# COMBINED NON-ORTHOGONAL JOINT ZERO-DIAGONALIZATION AND JOINT DIAGONALIZATION FOR SOURCES SEPARATION

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## ABSTRACT

This communication is concerned with blind separation of instantaneous mixtures of source signals based on the use of spatial quadratic time-frequency (spectrum) distributions. First, we propose a new algorithm to perform the non orthogonal joint zero-diagonalization and joint-diagonalization of given sets of matrices. We also present a selection procedure of useful time-frequency points in order to automatically determine which matrices have to be joint (zero-) diagonalized. One advantage of the proposed approaches is that they do not require any whitening stage and thus they are intended to work even with a class of correlated signals, rather than the classical independent one. Finally, computer simulations are provided in order to illustrate the effectiveness of the proposed methods and to compare them with more classical ones.

## 1. INTRODUCTION

This communication deals with the problem of blind separation of instantaneous mixtures of sources based on the use of spatial quadratic time-frequency distributions (SQTFD). Many works have been dedicated to that problem during the past seven years among which [1]-[10]. In fact, such an approach, by taking advantage of the non-stationary of the sources, makes it possible to consider a wider class of source signals than the classical one (statistically independent random source signals). Notice, however, that another approach can also be found in [11].

The use of SQTFD has led to numerous algorithms [1]-[10]. Most of them are based on the joint diagonalization of a particular set of matrices [1][4][8] and are applied after a preliminary whitening stage. More recently, two new research orientations have emerged: (i) to perform separation without any whitening stage [5]-[7] and (ii) to consider a new kind of SQTFD matrices having to be joint zero-diagonalized instead of joint diagonalized [2][3]. Whatever the considered method, a preliminary procedure of selection of "useful" time-frequency (*t-f*) points appears essential to the extent that it makes it possible to build matrices sets to be joint diagonalized and/or joint zero diagonalized. In fact, there exist four different types of *t-f* points: (1) those which correspond to sources auto-terms only (in such a *t-f* point the sources SQTFD matrix is diagonal), (2) those which correspond to sources cross-terms only (in such a *t-f* point the sources SQTFD matrix is zero-diagonal), (3) those which correspond to both sources cross-terms and sources auto-terms (in such a *t-f* point the sources SQTFD matrix has no particular algebraic structure), and finally (4) those which correspond to neither sources auto-terms nor sources cross-terms. Only the two first types of *t-f* points are particularly of interest with regard to blind sources separation.

As a first contribution, it was suggested in [1] to joint diagonalized SQTFD matrices calculated at *t-f* points corresponding to sources auto-terms only, this method being applied after pre-whitening of the observations. Latter, it was shown in [5]-[7], that the whitening stage could be dropped using non-orthogonal joint-diagonalization which makes it possible to consider "correlated" sources signals and which generally leads to better performances.

In the same time, a complementary approach was proposed in [2]. It consists in joint zero diagonalizing (after a whitening stage) another set of SQTFD matrices: those associated to *t-f* points corresponding to sources cross-terms only. Finally an algorithm combining joint diagonalization and joint zero diagonalization (after pre-whitening) has also been proposed in [3].

In this communication, we propose a new algorithm to perform the non-orthogonal joint zero diagonalization of a given set of matrices and, then, we show that the source separation can be realized applying this algorithm to a set of SQTFD matrices corresponding to sources cross-terms only. We also present the automatic selection procedure we use to determine which *t-f* points correspond to sources cross-terms only. Finally, we generalize the combination of joint zero diagonalization with joint diagonalization [3] to the non-orthogonal case. Computer simulations are provided in order to illustrate the effectiveness of the proposed approaches and to compare them with more classical ones.

## 2. MODEL AND ASSUMPTIONS

We consider the classical instantaneous blind sources separation problem where $N \in \mathbb{N} \setminus \{0, 1\}$ sources signals are received on $M \in \mathbb{N} \setminus \{0, 1\}$ sensors. We also suppose that $M \geq N$, the model is then called "overdetermined". In matrix and vector notations, the input/output relationship of the mixing model reads in the noiseless case:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) \tag{1}$$

with $\mathbf{A}$ the $(M, N)$ real mixing matrix which is assumed full column rank, $\mathbf{x}(t) = [x_1(t), \ldots, x_M(t)]^T$ the $(M, 1)$ observations vector ($(\cdot)^T$ denotes the transposition operator) and $\mathbf{s}(t) = [s_1(t), \ldots, s_N(t)]^T$ the $(N, 1)$ real sources vector.

The problem of blind sources separation consists in the estimation of a "separating" matrix, say $\mathbf{B}$, which applied to the observation as

$$\mathbf{y}(t) = \mathbf{B}\mathbf{x}(t) \tag{2}$$

yields an estimation of the source signals.

Defining $\mathbf{G} = \mathbf{B}\mathbf{A}$ as the matrix of the global system, the source separation problem is solved when one has found a separating matrix $\mathbf{B}$ in such a way that

$$\mathbf{G} = \mathbf{D}\mathbf{P} \tag{3}$$

where $\mathbf{D}$ is an invertible diagonal matrix which corresponds to arbitrary attenuations for the restored sources and $\mathbf{P}$ is a permutation matrix which corresponds to an arbitrary order of restitution of source signals.

Since the proposed developments are based on the use of Spatial Quadratic Transforms (SQT) of signals and their properties [1][12][13], let us now briefly recall the important points related to our utilization.

Considering a real random vectorial signal $\mathbf{z}(t)$, its correlation matrix $\mathbf{R}_z(t, \tau)$ is given by:

$$\mathbf{R}_z(t, \tau) = \mathsf{E}\{\mathbf{z}(t + \frac{\tau}{2})\mathbf{z}^T(t - \frac{\tau}{2})\} \ . \tag{4}$$

Its spatial quadratic time-frequency spatial spectrum (SQTFS) is then defined as:

$$\mathbf{D}_z(t, \nu) = \int_{\mathbb{R}^2} \mathbf{R}_z(\theta, \theta')K(\theta, \theta'; t, \nu)d\theta d\theta' \ . \tag{5}$$

The function $K(\theta, \theta'; t, \nu)$ which is generally a complex function is referred to as the *kernel* of the transform. Let us give the important example of the spatial Wigner-Ville spectrum (SWVS, noted $\mathbf{D}_{\mathsf{SWVS},z}(t, \nu)$):

$$\mathbf{D}_{\mathsf{SWVS},z}(t, \nu) = \int_{\mathbb{R}} \mathbf{R}_z(t, \tau)e^{-2\imath\pi\nu\tau}d\tau. \tag{6}$$

Considering now a real vectorial deterministic signal $\mathbf{z}(t)$ as a particular realization of a random one, the SQT is given by a matrix $\mathbf{D}_z(t, \nu) = \big(D_{z_i,z_j}(t, \nu)\big)$ written as

$$\mathbf{D}_z(t, \nu) = \int_{\mathbb{R}^2} \mathbf{z}(\theta)\mathbf{z}^T(\theta')R(\theta, \theta'; t, \nu)d\theta d\theta' \tag{7}$$

which is defined component-wise by

$$D_{z_i,z_j}(t, \nu) = \int_{\mathbb{R}^2} z_i(\theta)z_j(\theta')R(\theta, \theta'; t, \nu)d\theta d\theta' \tag{8}$$

for all $i$ and $j$. The diagonal terms of the SQT $\mathbf{D}_z(t, \nu)$ are called *auto-terms* while the off-diagonal ones are called *cross-terms*. The function $R(\theta, \theta'; t, \nu)$ which is generally a complex function is again referred to as the *kernel* of the transform. For physical reasons, this kernel is often constrained to satisfy the following property

$$R(\theta, \theta'; t, \nu) = R^*(\theta', \theta; t, \nu) \tag{9}$$

where $(\cdot)^*$ stands for the complex conjugate operator. Then, the SQT satisfies an *hermitian symmetry* as

$$\mathbf{D}_z(t, \nu) = \mathbf{D}_z^H(t, \nu) \tag{10}$$

where $(\cdot)^H$ stands for the complex conjugate and transpose operator. Notice that this is not a too restrictive condition to the extent that it is satisfied by many transformations among which the Wigner-Ville transformation (WV), the Choï-Williams transformation (CW), the Pseudo Wigner-Ville transformation (PWV), the Smoothed Pseudo Wigner-Ville transformation (with the help of certain conditions on the window which has to be real), the Born-Jordan distribution (BJ), etc...

We assume in what follows that the considered *t-f* representations of each source signals do not overlap too much.

## 3. PROPOSED APPROACH

It is necessary to consider two sets of matrices related to the joint diagonalization (JD) problem and to the joint zero-diagonalization (JZD) one. The first set denoted by $\mathcal{D}$ is a set of $N_D$, $N_D \in \mathbb{N}^*$, matrices $\mathbf{M}_{D,i}$, $i \in \{1, \ldots, N_D\}$ which all admits the following factorization: there exists an $(M, N)$, $M \geq N$, full column rank matrix $\mathbf{A}_D$ and a set $\mathcal{D}_d$ of $N_D$ $(N, N)$ diagonal matrices $\mathbf{D}_i$, $i \in \{1, \ldots, N_D\}$, such that

$$\mathbf{M}_{D,i} = \mathbf{A}_D\mathbf{D}_i\mathbf{A}_D^H \ , \qquad \forall i \in \{1, \ldots, N_D\} \ . \tag{11}$$

The second set denoted by $\mathcal{Z}$ is a set of $N_Z$, $N_Z \in \mathbb{N}^*$, matrices $\mathbf{M}_{Z,i}$, $i \in \{1, \ldots, N_D\}$ which all admits the following factorization: there exists an $(M, N)$, $M \geq N$, full column rank matrix $\mathbf{A}_Z$ and a set $\mathcal{Z}_z$ of $N_Z$ $(N, N)$ zero-diagonal matrices $\mathbf{Z}_i$, $i \in \{1, \ldots, N_Z\}$, such that

$$\mathbf{M}_{Z,i} = \mathbf{A}_Z\mathbf{Z}_i\mathbf{A}_Z^H \ , \qquad \forall i \in \{1, \ldots, N_Z\} \ . \tag{12}$$

A zero-diagonal matrix is a matrix whose all diagonal components are zero, *i.e.* $\mathbf{Z} = (Z_{i,j})$ is a zero-diagonal matrix if and only if $Z_{i,i} = 0$ for all $i$.

The JD problem consists in the estimation of matrices $\mathbf{A}_D$ and $\mathbf{D}_i$, $i \in \{1, \dots, N_D\}$ using only the matrices set $\mathcal{D}$. Similarly, the JZD problem consists in the estimation of matrices $\mathbf{A}_Z$ and $\mathbf{Z}_i$, $i \in \{1, \dots, N_Z\}$ using only the matrices set $\mathcal{Z}$. In case where $\mathbf{A}_D = \mathbf{A}_Z = \mathbf{A}_{DZ}$, we define the joint diagonalization/zero-diagonalization (JDZD) problem consisting in the estimation of matrices $\mathbf{A}_{DZ}$, $\mathbf{D}_i$ and $\mathbf{Z}_i$, $i \in \{1, \dots, N_Z\}$ using only the two matrices sets $\mathcal{D}$ and $\mathcal{Z}$.

For that task, we define the two following criteria

$$\mathcal{C}_D(\mathbf{B}) = \sum_{i=1}^{N_D} \|\mathsf{Offdiag}\{\mathbf{B}\mathbf{M}_{D,i}\mathbf{B}^H\}\|^2 \tag{13}$$

for the joint diagonalization part and

$$\mathcal{C}_Z(\mathbf{B}) = \sum_{i=1}^{N_Z} \|\mathsf{Diag}\{\mathbf{B}\mathbf{M}_{Z,i}\mathbf{B}^H\}\|^2 \tag{14}$$

for the joint zero-diagonalization one. The goal in now to find a matrix argument of the minimization of $\mathcal{C}_D(\mathbf{B})$ and/or $\mathcal{C}_Z(\mathbf{B})$. For generality, we introduce the combined cost function

$$\mathcal{C}(\mathbf{B}) = \alpha\mathcal{C}_D(\mathbf{B}) + (1-\alpha)\mathcal{C}_Z(\mathbf{B}) \;, \qquad \alpha \in [0,1] \tag{15}$$

which takes into consideration both a diagonalization aspect and a zero-diagonalization one. It is important to notice that the matrix $\mathbf{B}$ plays the role of the (pseudo) inverse of the matrices $\mathbf{A}_D$ and $\mathbf{A}_Z$ we are looking for.

Now, let us rewrite the cost function $\mathcal{C}(\mathbf{B})$. Because for an $(M,M)$ matrix $\mathbf{M}_i$ and for an $(N,M)$ matrix $\mathbf{B}$, we have $\left(\mathbf{B}\mathbf{M}_i\mathbf{B}^H\right)_{\ell,k} = \mathbf{b}_\ell\mathbf{M}_i\mathbf{b}_k^H$ where $\mathbf{b}_\ell$, $\ell \in \{1, \dots, N\}$ stands for the row vectors of matrix $\mathbf{B}$, then

$$\begin{aligned}\mathcal{C}_D(\mathbf{B}) &= \sum_{i=1}^{N_D}\sum_{\ell=1}^{N}\sum_{\substack{k=1\\k\neq\ell}}^{N} \left|\mathbf{b}_\ell\mathbf{M}_{D,i}\mathbf{b}_k^H\right|^2 \\ &= \sum_{\ell=1}^{N} \mathbf{b}_\ell\mathbf{R}_{D,\ell}(\mathbf{B})\mathbf{b}_\ell^H \end{aligned}\tag{16}$$

where

$$\mathbf{R}_{D,\ell}(\mathbf{B}) = \sum_{i=1}^{N_D}\mathbf{M}_{D,i}\sum_{\substack{k=1\\k\neq\ell}}^{N}\left(\mathbf{b}_k^H\mathbf{b}_k\right)\mathbf{M}_{D,i}^H \tag{17}$$

and

$$\begin{aligned}\mathcal{C}_Z(\mathbf{B}) &= \sum_{i=1}^{N_Z}\sum_{\ell=1}^{N} \left|\mathbf{b}_\ell\mathbf{M}_i\mathbf{b}_\ell^H\right|^2 \\ &= \sum_{\ell=1}^{N} \mathbf{b}_\ell\mathbf{R}_{Z,\ell}(\mathbf{B})\mathbf{b}_\ell^H \end{aligned}\tag{18}$$

where

$$\mathbf{R}_{Z,\ell}(\mathbf{B}) = \sum_{i=1}^{N_Z}\mathbf{M}_{D,i}\mathbf{b}_\ell^H\mathbf{b}_\ell\mathbf{M}_{D,i}^H \tag{19}$$

Finally using (16) and (18) in (15), leads to

$$\mathcal{C}(\mathbf{B}) = \sum_{\ell=1}^{N}\mathbf{b}_\ell\mathbf{R}_\ell(\mathbf{B})\mathbf{b}_\ell^H \tag{20}$$

where

$$\mathbf{R}_\ell(\mathbf{B}) = \alpha\mathbf{R}_{D,\ell}(\mathbf{B}) + (1-\alpha)\mathbf{R}_{Z,\ell}(\mathbf{B}) \;. \tag{21}$$

First notice that the minimization of $\mathcal{C}(\mathbf{B})$ in (20) can be realized row by row. For a given row, *i.e.* $\ell$ being fixed, one of the simplest way to find a minimum of the quadratic form in (20) consists in the determination of the normalized eigenvector of $\mathbf{R}_\ell(\mathbf{B})$ associated with the lowest (non zero) eigenvalue. However since matrix $\mathbf{R}_\ell(\mathbf{B})$ for a given $\ell$ also depends on the vector $\mathbf{b}_\ell$ when $\alpha \neq 1$, we propose the use of an iterative procedure as

Given $\mathbf{B}^{(0)}$ a (good) initial matrix with unit norm rows with $i \in \mathbb{N}_*$, for each $\ell \in \{1, \dots, N\}$, do *(a)* and *(b)*

*(a)* calculate $\mathbf{R}_\ell(\mathbf{B}^{(i-1)})$

*(b)* find the $N$-th lowest eigenvalue $\lambda_\ell^{(i)}$ and the associated unit-norm eigenvector $\mathbf{b}_\ell^{(i)}$ of matrix $\mathbf{R}_\ell(\mathbf{B}^{(i-1)})$.

Stop when $|\lambda_\ell^{(i)} - \lambda_\ell^{(i-1)}| \leq \varepsilon$ where $\varepsilon$ is a given small positive threshold.

## 4. *T-F* POINTS DETECTION CRITERIA

We now propose two *t-f* points detection procedures in order to build the matrices sets to be joint diagonalized and/or joint zero-diagonalized.

Considering the following assumptions: mixing matrix $\mathbf{A}$ is real, the used SQTFD exhibit hermitian symmetry, one can show that the SQTFD of the sources ($\mathbf{D}_s(t,\nu)$) and of the whitened observations ($\underline{\mathbf{D}}_x(t,\nu)$) are real and non null matrices when they correspond to sources auto-terms only while they are generally complex when they correspond to cross-terms only. The trace being invariant under unit transform, one also has $\mathsf{trace}\{\underline{\mathbf{D}}_x(t,\nu)\} = \mathsf{trace}\{\mathbf{D}_s(t,\nu)\}$. One of the most simple way to select matrices (associated to whitened observations SQTFD) to joint zero diagonalized consists in choosing those whose trace absolute value ($|\mathsf{trace}\{.\}|$) is small enough (lower than a given threshold $\varepsilon_2$) whereas their imaginary part euclidian norm ($\|\Im\{.\}\|$) is high enough (higher than a given threshold $\varepsilon_1$). It is summed up as follows:

For joint zero-diagonalization choose $(t,\nu)$ such as:

$$\begin{cases} \|\Im\{\underline{\mathbf{D}}_x(t,\nu)\}\| > \varepsilon_1 \\ |\mathsf{trace}\{\underline{\mathbf{D}}_x(t,\nu)\}| < \varepsilon_2 \end{cases} . \tag{22}$$

For joint diagonalization choose $(t, \nu)$ such as:

$$\begin{cases} \|\Im\{\underline{\mathbf{D}}_x(t,\nu)\}\| < \varepsilon_3 \\ |\mathsf{trace}\{\underline{\mathbf{D}}_x(t,\nu)\}| > \varepsilon_4 \end{cases} . \tag{23}$$

Notice, first, that the *t-f* points selection procedure we present is applied on pre-whitened observations SQTFD in order to compare unitary and non-unitary (zero-)diagonalization algorithms on the same sets of *t-f* points. In [5] and [7], we have also proposed *t-f* points selection procedures which can be used without pre-whitening.

Notice, also, that criterium given in (23) is not restrictive enough, in fact a problem may occur when sources cross-terms do exist (their real parts are not null) but with null imaginary parts (in such *t-f* points, if there are sources cross-terms only, the corresponding SQTFD should be joint zero-diagonalized, and if there are both sources auto and cross-terms, nothing should be done). To eliminate this ambiguity, an additional condition can be introduced thanks to a 2D-image filtering applied on each imaginary parts of the cross-terms of the SQTFDs, whose effect is to differentiate the *t-f* images. A fifth threshold $\varepsilon_5$ (regarding joint-diagonalization) is introduced: if the derivative in this *t-f* point is smaller than $\varepsilon_5$, there is no sources cross-terms but only sources auto-terms and the corresponding SQTFD has to be joint-diagonalized, in the other case nothing is done.

## 5. COMPUTER SIMULATIONS

The effectiveness of these algorithms is illustrated by comparing the obtained performances with those obtained thanks to more classical algorithms. This is realized through the use of a performance index which allows to quantify the performances of the proposed algorithms and to compare themselves with each other. The following performance index is thus used as a measure of the quality of the separation:

$$\begin{aligned} I(\widehat{\mathbf{B}}\mathbf{A}) &= \frac{1}{N(N-1)} \sum_{i=1}^{N} \left( \sum_{j=1}^{N} \frac{|(\widehat{\mathbf{B}}\mathbf{A})_{ij}|^2}{\max_l |(\widehat{\mathbf{B}}\mathbf{A})_{il}|^2} - 1 \right) \\ &+ \frac{1}{N(N-1)} \sum_{j=1}^{N} \left( \sum_{i=1}^{N} \frac{|(\widehat{\mathbf{B}}\mathbf{A})_{ij}|^2}{\max_l |(\widehat{\mathbf{B}}\mathbf{A})_{lj}|^2} - 1 \right) \end{aligned} \tag{24}$$

with $N$ the dimension of the considered matrix $\widehat{\mathbf{B}}\mathbf{A}$. This index is given in dB defined by $I(\cdot)$ dB $= 10\log(I(\cdot))$.

The Spatial Pseudo Wigner-Ville distribution is used in order to have a sufficient number of *t-f* points corresponding to cross-terms only to the extent that we intend to apply zero-diagonalization algorithms.

In a first example, we consider the case of 3 sources (a linear frequency modulation, a sum of 2 sinusoids, a sinusoidal frequency modulation) received on 4 sensors. The real part of the Spatial Pseudo Wigner-Ville distribution (SPWV) of the sources is displayed on Fig. 1 and its imaginary part on Fig. 2. It is computed over $64$ frequency bins and with a Hamming window of length 33. One can check that the diagonal terms of the SPWV are effectively real to the extent that they correspond to the QTFRs of each of the $4$ sources. With regard to the off-diagonal terms, they are complex: they correspond to the BTFRs of couples of different sources. These sources are mixed by the following mixing matrix:

$$\mathbf{A} = \begin{pmatrix} 1 & -0.1 & 0.2 \\ 0.5 & 1 & 0.3 \\ 0.2 & -0.75 & 1 \\ 0.5 & -0.4 & 0.25 \end{pmatrix} .$$

| Methods | Perf. (no filter) | Perf. (2D filter) |
|---|---|---|
| JD (dB) | -25.20 | -25.02 |
| JZD (dB) | -26.63 | -26.63 |
| JD/JZD (dB) | -25.8 | -25.71 |
| $\mathsf{JD}_{\mathsf{NO,ALG}}$ (dB) | -38.64 | -49.48 |
| $\mathsf{JZD}_{\mathsf{NO,ALG}}$ (dB) | -41.92 | -41.92 |
| $\mathsf{JD/JZD}_{\mathsf{NO,ALG}}$ (dB) | -41.08 | -48.86 |
| JADE (dB) | -20.26 | |

**Table 1**. Example 1, comparison of performances obtained thanks to the proposed methods .

The real part of the observations SPWV is given on Fig. 3. Using $\varepsilon_1 = 16.5$, $\varepsilon_2 = 4$, $\varepsilon_3 = 0.25$, $\varepsilon_4 = 16$, the set of detected and used *t-f* points is represented by a “plus” on Fig. 4, left for joint-diagonalization and right for joint zero-diagonalization. For an easier interpretation, with regard to joint-diagonalization, the 230 selected *t-f* points are superimposed with the trace of the sources SPWV while for joint zero-diagonalization the 226 selected *t-f* points are superimposed with the sum of the off-diagonal terms of the sources SPWV. Only 220 *t-f* points are selected when a 2D-image filtering is applied. Using the performance index we have given before, obtained performances are given in Table 1. We compare joint diagonalisation (JD) and joint zero-diagonalization (JZD) and their combination (JD/JZD) applied on sets matrices calculated after a whitening stage. The result obtained thanks to the classical JADE algorithm is also given. We compare them with the proposed algorithms: algebraic joint-diagonalization ($\mathsf{JD}_{\mathsf{NO,ALG}}$), zero-diagonalization ($\mathsf{JZD}_{\mathsf{NO,ALG}}$) and their combination ($\mathsf{JD/JZD}_{\mathsf{NO,ALG}}$) applied on matrices sets calculated without whitening stage.

In a second example, we consider the case of 3 sources (two linear frequency modulations and a sinusoid) received on 3 sensors. The real part of the SPWV distribution of

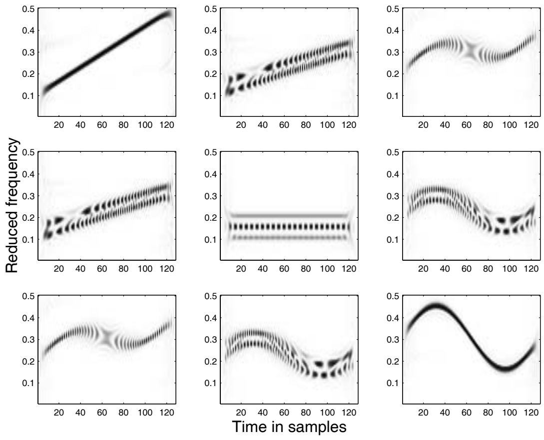


**Fig. 1**. Real part of the Spatial Pseudo Wigner-Ville distribution of the sources

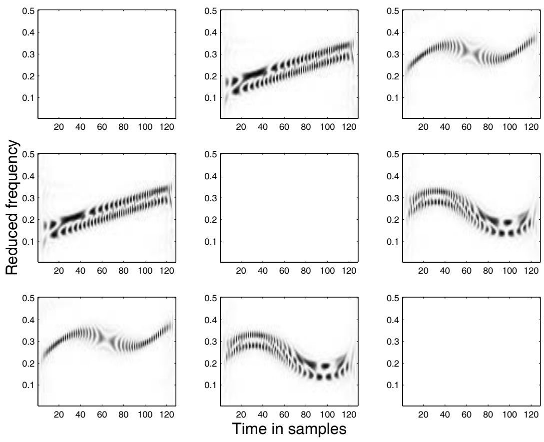


**Fig. 2**. Imaginary part of the Spatial Pseudo-Wigner-Ville distribution of the sources

the sources is displayed on Fig. 5 and the real part of the SPWV of the observed signals is given on Fig. 6. On Fig. 7, 27 *t-f* points are selected by hand for joint-diagonalization (left) and 26 for joint zero-digonalization (right). On Fig. 8, 546 *t-f* points have been automatically detected for joint-diagonalization (left) and 591 for joint zero-diagonalization (right), using $\varepsilon_1 = 10$, $\varepsilon_2 = 5$, $\varepsilon_3 = 0.5$, $\varepsilon_4 = 12$. The obtained performances using the different algorithms on both sets of *t-f* points are summed up in Table 2.

Results are better with the proposed algorithms *i.e.* when no pre-whitening stage is applied. Best results are generally obtained with non orthogonal zero-diagonalization (because *t-f* points are better selected).

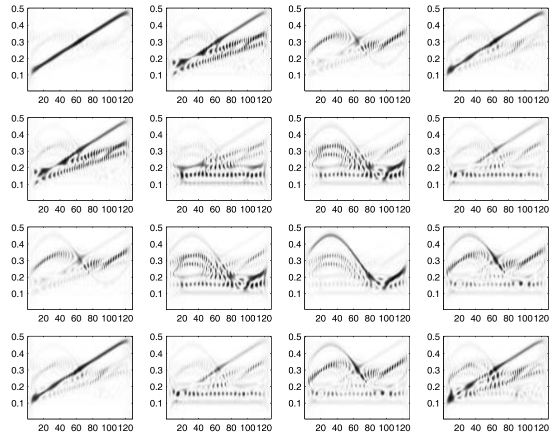

**Fig. 3**. Real part of the Spatial Pseudo Wigner-Ville distribution of the mixture

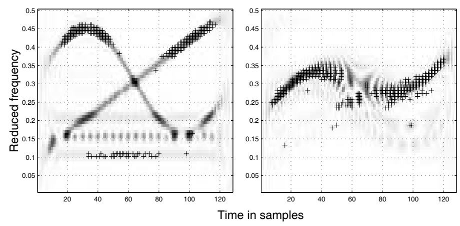


**Fig. 4**. The 230 *t-f* points automatically selected for joint-diagonalization (left) ; the 226 *t-f* points retained for joint zero-diagonalization (right)

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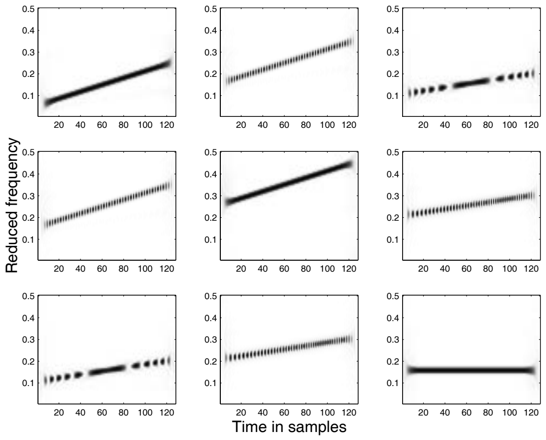


**Fig. 5**. Real part of the Spatial Pseudo-Wigner-Ville distribution of the sources

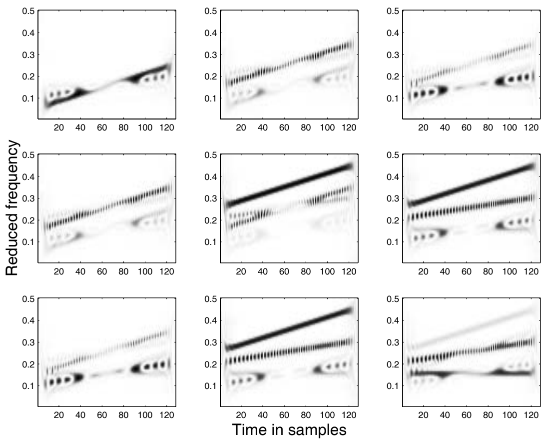


**Fig. 6**. Real part of the Spatial Pseudo Wigner-Ville distribution of the mixture

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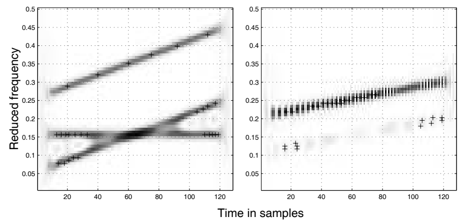


**Fig. 7**. The 27 *t-f* points selected by hand for joint-diagonalization (left) ; the 26 *t-f* points selected by hand for joint zero-diagonalization (right)

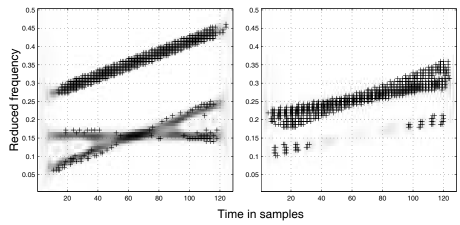


**Fig. 8**. The 546 *t-f* points automatically selected for joint-diagonalization (left) ; the 591 *t-f* points retained for joint zero-diagonalization (right)

[10] N. Linh-Trung, A. Belouchrani, K. Abed-Meraim and B. Boashash, "Separating more sources than sensors using time-frequency distributions", in Proc. *International symposium on Signal Processing and its applications (ISSPA'2001)*, Kuala-Lumpur, Malaysia, August 2001.

[11] Pham, D.-T. and Cardoso, J.-F, "Blind separation of instantaneous mixtures of non-stationary sources", In Proc. *Int. Workshop on Independent Component Analysis and Blind Signal Separation (ICA'2000)*, pages 187-193, Helsinki, Finland.

[12] P. Flandrin, *Time-Frequency/Time-Scale Analysis*, Academic Press, 1999.

[13] L. Cohen, *Time-Frequency Analysis*, Prentice Hall, 1995.

| Methods | Perf. (hand) | Perf. (auto.) |
|---|---|---|
| JD (dB) | -21.18 | -21.5 |
| JZD (dB) | -21.48 | -21.47 |
| JD/JZD (dB) | -21.21 | -21.47 |
| $JD_{NO,ALG}$ (dB) | -31.78 | -37.76 |
| $JZD_{NO,ALG}$ (dB) | -44.61 | -33.62 |
| $JD/JZD_{NO,ALG}$ (dB) | -33.86 | -40.31 |
| JADE (dB) | -20.57 | |

**Table 2**. Example 2, comparison of performances obtained thanks to the proposed methods on two different sets of *t-f* points.