Comparison of two IMM tracking and classifier architectures based on Extended and Unscented Kalman Filter with CRLB

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ABSTRACT

On the basis of the BT kinematics model it is possible to build up an interactive multiple model (IMM) for target tracking using EKFs and UKFs. The performance evaluation (accuracies and classification) of the designed IMM tracking algorithm is predicted via Monte Carlo simulation and compared with the CRLB achievable for the same study case.

1 INTRODUCTION

The paper deals with the problem of tracking of Ballistic Target (BT) with multifunctional radar. It is assumed that the radar acquires a limited number of measurements that do not encompass the whole target trajectory; thus the established target track has to be extrapolated ahead in time in order to predict, as an example, the coordinates of the impact point. A further function is the support to threats classification which allows to adopt the proper reaction to the target under tracking.

A model of BT kinematics is required to develop a successful tracking filter. Three main forces affect the BT motion: thrust, drag and gravity [4,5]. For the sake of this paper, it is assumed that the BT is in the cruise phase during the BT state vector estimation while drag and gravity are acting on the target body during the re-entry phase. The drag acceleration expression is

$$\mathbf{a}_{drag}=\begin{bmatrix} a_{dragx} \\ a_{dragy} \\ a_{dragz} \end{bmatrix}=-\frac{1}{2}\frac{\rho(z)\cdot g_0}{\beta}\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\cdot\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

where β is the ballistic coefficient (N/m^2), $\rho(z)$ is the air density function of the height:

$$\rho(z)=1.21907\cdot e^{-z/9146.64}$$

$\dot{x}, \dot{y}, \dot{z}$ are the velocity components of the BT along the three axes of a Cartesian reference system. The gravity acceleration is considered constant, g_0=9.8m/s^{2}and directed along the z-axis.

As shown above, the BT motion equations are highly non linear so that an optimum tracking filter cannot be found in principle; thus sub-optimum filters are the usual current practice. The techniques based on Kalman Filter (KF) fails in presence of BT, while tracking procedure based on: accurate description of BT kinematics, Extended Kalman Filter (EKF) [1], Unscented Kalman Filtering [2] and Interacting Multiple Models (IMM) [3,4,5], permit the accurate estimation of BT position and velocity. The first interest of this paper is the comparison of tracking performance of bank of EKFs and UKFs inter-connected in a IMM architecture. The bank of filters will be compared in terms of (i) accuracy of BT position and velocity, (ii) relation with the Cramer Rao Lower Bound (CRLB), (iii) probability of correct classification (Pcc) of the target under tracking and, finally (iv) rough estimation of the computational load.

The paper is organised as follows: next section 2 briefly recalls the theory of EKF and UKF and the working principle of the IMM-EKF and IMM-UKF tracking architectures; section 3 presents the procedure to compute the CRLB of the tracking performance in presence of a BT while section 4 reports on the comparison between the CRLB and the proposed tracking architectures. Conclusions and references are presented in sections 5 and 6. Appendix A contains the details of the derivation of EKF and UKF for the BT.

2 RECALL OF EKF, UKF AND IMM THEORY

A complete description of the EKF, UKF and IMM architectures is reported in [4,5]; appendix A of this paper details the derivation of EKF and UKF for the BT study case. This section briefly resumes the working principles of the filters and the rationale for the IMM selection.

2.1 EKF working principle

The EKF is basically an extension of the linear Kalman Filter and it is used to account for the non linear BT state equation due to the presence of drag in the re-entry phase and/or non linear measurement process. The EKF only uses the first order terms in the Taylor series expansion of the non linear state equation. In general, when the filtering problem is highly non linear and the local linearity assumption breaks down, the EKF may introduce large estimation errors due to filter divergence.

2.2 UKF working principles

Similar to EKF, the UKF is a recursive minimum mean square error (MMSE) estimator. But unlike the EKF, the UKF does not approximate the non linear equations (dynamic and/or measurement). Instead it uses the true non linear model and approximates the pdf of the state vector [6]. This density, however, is still assumed Gaussian (in reality it is not, because of non linear dynamics/measurement equations) and is specified by $2n_x+1$ deterministically chosen samples or sigma points (n_x being the dimension of the state vector). The choice of the sigma points guarantees an accurate prediction of the mean and covariance up to the third order for Gaussian priors. The prediction step of the UKF is performed as follows. The unscented transformation [6] first computes the sigma points based on the values of the filtered state vector and the corresponding covariance matrix: $\hat{\mathbf{x}}_{k/k}, \mathbf{P}_{k/k}$. The sigma points are then propagated through the non linear functions and from them the predicted state and its covariance are computed.

2.3 Why the IMM

The rationale for choosing the IMM approach for the tracking of a potential BT is mainly based on: 1) Radar systems operate in a "dense" environment, i.e. in presence of a number of different threats like ABT, anti-radiation missile (ARM), BT and others. The selection of the proper filter for each target can be managed by the IMM. 2) The BT characteristics are not generally "a priori" known, thus it is required to "on-line" estimate the BT parameters to maximise the tracker accuracy. The IMM offers the possibility of mixing the output of different filters designed for different BTs, thus permitting the correct tracking of BTs pertaining to different "classes". 3) The probability of selecting one of the filters existing in the bank of the IMM gives a clear indication of the confidence of the tracker on the type of target under analysis; the Pcc is computed starting from the information available from the bank of filters.

3 PROPOSED APPROACH FOR TRACKING ARCHITECTURES COMPARISON

3.1 CRLB Theory

The relation with the CRLB (i.e. ideal performance) defines the capability of a system to be compliant with important Customer requirement such as range, azimuth and elevation accuracies of BT and air breathing target (ABT). If the CRLB accuracies of the system under analysis do not reach the Customer requirements, then it is mandatory to define different system architecture because the CRLB represents the lower limit of the system performance.

The CRLB, which depends on the measurement model, sensor characteristics and the target state model, plays an important role in algorithm evaluation and assessment of the level of approximation introduced by a particular tracking filter. For a general non linear filtering problem, the optimal recursive state estimator (the tracker) in the Bayesian sense requires the complete posterior density of the state to be determined as a function of time. In the special case of linear/Gaussian estimation, the required density is Gaussian and the solution is the well known Kalman filter, which gives also the accuracy of the estimated track. In the general non linear/non-Gaussian case the problem is very difficult and has no analytic closed form solution. A theoretical formula has been found in [7,8]; for sake of simplicity, the simulations described in this paper refer to the case of radar detection probability equal to 1. The above mentioned reference gives the strategy to include in the CRLB computation a detection probability <1.

The findings of [7,8] allow the CRLB computation for the BM tracking accuracy by means of the following methodology:

(i) compute the trajectory for the BM using the equations of section 2 process noise is assumed to be absent;
(ii) derive the EKF as described in section 3; suppose the a-priori knowledge of the BM parameters (drag coefficient as an example);
(iii) run the EKF of step (ii) using the "nominal" trajectory of step (i) considering the radar measurement errors only in the definition of the **R** matrix. In other word, no Monte Carlo trials are required being the effect of the radar uncertainty considered in the computation of the Kalman gain matrix.

The tracker accuracies found with the above mentioned procedures are the CRLB for the BM trajectory under analysis. Once that the CRLB is obtained, it is possible to compare the performance of any other tracking architecture; the comparison with the IMMs that we have developed is the subject of next section 4.

4 RESULTS

The first study case selected for the CRLB, IMM-EKF & IMM-UKF comparison concerns a simulated BT with following characteristics: single stage, linear consumption of propellant vs. flight time, drag coefficient=25000 N/m^2, radar detection range=150 km, target radar cross section=1 m^2 in approaching flight.. The parameters of the selected notional radar are: range accuracy=25m, azimuth accuracy=0.2°, elevation accuracy =0.2°, data rate=2 seconds, either Pd=1. The IMM-EKF is constituted by two EKFs with different initial values of β (15000 and 50000 N/m^2); the IMM-UKF contains two UKF with the same beta values of the IMM-EKF. Each IMM have also two "conventional" Kalman Filter (KF) to track ABT. To avoid a large number of figures, it has been decided to plot the cubic root of volume of uncertainty ellipsoid (one sigma) of the target track (CRV). Next figure 4.1 contains four curves:

(i) CRV computed from for the radar plots (curve labeled with 'Rad. Meas. Accuracy');
(ii) CRV computed from for the IMM-UKF (curve labeled with 'UKF tracker accuracy');
(iii) CRV computed from for the IMM-EKF (curve labeled with 'EKF tracker accuracy');
(iv) CRLB of CRV (curve labeled with 'CRLB').

The CRVs are reported versus time expressed in seconds; it must be noted that time=0 second indicates the track initialization time.

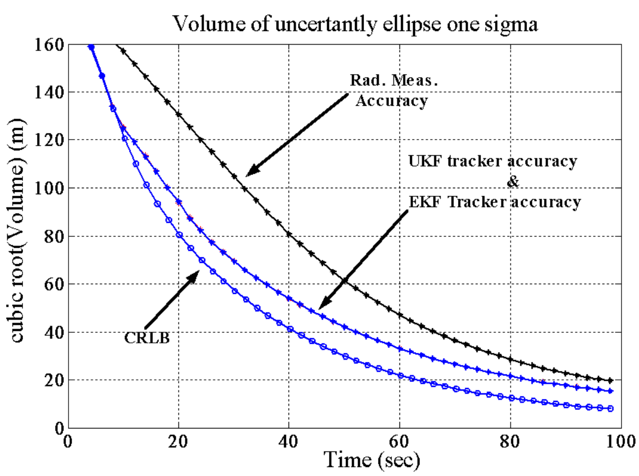

Figure 4.1 CRV for first study case

Subsequent figure 4.2 reports the classification probability achieved with the two IMM architecture; probability of correct classification (Pcc) of the threats under analysis higher than 0.9 is achieved after approx. 20 seconds.

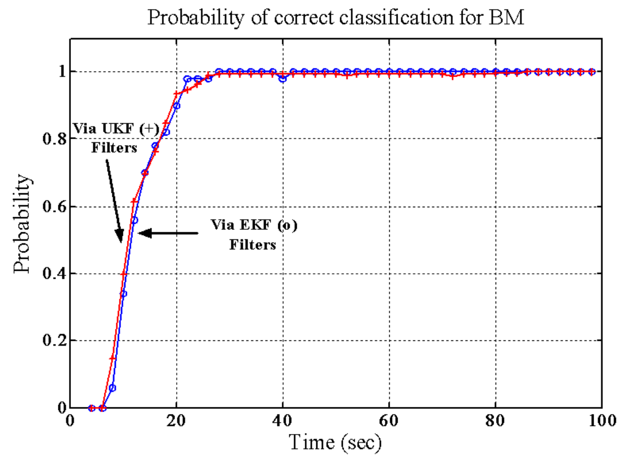

Figure 4.2 PCC of the two IMM architectures

A rough estimation of the computational costs required by the two trackers procedures is the following: said CC the computational cost of the IMM-EKF, the IMM-UKF requires approx. 5CC. From figures 4.1 and 4.2, it derives that for the study case under analysis, i.e. tracking of a BT with plot updating rate of 2 seconds, the IMM-UKF does not show substantial performance improvement with respect to the IMM-EKF; on the contrary, the IMM-UKF computational load is considerably higher than the IMM-EKF. For this reason, the selection of the IMM-EKF is recommended for this application.

Using the IMM-EKF approach, the BT typing (i.e. sub-classification between different BTs based on the different drag coefficients) has been attempted using the tracker beta estimation. The following study case has been selected: two BTs, single stage, linear consumption of propellant vs. flight time, drag coefficient=10000 N/m^2 (BT1) and 25000 (BT2), radar detection range=150 km, targets radar cross section=1 m^2 in approaching flight. The IMM-EKF above described has been selected for the BT typing. The achieved results are reported in next figure 4.3 presenting the estimated beta values vs. time. The two curves report the mean values of the beta estimations together with a confidence interval equal to the estimated standard deviations (±one sigma).

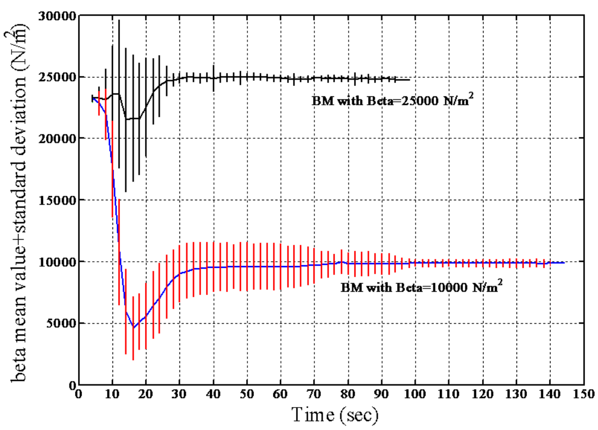

Figure 4.3 BT typing

Figure 4.3 demonstrates that the proposed approach for BT typing achieves reliable results after 30 seconds from the track initialization time.

5 CONCLUSIONS

The comparison between the IMM-UKF, the IMM-EKF and the CRLB has been presented and the results achieved in the classification and typing of BTs have been briefly discussed. It must be noted that in [9] it has been shown that the IMM-UKF presents performance advantages with respect to the IMM-EKF in a typical surveillance radar case (i.e. BT at distance >300 Km, updating plot rate in the order of 10 seconds). In a different radar application (i.e. BT at shorter distance and plot updating rate of 2 seconds) the same improvements have not been found. The main reason is that the first order approximation of the EKF (see next appendix A) guarantees performance near to the CRLB in case of a higher plot rate when the non linearity of the BT motion (see section 1) are well approximated by first order terms in the Taylor series expansion of the non linear motion equation. The tracker is able, using the update probability of IMM theory, to support the classification among all possible threats whose kinematics model are matched with the filters inside the bank and, in some cases, it is possible to perform a sub-classification using the tracker estimated parameter.

6 REFERENCES

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APPENDIX A

EKF theory

This section describes the procedure to derive the EKF starting from the equations describing the forces acting on a BT. For sake of simplicity, the hypothesis of flat earth is done and all the mathematical details connected to the change of co-ordinate system are not reported here. The theory of EKF is widely detailed in [3]. The EKF is used to account for the non linear target state equation due to the presence of drag [6]. At the k-th time instant the state vector $\mathbf{s}_k$ contains the position, the speed and acceleration components of target with respect to the Cartesian axes and the ballistic coefficient:

$$\mathrm{s}_k = \begin{bmatrix} x_k & \dot{x}_k & y_k & \dot{y}_k & z_k & \dot{z}_k & \beta_k \end{bmatrix}^T$$

The evolution of the state in time is [7]:

$$\mathbf{s}_{k+1} = \mathbf{\Phi s_k} + \mathbf{G} \cdot \left[\mathbf{a}_g + \mathbf{a}_{drag} \right] + \mathbf{w}_k = \mathbf{\Phi s}_k + \mathbf{G} \cdot \mathbf{a}_g + \mathbf{f}_k(\mathbf{s}_k) + \mathbf{w}_k$$

where $\mathbf{\Phi}$ is the 7×7 state transition matrix:

$$\mathbf{\Phi} = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

G is a 7×3 matrix where T is the radar scan time:

$$\mathbf{G} = \begin{bmatrix} T^2/2 & T & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & T^2/2 & T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & T^2/2 & T & 0 \end{bmatrix}^T$$

The column vectors $\mathbf{a}_g$ and $\mathbf{a}_{drag}$ contain respectively the gravity and the drag acceleration components along the axes *x, y* and *z*:

$$\mathbf{a}_g = \begin{bmatrix} a_{gx} \\ a_{gy} \\ a_{gz} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g_0 \end{bmatrix}$$

$$\mathbf{a}_{drag} = \begin{bmatrix} a_{drag\ x} \\ a_{drag\ y} \\ a_{drag\ z} \end{bmatrix} = -\frac{1}{2} \cdot \frac{\rho(z) \cdot g_0}{\beta} \cdot \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \cdot \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} =$$

$$\mathbf{f}_k(\mathbf{s}_k) = -\frac{1}{2} \cdot \frac{\rho(s_{k5}) \cdot g_0}{s_{k8}} \cdot \sqrt{s_{k2}{}^2 + s_{k4}{}^2 + s_{k6}{}^2} \cdot \begin{bmatrix} s_{k2} \\ s_{k4} \\ s_{k6} \end{bmatrix}$$

$\mathbf{f}_k(\mathbf{s}_k)$ is the non linear function which embeds the drag contribution; this function depends on the state components. $\mathbf{w}_k$ is the process noise: it has Gaussian pdf, with zero mean and non singular covariance matrix:

$$\mathbf{Q} = q \begin{pmatrix} \boldsymbol{\psi} & 0 & 0 & 0 \\ 0 & \boldsymbol{\psi} & 0 & 0 \\ 0 & 0 & \boldsymbol{\psi} & 0 \\ 0 & 0 & 0 & \sigma_\beta^2 \end{pmatrix}$$

with

$$\boldsymbol{\psi} = \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix}$$

where q is a scalar quantity accounting for the uncertainty on the target model; the variance σ_β^2 expresses the uncertainty on the ballistic coefficient. The range of values assumed by β is wide: from about 4000 to about 400000 and it has a relevant impact on the trajectory shape. Given the state $\hat{\mathbf{s}}_{k/k}$ estimated at the *k*-th time instant given all the measurements up to time instant k, with the corresponding estimation covariance matrix $\mathbf{P}_{k/k}$, the prediction at time instant k+1 is:

$$\hat{\mathbf{s}}_{k+1/k} = \mathbf{\Phi}\mathbf{s}_k + \mathbf{G} \cdot \left[\mathbf{a}_g + \mathbf{f}_k(\mathbf{s}_k)\right]$$

and the covariance matrix of the predicted state is:

$$\mathbf{P}_{k+1/k} = (\mathbf{\Phi} + \mathbf{G} \cdot \mathbf{J}_k)\mathbf{P}_{k/k}(\mathbf{\Phi} + \mathbf{G} \cdot \mathbf{J}_k)^T + \mathbf{Q}$$

where $\mathbf{J}_k$ (3×8 matrix) is the sum of Jacobian of the non linear functions $\mathbf{f}_k(\mathbf{s}_k)$ and $\mathbf{g}_k(\mathbf{s}_k)$ calculated at the state $\hat{\mathbf{s}}_{k/k}$ estimated at the previous step. The Jacobian is:

$$\mathbf{J}_k = \mathbf{F}_k + \mathbf{G}_k = [\nabla_{\mathbf{s}_k} \mathbf{f}_k^T(\mathbf{s}_k)]^T$$

For sake of brevity, the Jacobian of above equation is not detailed here.

UKF Theory

Same as the EKF, the UKF is a recursive MMSE (Minimum Mean Square Error) estimator. But unlike the EKF, the UKF [2] does not approximate the non linear state and measurement equations. It uses the true non linear model of state and/or measurements equation but approximates the pdf of the state vector. This density is still Gaussian, but is specified by a set of deterministically chosen sample (or *sigma*) points. The sigma points completely capture the true mean and covariance of the Gaussian density and when propagated through the non linear system, capture the posterior mean and covariance accurately to the second order for any non linearity [2].

The unscented transform (UT) is a method for calculating the statistics of a random vector which undergoes a non linear transformation. Let **x** be the n_x dimensional random vector, $\mathbf{g}: \mathrm{R}^{n_x} \rightarrow \mathrm{R}^{n_y}$ a non linear function and $\mathbf{y} = \mathbf{g}(\mathbf{x})$. Assume the mean and the covariance of **x** are $\bar{\mathbf{x}}$ and $\mathbf{P}_x$ respectively. The simple procedure for the calculation of the first two moments of $\mathbf{y}$ using the UT is as follows[6].

1. Compute $2n_x$ sigma points χ_i and their weights W_i

$$\chi_i = \bar{\mathbf{x}} + \left(\sqrt{n_x \mathbf{P}_x}\right)_i$$

$$i = 1, \ldots, n_x$$

$$W_i = 1/(2n_x)$$

$$\chi_i = \bar{\mathbf{x}} - \left(\sqrt{n_x \mathbf{P}_x}\right)_i$$

$$i = n_x + 1, \ldots, 2n_x$$

$$W_i = 1/(2n_x)$$

where $\left(\sqrt{(n_x \mathbf{P}_x}\right)_i$ is the i-th row or column of the matrix square root of $n_x \mathbf{P}_x$. The weights are normalised (i.e. add up to 1).

2. Propagate each sigma point through the non linear function

$$y_i = \mathbf{g}(\chi_i) \qquad i = 0, \ldots 2n_x$$

Estimated mean and covariance of $\mathbf{y}$ are computed as

$$\bar{\mathbf{y}} = \sum_{i=1}^{2n_x} W_i y_i$$

$$\mathbf{P}_y = \sum_{i=1}^{2n_x} W_i \left(y_i - \bar{\mathbf{y}}\right)\left(y_i - \bar{\mathbf{y}}\right)^T$$

Next we describe the implementation of the UKF assuming that at time K the state estimate and its covariance are $\mathbf{s}_{k|k}$ and $\mathbf{P}_{k|k}$ respectively.

- Compute sigma points $\xi_{k|k}(i)$ and weights W_i ($i = 1, \ldots, 12$) corresponding to $\mathbf{s}_{k|k}$ and $\mathbf{P}_{k|k}$;
- Propagate sigma points using state equation (described in the previous section) as follows

$$\xi_{k+1|k}(i) = \Phi_k\left[\xi_{k|k}(i)\right] + \mathbf{G}\begin{pmatrix} 0 \\ -g_0 \end{pmatrix}$$

- Compute the mean and covariance of the predicted state $\mathbf{s}_{k+1|k}$ and $\mathbf{P}_{k+1|k}$ using predicted sigma points $\xi_{k+1|k}(i)$, weights W_i as follows

$$\hat{\mathbf{s}}_{k+1|k} = \sum_{i=0}^{8} W_i \xi_{k+1|k}(i)$$

$$\mathbf{P}_{k+1|k} = \mathbf{Q} + \sum_{i=0}^{8} W_i \left[\xi_{k+1|k}(i) - \hat{\mathbf{s}}_{k+1|k}\right] \cdot \left[\xi_{k+1|k}(i) - \hat{\mathbf{s}}_{k+1|k}\right]^T$$

- Predict measurements derived from sigma points, that is

$$\varsigma_{k+1|k}(i) = \mathbf{H}\xi_{k+1|k}(i)$$

- Predict measurement and covariances

$$\hat{\mathbf{z}}_{k+1|k} = \sum_{i=0}^{8} W_i \varsigma_{k+1|k}(i)$$

$$\mathbf{P}_{zz} = \mathbf{R}_{k+1} + \sum_{i=0}^{8} W_i \left[\varsigma_{k+1|k}(i) - \hat{\mathbf{z}}_{k+1|k}\right] \cdot \left[\varsigma_{k+1|k}(i) - \hat{\mathbf{z}}_{k+1|k}\right]^T$$

$$\mathbf{P}_{sz} = \sum_{i=0}^{8} W_i \left[\xi_{k+1|k}(i) - \hat{\mathbf{s}}_{k+1|k}\right] \cdot \left[\varsigma_{k+1|k}(i) - \hat{\mathbf{z}}_{k+1|k}\right]^T$$

where $\mathbf{P}_{zz}$, $\mathbf{P}_{sz}$ are, respectively, the covariance matrix of the measurement and the cross-covariance of the measurement and the state variable.

- Compute the UKF gain and update state and covariance

$$\mathbf{K}_{k+1} = \mathbf{P}_{sz} \mathbf{P}_{zz}^{-1}$$

$$\mathbf{s}_{k+1|k+1} = \mathbf{s}_{k+1|k} + \mathbf{K}_{k+1}\left(\mathbf{z}_{k+1} - \hat{\mathbf{z}}_{k+1|k}\right)$$

$$\mathbf{P}_{k+1|k+1} = \mathbf{P}_{k+1|k} - \mathbf{K}_{k+1} \mathbf{P}_{zz} \mathbf{K}_{k+1}^T$$