

A FIRST LOOK AT THE HIDDEN COST OF RANDOM NODE PLACEMENT IN WIRELESS SENSOR NETWORKS

Jean-François Chamberland

Department of Electrical Engineering
Texas A&M University

ABSTRACT

While wireless sensor networks are often envisioned to be composed of a collection of nodes scattered inadvertently in an environment, the implications of random node placement in terms of system resource requirements remain mostly unexplored. This work studies resource consumption in wireless sensor networks in the context of decentralized detection. It is argued that, for most detection problems, the number of bits necessary to capture most of the information contained in an observation is small and insignificant when compared to the communication overhead associated with data transfer. Message compression therefore plays a limited role in minimizing resource consumption at each node. The communication rate and the positioning of individual nodes are likely to have a much greater impact overall. In particular, it is shown that for typical dispersive environments, the random placement of wireless nodes may result in an average transmit power that is 6 to 24 times the power required to communicate on a regular lattice. This excessive power consumption may affect the coverage of a system, its life expectancy, the probability of error at the fusion center, and the number of nodes required for a system to operate properly. These preliminary results underscore the importance of planning in the deployment of sensor systems.

1. INTRODUCTION

Networks of wireless nodes vary in flavor and purpose. The precise pattern in which information flows on a wireless network may depend on the architecture of the system, the capabilities of the nodes, and the communication protocol used. For example, a self-configuring wireless network is a distributed system where nodes are scattered randomly in an environment and then cooperate with one another to establish a communication infrastructure. Self-configuring networks are especially useful in hostile or vast environments, where the precise positioning of individual sensor nodes is impossible or impractical. Such systems are flexible in operation, enabling scalability in the network and allowing for nodes to be scattered inadvertently in a designated area. The price for this greater flexibility is a much

more involved communication mechanism, with substantial overheads. Communication aspects of self-configuring networks include topology management, node identification, and the choice of a routing policy. For instance in ad hoc networks, nodes may successively assume the roles of sensors, relays, and routers. Of course, the choice of a strategy for multihop routing between a source and its destination may depend on the ultimate goal of the system.

Alternatively, a sensor network can be deployed with precision, positioning every node separately. Carefully deployed sensor systems form a connected graph where nodes can communicate with one another through various paths. A communication infrastructure is then established by fixing a specific route through the graph for every source-destination pair. In a network where data is to converge from the sensors to a single destination, this communication infrastructure is greatly simplified. The information flows from the nodes to the fusion center in a unique, straightforward manner.

For static wireless sensor networks, the overhead associated with establishing a communication infrastructure is typically negligible since routing tables are set during the initialization phase of the system and remain essentially fixed thereafter. It is therefore often argued or implied that node placement only plays a minor role in determining the performance of a system. Moreover, in decentralized detection, the communication infrastructure is frequently simplified or abstracted to the most basic topology, namely the parallel architecture. Indeed, the parallel architecture where each node communicates directly with the fusion center has received much attention in the literature [1, 2, 3]. While this framework leads to significant insights about the design of decentralized sensor systems, it may be too simplistic for a thorough performance analysis of resource constrained wireless sensor systems in the context of inference problems. This is precisely the problem we consider next.

The goal of this paper is twofold. First, we argue that communication overheads such as packet headers play an important role in the design of quantization strategies for decentralized detection. Second, we study the impact that random node placement may have on the performance of wireless sensor systems. More specifically, we examine the

relation between node placement and overall performance for decentralized detection problems in power constrained wireless sensor systems. Power consumption has been recognized as one of the key aspects in the conception of sensor networks [4, 5, 6]. Wireless nodes may be powered by small batteries or photocells. Alternatively, they may have to generate their own power from ambient vibrations and acoustic waves through the use of micro electro-mechanical resonators and piezoelectrics. Regardless of their power source, nodes tend to operate on small energy budgets. This underlines the importance of energy awareness in the design of wireless sensor networks. Energy-efficient designs translate into prolonged lifetimes for the corresponding systems and, potentially, more information at their fusion centers.

2. SENSOR NODES

The detailed operation of a sensor node is slightly involved and may vary across platforms. Fortunately, the power consumption characteristics of a generic node can be abstracted to a few elementary considerations [7]. When the communication unit of a sensor node is powered up, a significant amount of energy is expended in the process. The transmission of a message typically draws much more power from the battery than local computations. The operations that take place at the onset of a wireless data transfer further exacerbate this demand on sensor resources. Indeed packet transmission involves acquisition, synchronization, and the transfer of information; each of these operations requires energy.

In the context of decentralized detection, several studies point to the fact that most of the information provided by an observation can be compressed to a very few bits [8, 9, 10, 11]. In other words, the performance loss due to quantization decays rapidly as the number of information bits increases. Packet can therefore be assumed to carry only a few information bits. The exact number of bits per packet is unlikely to be a significant factor in energy consumption in view of the operations that take place when a wireless connection is established, and also taking into consideration the size of a packet header. A similar phenomenon can be observed in other services and applications such as voice over IP, interactive games, and messaging, where the payload of a packet is nearly of the same size or even smaller than its header [12, 13]. It is safe to assume that once a communication link is established between two sensor nodes, the information content of an observation can be transferred essentially unaltered. This characteristic leads to an all-or-nothing model akin to that proposed by Rago et al [14].

Significant energy savings can be achieved, not so much by quantizing observations coarsely, but rather by reducing the energy cost of every transmission. Below, we adopt an all-or-nothing transmission model with the understanding

that only a few bits are necessary to carry almost all the information content of an observation.

3. WIRELESS CHANNEL

In this work, sensor nodes are assumed to communicate with one another over wireless channels. Our goal is to study how fluctuations in channel gains attributable to random node placement impact the expected power consumption of a sensor network. Channel variations in a wireless environment are primarily due to the distance between the transmit and receive antennas, the presence of scatterers, and multipath effects. To characterize these variations, we adopt a frequency non-selective (flat) fading channel model, which requires that the bandwidth of the transmitted signal be small [15]. The received signal is given by

$$y(t) = g(d)\alpha(t)s(t) \quad (1)$$

where $g(d)$ represents the mean path gain and $\alpha(t)$ is a random variable that accounts for the unpredictable nature of a wireless connection. Note that in static sensor networks, as opposed to mobile communication, the value of $\alpha(t)$ will only vary with time if the structure of the environment is changing. In particular, variations are not induced by the mobility of the nodes and, as such, $\alpha(t)$ is more likely to stay constant than to vary with time.

For a specific system implementation which may include a standard packet format, a bandwidth constraint, and a modulation and error correcting scheme; the reliable operation of the system can be insured by transmitting with enough power for the received signal to exceed a minimum signal-to-noise ratio (SNR) threshold. Given channel state information (CSI) and the ability to control power, this requirement translates into a power control strategy for which the minimum admissible power level is used during every transmission.

In the context of static sensor networks, it is practically impossible to use opportunistic transmission schemes for which data is sent only when the corresponding wireless channel is good. The communication channel between two sensor nodes remains essentially constant over long periods of time since nodes are immobile. Furthermore, adaptive compression schemes where the quality of the wireless channel determines the level of compression applied to a packet only offer minimum gain in decentralized detection because packet payloads are already small compared to headers, as discussed above. A good power control strategy for a sensor network where every transmitter has CSI is to invert the effect of the wireless channel to meet a minimum SNR requirement at the receiver. Efficient routing may help save power, whereas delaying transmissions in the hope of a better channel quality in the future does not. When CSI is not available at the transmitters, the power control problem

becomes far more complex [16]. Nevertheless, given the static nature of the networks under consideration, the availability of CSI at the transmitters seems to be a reasonable assumption.

4. PERFORMANCE ANALYSIS

Suppose that the system under consideration is a static wireless sensor network where CSI is available at every node. Then, arguably, each node will attempt to reduce its power consumption by using the minimum transmit power level necessary to meet a prescribed SNR threshold γ at the antenna of the destination node. Under the propagation model of (1), the expected power used by the transmitting node is given by

$$\bar{P}_t(d) = \mathbb{E} \left[\frac{\gamma N_0}{g^2(d)\alpha^2(t)} \right] = \frac{1}{g^2(d)} \mathbb{E} \left[\frac{\gamma N_0}{\alpha^2(t)} \right] \quad (2)$$

where N_0 denotes the variance of the communication noise. Interference among nodes is assumed to be maintained at a negligible level by time-division or frequency-division multiplexing. While the second term on the right-hand-side of (2) depends on the characteristics of the wireless environment, the mean path gain $g(d)$ is determined solely by the relative location of the communicating nodes. If a node and its neighbors are in close proximity then $g(d)$ is large and, consequently, little transmit power is required. On the other hand, if the distance between two nodes increases, then the gain $g(d)$ becomes smaller and much higher transmit power levels are necessary. It is this term, $g(d)$, that the random placement of nodes affects the most. Moreover, because of the strong non-linearity intrinsic to wireless communication, random node placement affects power consumption adversely. This is exemplified below.

Consider a wireless environment with a mean path gain given by $g(d) = ad^{-b}$, where a and b are constants. This mathematical representation is frequently used to model the path gain of indoor and outdoor wireless channels [15]. In free space, signal strength decays as distance squared; in dispersive environments, the decay in mean signal power is even more dramatic and typically found to be $g^2(d) = a^2d^{-2b}$, where $2b \in [3, 4]$. If sensor nodes are placed randomly along a communication path, the distance between consecutive nodes becomes a random variable with probability distribution function $f(d)$. The expected power per transmission can be computed as follows;

$$\begin{aligned} \bar{P}_{\text{ran}} &= \mathbb{E} [\bar{P}_t(d)] = \mathbb{E} \left[\frac{1}{g^2(d)} \right] \mathbb{E} \left[\frac{\gamma N_0}{\alpha^2(t)} \right] \\ &= \int_0^\infty \frac{\xi^{2b}}{a^2} f(\xi) d\xi \mathbb{E} \left[\frac{\gamma N_0}{\alpha^2(t)} \right]. \end{aligned}$$

The additional cost associated with the random placement of wireless nodes can be characterized by comparing the ex-

pected power per transmission for a given distribution $f(d)$ with the expected power used when all the nodes are placed at an equal distance from their nearest neighbors.

4.1. One-Dimensional Model

Consider a scenario where $n + 2$ nodes are placed on an interval of length L . One node is placed at 0, one node is placed at L , and the remaining nodes are dropped independently along the interval $[0, L]$ according to a uniform distribution. The distance between two consecutive nodes has a probability density function given by

$$f_n(d) = \frac{n}{L} \left(1 - \frac{d}{L}\right)^{n-1} \quad 0 \leq d \leq L.$$

The expected transmit power per data transfer for this sensor system is

$$\begin{aligned} \bar{P}_{\text{ran}} &= \int_0^1 \frac{n}{a^2} (L\xi)^{2b} (1 - \xi)^{n-1} d\xi \mathbb{E} \left[\frac{\gamma N_0}{\alpha^2(t)} \right] \\ &= \frac{L^{2b}}{a^2} \sum_{k=0}^{n-1} \binom{n-1}{k} (-1)^k \frac{n}{2b+k+1} \mathbb{E} \left[\frac{\gamma N_0}{\alpha^2(t)} \right]. \end{aligned}$$

In contrast, when the wireless nodes are positioned carefully at a distance $L/(n+1)$ from adjacent nodes, the expected transmit power per data transfer is equal to

$$\bar{P}_{\text{det}} = \bar{P}_t \left(\frac{L}{n+1} \right) = \frac{L^{2b}}{a^2(n+1)^{2b}} \mathbb{E} \left[\frac{\gamma N_0}{\alpha^2(t)} \right].$$

This yields an excess power consumption factor of

$$\frac{\bar{P}_{\text{ran}}}{\bar{P}_{\text{det}}} = \sum_{k=0}^{n-1} \binom{n-1}{k} (-1)^k \frac{n(n+1)^{2b}}{2b+k+1}.$$

Note that this expression is independent of the length of the sensing interval. In other words, the excess power consumption factor associated with the random placement of nodes does not depend on sensor density. As the number of nodes in the system grows larger, this ratio converges to

$$\begin{aligned} \frac{\bar{P}_{\text{ran}}}{\bar{P}_{\text{det}}} &= \int_0^1 n(n+1)^{2b} \xi^{2b} (1 - \xi)^{n-1} d\xi \\ &= \frac{n}{n+1} \int_0^{n+1} \mu^{2b} \left(1 - \frac{\mu}{n+1}\right)^{n-1} d\mu \\ &\rightarrow \int_0^\infty \mu^{2b} e^{-\mu} d\mu = \Gamma(2b+1) \quad \text{as } n \rightarrow \infty, \end{aligned}$$

where the limit follows from Lebesgue's Dominated Convergence Theorem.

For a sensor system with a large number of nodes, the excess power consumption factor is

$$\frac{\bar{P}_{\text{ran}}}{\bar{P}_{\text{det}}} \approx \Gamma(2b+1) \in [6, 24],$$

depending on the exponential decay parameter b of the mean path gain. That is, the random placement of nodes studied in this example leads to an expected power consumption per transmission that is 6 to 24 times larger than the average transmit power corresponding to a uniform array of sensors. This factor is very significant given the small energy budget of a typical sensor nodes. The excess power consumption factor for various network sizes appears in Figure 1.

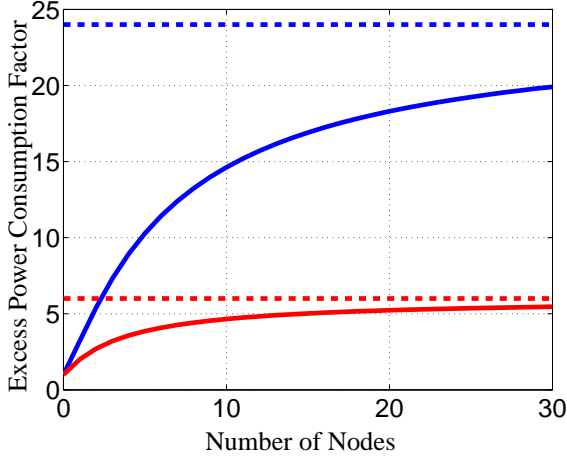


Fig. 1. Excess power consumption factor as a function of number of wireless sensor nodes.

4.2. Multihop Communication

In the classical decentralized detection problem with fusion, all the nodes transmit information to their fusion center. Based on the received data, the fusion center selects one of the possible hypotheses. To save energy, nodes may elect to exploit multihop communication rather than transmit their information directly to the fusion center. In multihop communication, data packets are carried over to their final destinations through multiple relays, essentially limiting the power level required for individual transmissions. Although this strategy necessitates more transmissions, the non-linear nature of the wireless channel insures overall gains.

Again suppose $n + 2$ nodes are placed on an interval of length L , and assume that the node located at 0 acts as a fusion center. There will be a total of $n + 1$ transmissions if nodes transmit their information directly to the fusion center, compared to $(n + 1)(n + 2)/2$ transmissions if the nodes are using multihop communication. However, the average expected power required for individual transmissions in the latter paradigm is much smaller. This leads to a total power ratio equal to

$$\frac{\bar{P}_{\text{single}}}{\bar{P}_{\text{multi}}} = \frac{2 + \frac{2n}{2b+1}}{(n+1)(n+2) \sum_{k=0}^{n-1} \binom{n-1}{k} (-1)^k \frac{n}{2b+k+1}}.$$

for randomly scattered nodes, and equal to

$$\frac{\bar{P}_{\text{single}}}{\bar{P}_{\text{multi}}} = \frac{2}{(n+1)(n+2)} \sum_{k=1}^{n+1} k^{2b}.$$

when sensor nodes are equally spaced. Multihop communication results in substantial energy savings for both stochastic and deterministic systems. This can be seen in Figure 2 for power exponential parameters $2b \in \{3, 4\}$.

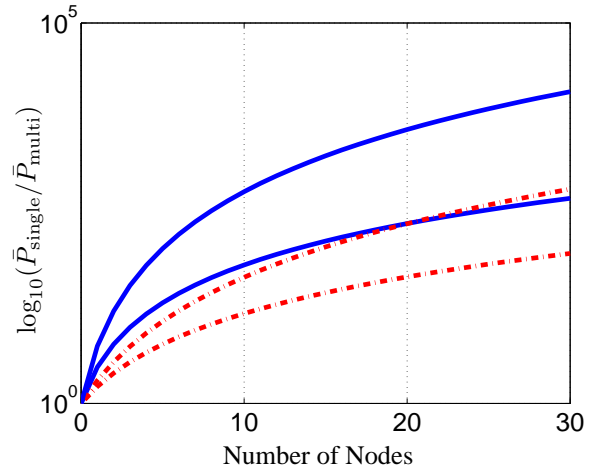


Fig. 2. Comparison of the total power required to send data from every node to the fusion center for a single-hop and a multihop communication paradigm.

The benefits of communicating over short distances are evident. The downside of multihop communication is a longer delay at the fusion center. Energy savings are only possible to the extent that the fusion center is willing to tolerate delays in acquiring environmental data.

4.3. Two-Dimensional Model

A similar performance analysis can be carried in the two-dimensional case. However, this analysis requires that a routing protocol be specified. For simplicity, we assume that a packet is sent to the nearest node located within a 90-degree aperture in the direction of the destination node. If wireless sensors are dropped uniformly and independently over a large two-dimensional surface, the distribution of the minimum distance between a node and its nearest admissible neighbor (90-degree aperture) is well-approximated by

$$f(r) \approx \frac{\lambda \pi r}{2} e^{-\frac{\lambda \pi r^2}{4}} \quad r \geq 0,$$

where λ is the overall node density. The expected transmit power per transmission is

$$\bar{P}_{\text{ran}} \geq \frac{2^{2b} \Gamma(b+1)}{a^2 \lambda^b \pi^b} \text{E} \left[\frac{\gamma N_0}{\alpha^2(t)} \right]. \quad (3)$$

On the other hand, if nodes are positioned on a uniform lattice to minimize the distance between a node and its closest neighbor, the expected transmit power per data transfer becomes

$$\bar{P}_{\text{det}} = \bar{P}_t(r^*) = \frac{(r^*)^{2b}}{a^2} \mathbb{E} \left[\frac{\gamma N_0}{\alpha^2(t)} \right] \quad (4)$$

where r^* is the distances between adjacent vertices on the optimal regular two-dimensional lattice. Note that the lattices must be scaled to have a density of λ vertices per unit area. The minimum distance can be computed explicitly as

$$r^* \triangleq \min \left\{ \frac{2}{\sqrt{3\lambda \tan \frac{\pi}{3}}}, \frac{1}{\sqrt{\lambda}}, \frac{2}{\sqrt{6\lambda \tan \frac{\pi}{6}}} \right\}.$$

Combining expressions (3) and (4) yields a lower bound on the excess power consumption factor,

$$\frac{\bar{P}_{\text{ran}}}{\bar{P}_{\text{det}}} \geq \frac{3^{3b/2} \Gamma(b+1)}{\pi^b} \in \left[\frac{3^{13/4}}{4\pi}, \frac{54}{\pi^2} \right] \\ \approx [2.83, 5.47].$$

In either scenario considered above, the excess power consumption factor attributable to random node placement is substantial. Overlooking the process by which nodes are positioned may lead to a dramatic increase in power consumption. For a deployed network, randomly positioned nodes may cause the premature death of wireless agents. In power constrained systems, higher averaged power per data transfer may limit the capacity of the network and the area covered by the system.

5. DISCUSSION

The results presented above have interesting implications for the design of wireless sensor networks in the context of decentralized detection. The excess transmit power associated with random node placement may have various consequences depending on the design goal of a system. For instance, under a total power constraint, random node placement restricts the area covered by the sensor network because it limits the total number of nodes contained in the system. This in turn results in a performance loss at the fusion center. Alternatively, the node density of a random network can be increased to limit the average transmit power. The extra system cost can then be computed in terms of additional nodes per area.

The analysis carried in this paper is based on an expected cost per data transfer. It can be extended to the case where the cost is based on expected energy consumption per packet transmission from a node to the fusion center.

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