# TRACKING VARIABLE NUMBER OF TARGETS USING SEQUENTIAL MONTE CARLO METHODS

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**ABSTRACT**

In this paper, we present a simulation-based method for multitarget tracking and detection using sequential Monte Carlo (SMC), or particle filtering (PF) methods. The proposed approach is applicable to nonlinear and non-Gaussian models for the target dynamics and measurement likelihood, where the environment is characterised by high clutter rate and low detection probability. The number of targets is estimated by continuously monitoring the events being represented by the regions of interest (ROIs) in the surveillance region. Subsequent to target detection, the sequential importance sampling filter is employed for recursive target state estimation, in conjunction with a 2-D data assignment method for measurement-to-target association. Computer simulations are also included to demonstrate and evaluate the performance of the proposed approach.

## 1. INTRODUCTION

Multitarget tracking (MTT) [1, 2] using one or multiple sensors that deals with the multitarget state estimation of moving targets may find applications in radar and sonar based tracking of objects, for example. Online joint multitarget detection and tracking remain a challenging problem, especially when the environment is hostile with a high degree of clutter and a low target detection probability.

Recently, the sequential Monte Carlo (SMC) methods [3, 4, 5], otherwise known as Particle Filtering (PF) [6, 7], have drawn attention in solving the MTT problem, since these methods are able to perform well even when the data models are nonlinear and non-Gaussian. The Probability Hypothesis Density (PHD) filter [8, 9, 10] is proposed to estimate the number of targets by modelling it as a random set. Its performance in terms of target detection and estimation is, however, significantly degraded when the environment is hostile [11, 12], and target identity is not preserved [11]. Existence JPDAF (E-JPDAF) [11], an extension of the JPDAF to the SMC framework, introduces an existence variable to indicate whether a track is active or inactive, and hence estimates the number of active tracks. This approach, however, imposes a bound on which the maximum targets can be estimated, which may be an unrealistic assumption for real-life applications.

In this paper, we present a SMC-based method for joint multitarget detection and tracking algorithms for a single observer, but can be readily extended for multiple observers. To estimate the number of targets, we first locate the regions of interest (ROIs) in the surveillance region which indicate where some events of interest may have happened, including the motion of true targets or spurious objects. Depending on the persistence these events are located, we may propose to execute either a track initiation, removal, or maintenance, respectively. With an estimated number of targets, the approach recursively estimates the target state, in conjunction with the very efficient and flexible 2-D data assignment strategy [1, 2, 12, 13, 14], which computes a set of feasible measurement-to-target assignments using a constrained optimisation approach.

The paper is organised as follows. In Section 2, we introduce the general state-space model for the MTT problem, and discuss the data association problem. We then formulate the likelihood functions for the observations, and introduce the necessary prior functions for the parameters of interest in order to construct the posterior distrubution function. In Section 3, we briefly present the general SMC framework and the sequential importance sampling filter for state estimation. Computer simulations are presented in Section 4, followed by the conclusions in Section 5.

## 2. DATA MODEL

### 2.1. State-Space and Dynamics

Let $\boldsymbol{\varphi}_t \triangleq \{\boldsymbol{x}_t, K_t\}$ be our primary parameter of interest in question, where $\boldsymbol{x}_t = [\boldsymbol{x}_{1,t}^T, ..., \boldsymbol{x}_{K_t,t}^T]^T$ is the combined state vector for $K_t$ unknown and time-varying targets. The state evolution equation for the $k$th target is

$\boldsymbol{x}_{k,t} = \boldsymbol{f}(\boldsymbol{x}_{k,t-1}) + \boldsymbol{v}_{k,t}$, $k \in \{1, ..., K_t\}$, where $\boldsymbol{f}(\cdot)$, which models the maneuvering of the target, can be a linear or nonlinear function. The noise $\boldsymbol{v}_{k,t}$ is a zero-mean random variable with a fixed and known covariance matrix $\boldsymbol{\Sigma}_v$. The number of targets is stochastically modelled as $K_t = K_{t-1} + \epsilon_{K_t}$, where $\epsilon_{K_t}$ is a discrete $iid$ random variable such that

$$\begin{aligned} \mathbf{Pr}(\epsilon_{K_t} = -1) &= h_d, \\ \mathbf{Pr}(\epsilon_{K_t} = 0) &= 1 - h_b - h_d, \\ \mathbf{Pr}(\epsilon_{K_t} = 1) &= h_b, \end{aligned} \tag{1}$$

where $h_b, h_d \in \{0, 1\}$, implying that the number of targets can change by no more than one at a given time. In general, we set $h_d = h_b = h/2$, where $h \in \{0, 1\}$, but when $K_{t-1} = 0$, we set $h_d = 0$ and $h_b = h$. Assuming that all targets are moving independently according to Markovian dynamics [15] and that they share the same dynamic model $\boldsymbol{f}(\cdot)$, we have the following joint dynamics for $\boldsymbol{\varphi}_t$

$$\begin{aligned} p(\boldsymbol{\varphi}_t|\boldsymbol{\varphi}_{t-1}) &= p(\boldsymbol{x}_t, K_t|\boldsymbol{x}_{t-1}, K_{t-1}), \\ &= p(\boldsymbol{x}_t|\boldsymbol{x}_{t-1}, K_t, K_{t-1})\, p(K_t|K_{t-1}), \end{aligned} \tag{2}$$

where the dynamic prior function for $\boldsymbol{x}_t$ is given by

$$\begin{aligned} &p(\boldsymbol{x}_t|\boldsymbol{x}_{t-1}, K_t, K_{t-1}) \\ &= \begin{cases} p_0(\boldsymbol{x}_{K_t,t}) \prod_{k=1}^{K_{t-1}} p(\boldsymbol{x}_{k,t}|\boldsymbol{x}_{k,t-1}), & \text{if } K_t = K_{t-1} + 1, \\ \prod_{k=1}^{K_t} p(\boldsymbol{x}_{k,t}|\boldsymbol{x}_{k,t-1}), & \text{if } K_t = K_{t-1}, \\ \prod_{k=1, k\neq k^*}^{K_t-1} p(\boldsymbol{x}_{k,t}|\boldsymbol{x}_{k,t-1}), & \text{if } K_t = K_{t-1} - 1, \end{cases} \end{aligned} \tag{3}$$

with $p_0(\boldsymbol{x}_{K_t,t})$ being the distribution function for a new target and $k^*$ being the target to be removed. The quantity $p(K_t|K_{t-1})$ in (2) is the prior distribution of $K_t$.

### 2.2. Observation Model and Likelihood

We consider a single observer $(x_o, y_o)$, whose position is known at every instant, scanning within the surveillance region $\mathcal{R}_V$ with volume $V$. Denoting an observation vector by $\boldsymbol{y}_t = [\boldsymbol{y}_{1,t}^T, ..., \boldsymbol{y}_{M_t,t}^T]^T$ with $M_t$ independent observations, we assume that the measurements may originate from true targets or from clutter, and that each of the true targets can generate at most one measurement at a given time but may go undetected. If the $m$th measurement $\boldsymbol{y}_{m,t}$ originates from the $k$th target, it follows [15] that $\boldsymbol{y}_{m,t} = \boldsymbol{g}(\boldsymbol{x}_{k,t}) + \boldsymbol{w}_{m,t}$, $m \in \{1, ..., M_t\}$, where $\boldsymbol{g}(\cdot)$ may be a nonlinear function and $\boldsymbol{w}_{m,t}$ is a zero-mean random variable with covariance $\boldsymbol{\Sigma}_w$. If, on the contrary, the measurement is due to clutter, we will assume it to be distributed uniformly over $R_V$.

To properly deal with the data association problem, i.e., measurement-to-target assignment, prior to target tracking, we denote a *measurement-to-target association* hypothesis by $\lambda_t = (\boldsymbol{\alpha}_t, N_{C_t}, N_{D_t})$, which is a stochastic variable [16], with the knowledge of $K_t$. The vector $\boldsymbol{\alpha}_t \in \mathbb{I}^{M_t}$ is known as the association vector, whose $m$th element is $\alpha_{m,t} = k$ if $\boldsymbol{y}_{m,t}$ originates from target $k$; otherwise $\alpha_{m,t} = 0$. The quantities $N_{C_t}$ and $N_{D_t}$ are the number of clutter measurements and detected targets, respectively.

Given the association hypothesis the likelihood for the observation $\boldsymbol{y}_t$ becomes [16, 17]

$$p(\boldsymbol{y}_t|\boldsymbol{\varphi}_t, \lambda_t) = V^{-N_{C_t}} \prod_{l\in\mathcal{I}_D} p(\boldsymbol{y}_{l,t}|\boldsymbol{x}_{\alpha_{l,t},t}), \tag{4}$$

where $\mathcal{I}_D$ is the set of observation indices corresponding to the $N_{D_t}$ detected targets, and $p(\boldsymbol{y}_{l,t}|\boldsymbol{x}_{\alpha_{l,t},t})$ is the likelihood function for measurement $l$ due to target $\alpha_{l,t}$.

### 2.3. Association Prior

Here we follow the definitions in [16] for the association hypothesis $\lambda_t = (\boldsymbol{\alpha}_t, N_{C_t}, N_{D_t})$, whose prior function, conditioned on $K_t$, has the following hierarchical structure

$$p(\lambda_t|K_t) = p(\boldsymbol{\alpha}_t|K_t, N_{C_t}, N_{D_t})p(N_{C_t})p(N_{D_t}|K_t), \tag{5}$$

where $p(\boldsymbol{\alpha}_t|K_t, N_{C_t}, N_{D_t}) = \left[\binom{M_t}{N_{D_t}} \frac{K_t!}{(K_t-N_{D_t})!}\right]^{-1}$ gives the prior of the number of valid hypothesis, $p(N_{C_t}) = (\Lambda_C)^{N_{C_t}} \exp(-\Lambda_C)/N_{C_t}!$, a Poisson distribution, is the prior for $N_{C_t}$ with a fixed and known expected value $\Lambda_C$, and $p(N_{D_t}|K_t) = \binom{K_t}{N_{D_t}} P_D^{N_{D_t}} (1 - P_D)^{K_t - N_{D_t}}$ is the binomial prior with a fixed and known $P_D$, target detection probability, shared by all targets.

## 3. SEQUENTIAL MONTE CARLO METHODS

Let the posterior distribution function of $\boldsymbol{\varphi}_t$ be

$$p(\boldsymbol{\varphi}_t|\boldsymbol{y}_{1:t}) \propto p(\boldsymbol{y}_t|\boldsymbol{\varphi}_t) \int p(\boldsymbol{\varphi}_t|\boldsymbol{\varphi}_{t-1})p(\boldsymbol{\varphi}_{t-1}|\boldsymbol{y}_{1:t-1})\, d\boldsymbol{\varphi}_{t-1}, \tag{6}$$

where $p(\boldsymbol{\varphi}_{t-1}|\boldsymbol{y}_{1:t-1})$ is the posterior distribution at $t-1$, and $p(\boldsymbol{\varphi}_t|\boldsymbol{\varphi}_{t-1})$ is the prior in (2). The notation $(\cdot)_{1:t}$ indicates all the elements from time 1 to time $t$. The recursion in (6) is initialised with some distribution, say $p(\boldsymbol{\varphi}_0) = p(\boldsymbol{x}_0, K_0)$. Once the sequence of filtering distributions is known, statistical inferences, like expectation, maximum *a posteriori* (MAP) estimates, and minimum mean square error (MMSE), etc., can be computed.

### 3.1. Sequential Importance Sampling (SIS)

Assuming that a set of $N$ particles $\{\boldsymbol{\varphi}_{t-1}^{(i)}\}_{i=1}^N$ and associated weights $\{w_{t-1}^{(i)}\}_{i=1}^N$ approximate $p(\boldsymbol{\varphi}_{t-1}|\boldsymbol{y}_{1:t-1})$, we sample a set of new particles $\{\boldsymbol{\varphi}_t^{(i)}\}_{i=1}^N$ from a proposal function $\boldsymbol{\varphi}_t^{(i)} \sim q(\boldsymbol{\varphi}_t|\boldsymbol{\varphi}_{t-1}^{(i)}, \boldsymbol{y}_{1:t})$, and associated weights $w_t^{(i)}$ may be updated as follows

$$w_t^{(i)} \propto w_{t-1}^{(i)} \frac{p_\varphi(\boldsymbol{y}_t|\boldsymbol{\varphi}_t^{(i)})\, p(\boldsymbol{\varphi}_t^{(i)}|\boldsymbol{\varphi}_{t-1}^{(i)})}{q(\boldsymbol{\varphi}_t|\boldsymbol{\varphi}_{t-1}^{(i)}, \boldsymbol{y}_{1:t})}, \ \sum_{i=1}^N w_t^{(i)} = 1, \tag{7}$$

where $p_\varphi(\boldsymbol{y}_t|\boldsymbol{\varphi}_t^{(i)})$ is known as the marginalised likelihood and given by

$$p_\varphi(\boldsymbol{y}_t|\boldsymbol{\varphi}_t^{(i)}) \propto \sum_{\lambda^{(i_j)} \in \mathfrak{A}_t} p(\boldsymbol{y}_t|\boldsymbol{\varphi}_t^{(i)}, \lambda^{(i_j)}) p(\lambda^{(i_j)}|K_t^{(i)}), \tag{8}$$

where $\lambda^{(i_j)}$ is the $j$th feasible hypothesis of the $i$th particle, and $\mathfrak{A}_t$ is the space occupied by all feasible hypotheses. The particles $\{\boldsymbol{\varphi}_t^{(i)}\}_{i=1}^N$ with the weights $\{w_t^{(i)}\}_{i=1}^N$ are then approximately distributed according to $p(\boldsymbol{\varphi}_t|\boldsymbol{y}_{1:t})$.

### 3.2. Importance Sampling Function

Here we present a choice of importance sampling function $\boldsymbol{\varphi}_t^{(i)} \sim q(\boldsymbol{\varphi}_t|\boldsymbol{\varphi}_{t-1}^{(i)}, \boldsymbol{y}_{1:t})$, which may be expressed as

$$\begin{aligned} q(\boldsymbol{\varphi}_t|\boldsymbol{\varphi}_{t-1}^{(i)}, \boldsymbol{y}_{1:t}) =& q(\boldsymbol{x}_t|\boldsymbol{x}_{t-1}^{(i)}, K_t^{(i)}, K_{t-1}^{(i)}, \boldsymbol{y}_{1:t})\times \\ & q(K_t|K_{t-1}^{(i)}, \boldsymbol{y}_{1:t}), \end{aligned} \tag{9}$$

where the terms on the right-hand side are the proposal functions for $\boldsymbol{x}_t$ and $K_t$, respectively.

In this paper we only briefly describe the idea of the target detection method. For details please refer to [12]. We first use an observation clustering algorithm to locate the ROIs, given a buffer of observations $\{\boldsymbol{y}_{t'}\}_{t'=t-\tau}^t$, where $\tau$ is the buffer size. Each ROI is a cluster of observations collected successively that may have originated from a true target. Associating these ROIs with the existing tracks, we may propose a birth (death) move if an ROI (a track) cannot be associated with a track (an ROI), or a maintenance move otherwise. For instance, if an event is being represented by a set of ROIs found at different times, we may initiate a new track. Likewise, if a track has failed to be associated with an observation over a long period of time, we may drop this track. If all ROIs found can be associated with the existing tracks, a maintenance move is used instead. Accordingly, we may *deterministically* obtain an estimate of the number of targets, say $K_t = K^*$, with probability one. Since all particles are to share the same value of $K_t$, the proposal function $q(K_t|K_{t-1}^{(i)}, \boldsymbol{y}_{1:t})$ is not applicable in this paper. From this point onward, as we intend to remain the notation $K_t^{(i)}$ intact in the subsequent development, the particles $\{K_t^{(i)}\}_{i=1}^N$ will share the same value.

The choice of $q(\boldsymbol{x}_t|\boldsymbol{x}_{t-1}^{(i)}, K_t^{(i)}, K_{t-1}^{(i)})$ depends on the value of $K_t^{(i)}$, indicating which move is selected. When track maintenance is proposed, i.e., $K_t^{(i)} = K_{t-1}^{(i)} = \bar{K}^{(i)}$, we sample $\boldsymbol{x}_t^{(i)}$ according to $q(\boldsymbol{x}_t|\boldsymbol{x}_{t-1}^{(i)}, K_t^{(i)}, K_{t-1}^{(i)}) = p(\boldsymbol{x}_t|\boldsymbol{x}_{t-1}^{(i)}, \bar{K}_t^{(i)})$, which is the dynamic prior with $\bar{K}_t^{(i)}$ targets. When the death move is proposed, i.e., $K_t^{(i)} = K_{t-1}^{(i)} - 1$, where target $k^* \in \{1, ..., K_{t-1}^{(i)}\}$ is selected for removal, we first kill all the particles corresponding to that target, and then sample the remaining targets $K_{t-1}^{(i)} - 1$ targets via the track maintenance move. Finally, when the birth move is proposed, i.e., $K_t^{(i)} = K_{t-1}^{(i)} + 1$, we first initialise the target state of the initiated target, followed by the track maintenance move for the existing $K_{t-1}^{(i)}$ targets. For the target initialisation, we may sample uniformly from the measurements of $\boldsymbol{y}_t$ that are within the region of interest, for example. The following schema briefly summarise the steps in the birth and death moves.

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**Birth Move**

1. Assume that an unassociated ROI whose persistence is the largest is selected for initiation, say $\mathcal{S}_t$, which consists of a buffer of measurements clustered over a period of time [12], we initialise the particles for the new track, for $i \in \{1, ..., N\}$, by $\tilde{\boldsymbol{x}}_0^{(i)} \sim q(\tilde{\boldsymbol{x}}_0|\mathcal{S}_t)$.

2. Sample the $K_{t-1}^{(i)}$ tracks $\tilde{\boldsymbol{x}}_t^{(i)} \sim q(\tilde{\boldsymbol{x}}_t|\boldsymbol{x}_{t-1}^{(i)}, K_{t-1}^{(i)})$.

3. Insert the particles to form an augmented state vector $\boldsymbol{x}_t^{(i)} = [\tilde{\boldsymbol{x}}_t^{(i)^T}, \tilde{\boldsymbol{x}}_0^{(i)^T}]^T$.

4. Increment the number of targets to $K_t^{(i)} = K_{t-1}^{(i)} + 1$.

**Death Move**

1. Assume that track $k^*$ has failed to associate with a measurement for a period of time, we will remove this track as well as its particles $i \in \{1, ..., N\}$, i.e., $\tilde{\boldsymbol{x}}_{t-1}^{(i)} = [\boldsymbol{x}_{1,t-1}^{(i)^T}, ..., \boldsymbol{x}_{k^*-1,t-1}^{(i)^T}, \boldsymbol{x}_{k^*+1,t-1}^{(i)^T}, ..., \boldsymbol{x}_{K_{t-1}^{(i)},t-1}^{(i)^T}]^T$.

2. Decrement the number of targets to $K_t^{(i)} = K_{t-1}^{(i)} - 1$.

3. Sample the $K_t^{(i)}$ tracks $\tilde{\boldsymbol{x}}_t^{(i)} \sim q(\tilde{\boldsymbol{x}}_t|\tilde{\boldsymbol{x}}_{t-1}^{(i)}, K_t^{(i)})$, and then $\boldsymbol{x}_t^{(i)} = \tilde{\boldsymbol{x}}_t^{(i)}$.

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Finally, given $\boldsymbol{\varphi}_t^{(i)}$ and $\boldsymbol{y}_t$ we employ the efficient 2-D data assignment algorithm [1, 2] to compute $\mathfrak{M}_t^{(i)}$ feasible association hypotheses, i.e., $\lambda_t^{(i_j)} \in \mathfrak{A}_t$ and the associated

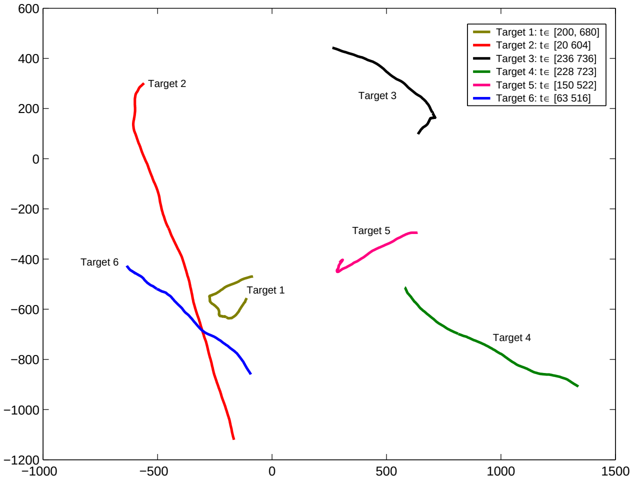


**Fig. 1**. Synthesised targets within $\mathcal{R}_V = [-1000, 1500] \times [-1200, 600]$.

| Parameters | Values |
|---|---|
| $(x_o, y_o)$ | $(0, 0)$ |
| $\sigma_x, \sigma_y$ | $8 \times 10^{-2}$ |
| $T_s$ (sampling interval) | 1 |
| $T$ (no. of scans) | 1000 |
| $P_D$ | 0.5 |

**Table 4.1**. Other parameters for computer simulations.

probabilities [1] $\mathfrak{U}_t^{(i_j)}$ for $j = 1, ..., \mathfrak{M}_t^{(i)}$ in order to compute the marginalised likelihood [13] in (8). Other association hypotheses are excluded as their probabilities are negligible when compared to those of the included.

## 4. COMPUTER SIMULATIONS

In this experiment, we evaluate the performance of the proposed method in target detection and estimation for 6 synthesised targets, appearing at different times as shown in Figs. 1, under different conditions. The targets are systhesised using a near constant velocity dynamic model [2] with state noise standard deviations $\sigma_x$ and $\sigma_y$, and the observation model is a nonlinear model with bearing and range measurements whose standard deviations are $\boldsymbol{\sigma} = [\sigma_\phi, \sigma_r]$. Other parameters for the simulations are listed in Table 4.1.

In the evaluation, we compute the Root Mean Square Error (RMSE) for $L = 20$ independent runs, defined as $RMSE = \frac{1}{L}\sum_{l=1}^{L} RMSE_l$, where $RMSE_l = \sqrt{\frac{1}{T}\sum_{t=1}^{T} ||\boldsymbol{x}_t - \hat{\boldsymbol{x}}_t^l||^2 / K_t}$, where $l = 1, ..., L$ and $\hat{\boldsymbol{x}}_t^l$ is a posterior mean estimate of $\boldsymbol{x}_t$ for $l$th run. We also compute the average probability of true and false targets detected by the algorithm for $L$ runs.

[1]Note that $\sum_{j=1}^{\mathfrak{M}_t^{(i)}} \mathfrak{U}_t^{(i_j)} = 1$ with $\mathfrak{U}_t^{(i_a)} \geq \mathfrak{U}_t^{(i_b)}$ for $a < b$ and $a, b \in \{1, ..., \mathfrak{M}_t^{(i)}\}$.

| $\boldsymbol{\sigma}, \Lambda_C, N$ | $P_0(\%)$ | $P_1(\%)$ | **RMSE** (1-$\sigma$) |
|---|---|---|---|
| $[1°, 25], 3, 5000$ | 100.0 | 100.0 | 13.2 (3.4) |
| $[1°, 40], 3, 5000$ | 100.0 | 100.0 | 16.0 (2.6) |
| $[2°, 25], 10, 5000$ | 95.0 | 100.0 | 18.5 (4.3) |
| $[2°, 40], 10, 5000$ | 85.0 | 100.0 | 22.4 (3.0) |
| $[5°, 25], 15, 5000$ | 67.0 | 100.0 | 28.8 (8.2) |
| $[5°, 40], 15, 5000$ | 64.0 | 100.0 | 33.1 (7.6) |

**Table 4.2**. Evaluation results with $L = 20$ runs for different values of $\boldsymbol{\sigma}$ and $\Lambda_C$, where $P_0$ is the number of true targets out of total number of detected tracks, and $P_1$ is the probability of true targets correctly detected, respectively.

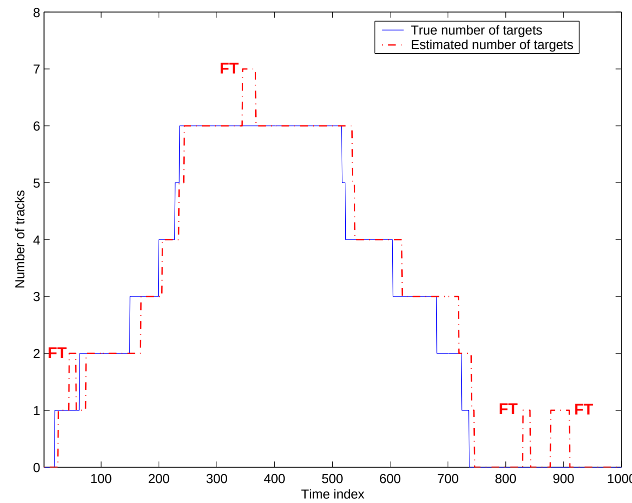


**Fig. 2**. A comparison between the true and estimated number of targets with the symbol "**FT**" referring to false tracks.

Fig. 2 shows a comparison between the true and estimated number of targets for one run with parameters $\boldsymbol{\sigma} = [2°, 25]$, $\Lambda_C = 10$, and $N = 5000$. Except the false tracks and the time delays, the proposed method tracks the number of targets very well. Apparently, the durations of the false tracks are relatively shorter than that of those true tracks. According to Table 4.2, the true targets are all correctly detected by the algorithm for all conditions under consideration, but when $\Lambda_C \geq 10$ the number of false tracks increases. Since the clustering approach the proposed method uses is sensitive to the density of spurious measurements, the frequency of birth proposal increases as $\Lambda_C$ becomes larger. Furthermore, it appears that according to the RMSE the impact of $\sigma_r$ to the tracking performance is less significant than that of $\sigma_\phi$.

## 5. CONCLUSIONS

In this paper, we presented a SMC-based method for joint multitarget detection and tracking. It estimated the number of targets by continuously monitoring the conditions of the events represented by the ROIs in the surveillance region, followed by a sequential importance sampling filter for the multitarget state in conjunction with the flexible 2-D data assignment algorithm for data association. Computer simulations demonstrated that the proposed approach performed well for target detection and estimation, even though the environment is very hostile.

## 6. ACKNOWLEDGEMENTS

The research of Ng, Li and Godsill was sponsored by the Data and Information Fusion – Defence Technology Centre, UK, Research Grant 10.2. Vermaak's work was funded by QinetiQ under contract QinetiQ Contract CU006 0000014890. The authors thank these parties for funding this work.

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