C}^{L_\mathrm{w} \times P} \quad ,$$

and

$$\mathbf{d}_n = [\, d_n, d_n, \cdots, d_n \,]^T \in \mathbb{C}^{P \times 1} \quad , \tag{7}$$

so that the corresponding error vector is given by

$$\begin{aligned} \mathbf{e}_n^* &= \mathbf{d}_n^* - \mathbf{R}_n^H \mathbf{w}_n \quad , \\ &= \mathbf{R}_n^H \Delta \mathbf{w}_{n+1} \quad , \end{aligned}$$

The minimum norm solution as demanded in (3) is given by the pseudo-inverse of $\mathbf{R}_n^H$ [25, 13]. If the system of equations is underdetermined $(P < L_\mathrm{w})$ the right sided pseudo-inverse

$$(\mathbf{R}_n^H)^\dagger = \mathbf{R}_n (\mathbf{R}_n^H \mathbf{R}_n)^{-1} \quad , \tag{8}$$

is required, otherwise the left sided pseudo-inverse

$$(\mathbf{R}_n^H)^\dagger = (\mathbf{R}_n \mathbf{R}_n^H)^{-1} \mathbf{R}_n \quad , \tag{9}$$

is appropriate. Hence the equaliser update equation is given by

$$\begin{aligned} \mathbf{w}_{n+1} &= \mathbf{w}_n + \mu \Delta \mathbf{w}_{n+1} \quad , \\ &= \mathbf{w}_n + \mu (\mathbf{R}_n^H)^\dagger \mathbf{e}_n^* \quad , \end{aligned} \tag{10}$$

where $\mu \in [0, 1)$ is an iteration step size.

To overcome the assumption of the availability of a reference signal at the receiver, which was necessary for the derivation above, we now relax the constraints of (4)-(6) and only insist on a constant modulus, such that $|d_n| = 1$. Hence, we set $d_n = \frac{z_n}{|z_n|}$, where z_n is the equaliser output at the nth time instance. The NSWCMA uses this result for d_n as the elements of $\mathbf{d}_n$ in (7).

3.3. Regularisation of the NSWCMA

It is obvious from (8), (9) and (10) that implementing the NSWCMA involves inversion of matrix $\boldsymbol{R}_n = \mathbf{R}_n^H \mathbf{R}_n \in \mathbb{C}^{P \times P}$ or $\boldsymbol{R}_n = \mathbf{R}_n \mathbf{R}_n^H \in \mathbb{C}^{L_\mathrm{w} \times L_\mathrm{w}}$, once per iteration. However, the correlated nature of $r[n]$ means that this matrix is ill-conditioned [12]. Fortunately a reasonable solution is possible if regularisation is applied so that

$$\begin{aligned} \boldsymbol{R}_n &= \mathbf{R}_n^H \mathbf{R}_n + \rho \mathbf{I}_P \quad , \quad \text{or} \\ \boldsymbol{R}_n &= \mathbf{R}_n \mathbf{R}_n^H + \rho \mathbf{I}_{L_\mathrm{w}} \quad , \end{aligned}$$

where ρ is a real and constant regularisation factor.

Regularisation stabilises the solution of $(\boldsymbol{R}_n)^{-1}$, but the choice of ρ will influence the accuracy of the result [25]. Efforts to compute a pseudo-optimal regularization factor for the APA are reported in [26], but this is mainly theoretical because it involves the complex task of estimating system mismatch, while a reduced-cost implementation in [27], maintains the assumption that $s[n]$ is white, which is not the case in Bluetooth. Therefore an appropriate range for ρ will be determined by experiments performed in Sec. 4.

4. SIMULATION RESULTS AND DISCUSSION

The importance of selecting an appropriate regularisation factor is established in this section, and this is exemplary shown for a baseband model of a Bluetooth signal.

4.1. Simulation Model

Transmit signal generation and signal flow is according to Sec. 2 and portrayed in Fig.1, with $N = 2$, $K_{BT} = 0.5$ and $h = 0.35$ in order to simulate a Bluetooth signal [4]. The channel $c[n]$ was derived via discretisation of the Saleh-Valenzuela indoor propagation model [28], and has an approximate RMS value of 300 ns, thereby typifying a medium to large sized office [15, 16], in which Bluetooth transceivers would be expected to operate. The MMSE solution was derived as in Sec. 3.1, it was equal in length to the NSWCMA equaliser, with i being its central index. When applicable the maximum normalised carrier frequency and modulation index offsets, acceptable in Bluetooth networks, of $\Delta\Omega = 0.075\pi$ and $\Delta h = 0.07$ respectively are employed. Equaliser learning curves plotted in this section are an ensemble of several simulation runs, while BER performance was determined using an efficient realisation of the MFB receiver [1], with a 3-bit observation interval.

4.2. Results and Discussion

Justification for a CM equalisation algorithm as opposed to a training based phase sensitive alternative is provided in Fig. 2, which shows that the normalised least mean square (NLMS) algorithm collapses even when operating within limits of carrier frequency offset permitted in Bluetooth. Therefore, a high integrity receiver cannot rely on NLMS if there is a possibility that such an offset can exist. On the other hand, the same figure demonstrates that NSWCMA operates equally irrespective of $\Delta\Omega$.

Unfortunately NSWCMA with $P = 1$ requires almost 10^4 symbol periods to converge, this is too slow taking into account the limited burst lengths in a Bluetooth network (even for DH5 packets) [4]. A larger P results in a quicker convergence speed,

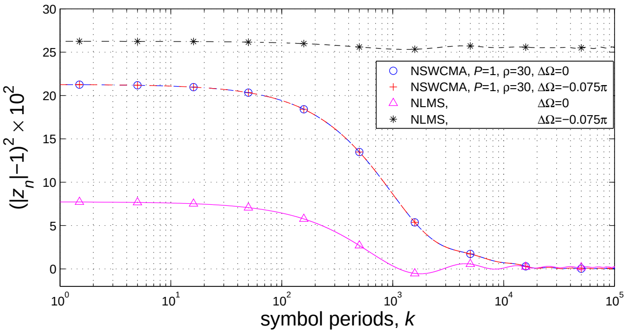

Fig. 2. Equaliser error plots when $L_{\mathrm{w}} = 64$ and $\mu = 0.5$, for NSWCMA ($P = 1$) vs. NLMS.

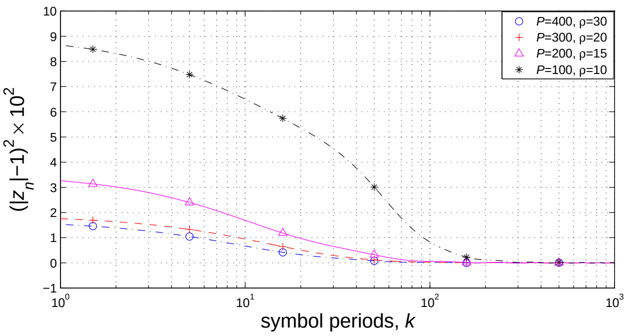

Fig. 3. Increase in equaliser convergence speed with P, when $L_{\mathrm{w}} = 64$, and $\mu = 0.5$.

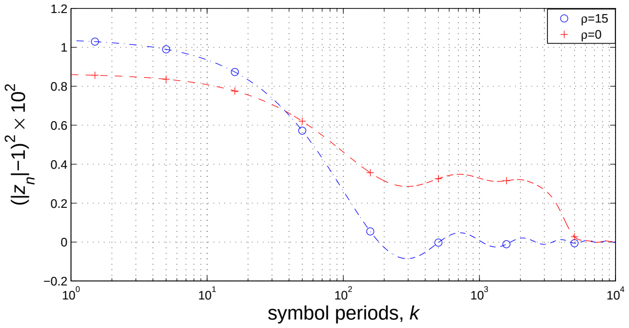

Fig. 4. Effect of regularisation on the NSWCMA equaliser when $P = 200$, $L_{\mathrm{w}} = 64$, and $\mu = 0.5$.

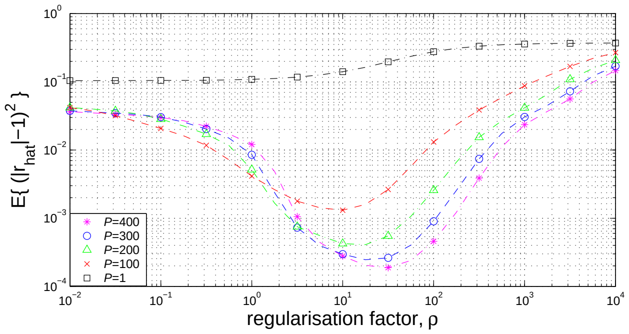

Fig. 5. Relationship between mean equaliser error and ρ after 500 bit periods when $L_{\mathrm{w}} = 64$ and $\mu = 0.1$.

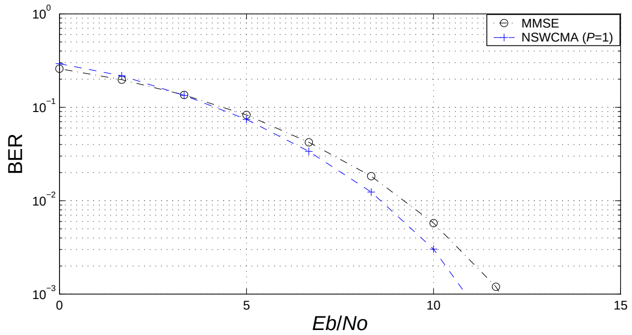

Fig. 6. BER performance with equalisation.

and this is portrayed in Fig. 3. An SDR hardware platform implementing relatively complex standards such as IEEE 802.11b or HiperLAN/2, will have excess computational capacity when running Bluetooth [1, 29]. Therefore in such cases, within limits, the added complexity due to large P is negligible, and the desired speed can be achieved.

Equaliser error plots in Fig. 4 give an idea of the magnitude of the problem that results from the attempt to invert the ill-conditioned matrix $\mathbf{R}_n^H\mathbf{R}_n$ or $\mathbf{R}_n\mathbf{R}_n^H$ in (8) or (9) respectively. The instability of the solution prevents a quick convergence. In this example $P = 200$, and it takes approximately 5000 bit periods to converge without regularisation, or about 25 times what it takes when $\rho = 15$. This will degrade performance, especially in cases where packet lengths are short, and the channel is time varying.

Bearing in mind the short burst lengths of Bluetooth signals, and the necessity that parameter synchronising blocks are given a sufficiently long equalised signal to converge, it is critical to select a value for ρ that ensures fastest convergence. Results in Fig. 5 depict an ensemble of the equaliser error plotted against ρ for a variety of window lengths. In this case only 500 bit periods where permitted for adaptation before computing the error. It is observable from Fig. 5 that the value of ρ that represents the minimum error increases with P, and is roughly 10, 15, 20 and 30 when P is 100, 200, 300 and 400 respectively. The range of acceptable ρ will depend on the desired speed and the value of P. In Fig. 5 a higher P offers lower equaliser error because it would have converged more completely after 500 bit periods.

Another interesting finding in our experiments, depicted in Fig. 6, is that at the NSWCMA requires 1 dB less than the MMSE solution to attain a BER of 10^{-3}. This result supports the argument that the minimum bit error ratio (MBER) cost function is totally different from the mean square error ξ_{MSE} or CM cost function, and the criterion with a minimum closest to that of the MBER will do better in terms of BER [30]. To date no theoretical MBER cost function has been derived for GFSK to confirm this finding, but is likely to be a focus of our future research.

5. CONCLUSION

A high integrity receiver for Bluetooth signals will require rapid equalisation to ensure parameter synchronising algorithms downstream are able to converge quickly, and prevent information loss. The probability of a significantly large carrier frequency offset makes LMS and its derivatives unsuitable, while popular equalising procedures are too slow with correlated input signals like Bluetooth. The NSWCMA attains much quicker convergence, provided an appropriate regularisation factor is chosen. Furthermore, for Bluetooth, the NSWCMA exceeds the BER performance of the MMSE.

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