The use of Probability Control in CDMA systems

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Abstract—**CDMA systems commonly use power control mechanism to reduce the amount of interference between the users. We challenge this common practice by considering non-continuous transmission and probability control mechanism. We analyze the effect of the transmission probability on the average signal-to-noise plus interference ratio (SIR), and give a sufficient condition for CDMA systems in which probability control (and not power control) is the optimal method for balancing users' interference. We also demonstrate this condition on a CDMA system with a single RAKE finger. Results are applicable both for CDMA and for impulse radio (IR) systems.**

Index Terms—**Code division multiple access (CDMA), Impulse Radio (IR), Ultra-Wdieband (UWB), Power control, Probability control.**

I. Introduction

Direct sequence code division multiple access (CDMA) systems that use non-continuous transmission were considered throughout the history of CDMA systems, but gained renewed interest with the emergence of impulse radio (IR) technology [1],[2]. While IR technology had evolved independently of the CDMA technology, several authors had pointed out that an IR systems can be regarded as a type of CDMA system [3],[4].

In this paper we consider CDMA systems that achieve non-continuous transmission by puncturing of the coded data symbols. Puncturing is often used in CDMA systems, mostly for rate adjusting of the error control code (e.g. [5]). In contrast to existing work, we will not analyze the effect of the puncturing on specific codes, but rather analyze the effect of the "quiet" periods caused by this puncturing on the interference in the system.

The use of non-continuous transmission opens a new dimension for system optimization. Intuitively, decreasing the transmission probability reduces the amount of collisions between symbols of different users. On the other hand, to achieve the same performance, the transmission energy must be compensated and therefore increase the interference caused by the transmitted symbols.

This optimization problem had only been addressed for some specific special cases. For example, Sadler and Swami [6] used a random spreading approach, and found an upper bound on the BER that is independent of the chip transmission probability. But, this upper bound is not too tight and some results indicate that the transmission probability has affect on the performance. Fishler and Poor [7] had shown for the 2 user broadcast channel in which both users use the same parameters, that the BER is a non-increasing function of the transmission probability. In addition, in a previous work [8], we showed for the special case of IR multiple access channel, that maximizing user rates requires that all users transmit their maximal allowed average power. In such case, any tradeoff between the users (increasing one user rate on expense of other users) is controlled by changing the chip transmission probability while keeping all transmission powers at their maximum, i.e., the transmission probability has a major affect on the achievable performance. Note that this result counters the common practice of using continuous transmission CDMA and using power control to balance between user rates.

In this paper we extend this work to the optimization of the average SIR of each user in almost any CDMA system. For this end, we define the type of CDMA systems that we analyze as Generalized CDMA (GCDMA) systems. A GCDMA system is a CDMA system that uses random spreading and allows non-continuous transmission by random puncturing. Note that we make no assumption on the system topology, nor on the channel between transmitters and receivers in the system.

We consider the optimization of a GCDMA system in a $2\times K$ dimensional domain (where K is the number of users). For each user we search for the optimal chip energy and transmission probability. In practical system, the first part is termed power control [9], along the same line, we term the second part probability control. Obviously, this extended optimization, which includes both power control and probability control, cannot do worst than simpler optimizations that use either power control or probability control. In the sequel we show that for many systems, the probability control alone can achieve the optimal performance.

The exact definition of the analyzed system and the optimization problem is given in section II. In section III we define the probability controlled optimal systems and prove the main results. Section IV include the discussion of the results and numerical examples, and section V contains our conclusions.

Manuscript submitted May 29, 2005 .

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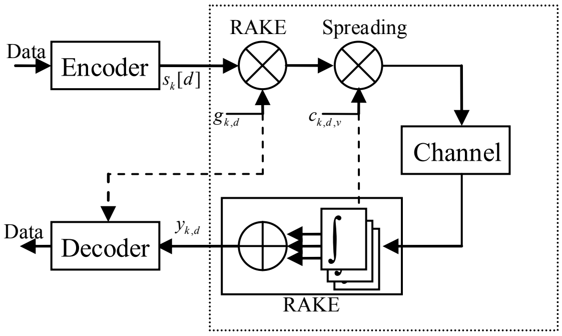

Fig. 1. Generalized CDMA System structure.

II. System Model

A. Generalized CDMA system

We use the term *Generalized CDMA system* for any CDMA system which employ random spreading and allows non-continuous transmission. The signal transmitted by the k[th] user is given by (e.g. [10],[11]):

$$x_k(t)=\sqrt{E_k}\sum_d s_k[d]g_{k,d}\sum_{v=0}^{N_k-1}c_{k,d,v}p(t-(dN_k+v)T_c)\ , \quad (1)$$

where $p(t)$ is the transmitted pulse shape, T_c is the chip time, E_k is the k^{th} user chip energy, N_k its spreading factor, $s_k[d]$ is its d^{th} data symbol and $c_{k,d,v}$ its spreading sequence which is different for each symbol. For normalization we set $\int p^2(t)dt=1$, and $E\left[s_k^2[d]\right]=1$.

The non-continuous transmission is generated by the iid binary gating sequence, $g_{k,d}\in\{0,1\}$, which "punctures" the CDMA signal, and determines whether each symbol is transmitted or not. The probability $p_k=\Pr(g_{k,d}=1)$ is termed the user transmission probability. The transmission probability determines the nature of the system: CDMA systems use $p_k=1$. The case of $p_k<1$ is termed non-continuous transmission, and IR systems use $p_k<<1$ (usually with $N_k=1$).

The compensation for the data lost in the punctured symbols is the task of the error control decoder. But, in this paper we wish to focus on the effect of the puncturing on the interference and not limit the discussion to any specific code. Therefore we only measure the performance at the output of the RAKE receiver. This is demonstrated in Fig. 1. In this paper we only analyze the block marked with the dotted line, including the puncturing, the spreading, the transmission through the channel and the dispreading in the RAKE receiver. We assume that the final performance of the system, including the error correction, can be specified by signal to noise plus interference ratio (SIR) at the RAKE receiver output, i.e., we assume that each decoder has a specified average SIR for which it can achieve the desired error rate. Although this is a somewhat simplified assumptions, its study is worthwhile as it give a good idea on system performance without specifying the error control code [12].

In this paper we make no assumptions on the communication network topography. The network includes K transmitters and K receivers, and the k[th] receiver is interested only in the information transmitted from the k[th] transmitter. The signal received by the k[th] receiver is given by:

$$r_k(t)=\sum_{j=1}^{K}\int_{-\infty}^{\infty}h_{kj}(\tau)x_j(t-\tau)d\tau+n_k(t)\ , \quad (2)$$

where $h_{kj}(\tau)$ is the impulse response of the channel from the j^{th} transmitter to the k^{th} receiver, and $n_k(t)$ is the noise measured at the receiver, modeled as white Gaussian noise with spectral density $N_0/2$. In order not to limit the paper scope to a specific channel model, we perform the analysis and state the proofs for any given set of K^2 channels. Since the proof holds for any set of channels, it also holds for any random channel model. Note that this model does not necessarily assume synchronous transmission, and the channel responses can contain different delay for each transmitter receiver pair.

In the sequel we need to write derivatives with respect to symbol parameters. For this reason we define a separate transmission probability and transmission power for each symbol. We rewrite (1) as:

$$x_k(t)=\sum_d\sqrt{E_{k,d}}s_k[d]g_{k,d}\sum_{v=0}^{N_k-1}c_{k,d,v}p(t-(dN_k+v)T_c)\ , \quad (1).\text{a}$$

where $E_{k,d}$ is the energy of a chip in the d^{th} symbol transmitted by the k^{th} user. The two formulations are identical because we set $E_{k,d}=E_k$. For the probability we term the symbol transmission probability $p_{k,d}=\Pr(g_{k,d,v}=1)$. And again, we set $p_{k,d}=p_k$.

B. Receiver structure

The receiver assumes a tapped delay line channel model and therefore employs a RAKE receiver [10]. The k^{th} user employs a RAKE receiver with L_k fingers. Each finger uses a spreading waveform correlator to despread the signal for each symbol, resulting in the vector of finger outputs $\mathbf{y}_{k,d}=[y_{k,d}\left(\tau_k(0)\right),\ldots,y_{k,d}\left(\tau_k(L_k-1)\right)]^T$ in which:

$$y_{k,d}(\tau)=\int_{-\infty}^{\infty}r_k(t)\sum_{v=0}^{N_k-1}c_{k,d,v}p(t-(dN_k+v)T_c-\tau)dt\ , \quad (3)$$

and $\tau_k(l)$ is the delay used by the l^{th} finger, chosen by the receiver to maximize the received signal power.

The vector of finger outputs can be expressed as:

$$\mathbf{y}_{k,d}=\boldsymbol{\mu}_{k,d}s_k[d]+\mathbf{u}_{k,d}\ , \quad (4)$$

where $s_k[d]$ is the symbol of interest, $\boldsymbol{\mu}_{k,d}$ is a vector of gains corresponding to the symbol of interest, and $\mathbf{u}_{k,d}$ is a vector that include the noise, multiple user interference (MUI) and inter symbol interference (ISI) terms.

The analysis of the received signal is performed in two stages. In the first stage we assume a specific set of spreading sequences, and derive an expression for the SIR at the output of the RAKE receiver given these spreading sequences (considering the random noise and interfering users). At the second stage we will consider the random nature of this SIR, resulting from the randomness of the spreading sequences.

For the first stage, as shown in appendix A, given the spreading sequences, $\boldsymbol{\mu}_{k,d}$ is a deterministic vector given by:

$$\boldsymbol{\mu}_{k,d} = \sqrt{E_{k,d}}\,\mathbf{r}_{k,k,d,d} \,, \tag{5}$$

while $\mathbf{u}_{k,d}$ is a random vector with zero mean and covariance matrix:

$$\begin{aligned}\mathbf{R}_{k,d} &= \sum_{j\neq k}\sum_{a} E_{j,a}\mathbf{r}_{k,j,a,d}\mathbf{r}_{k,j,a,d}^{T} \\ &+ \sum_{a\neq d} E_{k,a}\mathbf{r}_{k,k,a,d}\mathbf{r}_{k,k,a,d}^{T} \\ &+ \frac{N_0}{2}\mathbf{R}_{\mathbf{n}}\end{aligned} \,, \tag{6}$$

where the vectors $\mathbf{r}_{k,j,a,d}$ and the noise covariance matrix $\mathbf{R}_{\mathbf{n}}$ are defined in appendix A. Since the spreading sequence are random, the covariance matrix, $\mathbf{R}_{k,d}$, is also random with its distribution determined by the distribution of the spreading sequences through the random $\mathbf{r}_{k,j,a,d}$ and $\mathbf{R}_{\mathbf{n}}$.

The receiver uses linear combining of the RAKE finger output to produce the decision variable. Denoting the weight vector by $\mathbf{w}_{k,d}$, the decision variable is given by:

$$z_{k,d} = \mathbf{w}_{k,d}^{T}\mathbf{y}_{k,d} \,. \tag{7}$$

The SIR achieved for the reception of the d^{th} symbol by the k^{th} user is:

$$\rho_{k,d} = \frac{E\left[z_{k,d}\right]^2}{VAR\left[z_{k,d}\right]} = \frac{\mathbf{w}_{k,d}^{T}\boldsymbol{\mu}_{k,d}\boldsymbol{\mu}_{k,d}^{T}\mathbf{w}_{k,d}}{\mathbf{w}_{k,d}^{T}\mathbf{R}_{k,d}\mathbf{w}_{k,d}} \,, \tag{8}$$

where the expectation is taken over the noise and the transmitted symbols.

C. Optimization problem

The performance of each user is measured by the average SIR (ASIR) it achieves, given by:

$$\rho_k = E\left[\rho_{k,d}\right] , \tag{9}$$

where the expectation is taken over all possible spreading and puncturing sequences.

Using the formulation of the previous subsection, a system state can be defined by two vectors: the users transmission probabilities vector $\mathbf{p} = [p_1,...,p_K]^T$, and the users chip energies vector $\mathbf{E} = [E_1,...,E_K]^T$. The user ASIRs can then be written in a vector form: $\boldsymbol{\rho} = \boldsymbol{\rho}(\mathbf{p},\mathbf{E})$, where $\boldsymbol{\rho} = [\rho_1,...,\rho_K]^T$. The optimization domain is defined by the system constraints. Each user has constraints on both the peak power ($S_k^{\mathbf{pk}}$) and the average power ($S_k^{\mathbf{av}}$). The system constraints are summarized by:

$$\begin{aligned} &0 \le p_k \le 1 \\ &0 \le p_k E_k / T_c \le S_k^{\mathbf{av}} \quad , \quad k = 1,...,K \,. \\ &0 \le E_k / T_c \le S_k^{\mathbf{pk}} \end{aligned} \tag{10}$$

Generally, such a multi-objective optimization problem requires the definition of the user preferences in order to identify an optimal operating point. In this paper we avoid this by showing that for any user preferences it is always preferred for each user to transmit at its maximal allowed power with probability control (and not use a continuous transmission with power control).

III. PROBABILITY CONTROLLED OPTIMAL SYSTEMS

We begin by defining a *probability controlled optimal system*:

Definition 1: A GCDMA system is termed **Probability controlled optimal** if under any user preferences, any optimal solution satisfies: $E_k = T_{\mathrm{c}} S_k^{\mathbf{pk}}$ for all users ($k = 1,...,K$).

It is quite surprising to find a probability controlled optimal CDMA system, as it contradicts the popular notion of power control. A probability controlled optimal CDMA system does not require a power control mechanism, since it is always preferred that each user will transmit its maximal allowed peak power. Instead, in a probability controlled optimal system, the tradeoffs between users are controlled by the users transmission probabilities. Though, the power control mechanism should be replaced by a probability control mechanism.

Note that in a probability controlled optimal system, a user will not necessarily transmit its maximal allowed average power ($S_k^{\mathbf{av}}$). But, the control of the user average power is achieved by a reduction of the transmission probability without a change to the transmitted symbol energy.

The following proposition provides a sufficient condition for a GCDMA system to be probability controlled optimal.

Proposition 1: A sufficient condition for a GCDMA system (defined by equations (1)-(2)) to be a **probability controlled optimal system** is that:

$$\frac{\partial^2 \rho_k(\mathbf{p},\mathbf{E})}{\partial E_{j,a}^2} \ge 0 \,, \tag{11}$$

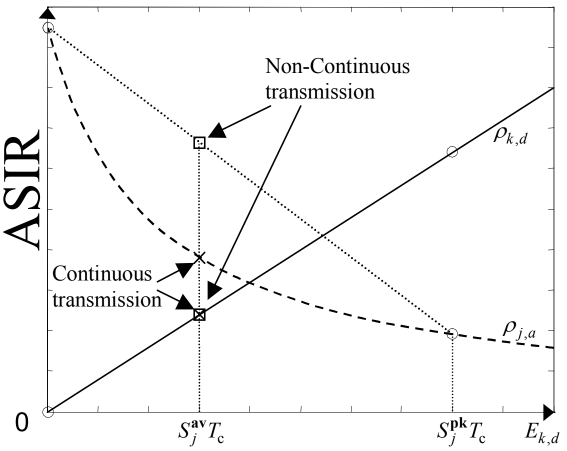

Fig. 2. Typical symbol average SNR vs. the d^th symbol energy transmitted by the k^th user.

for all integer a , $k = 1,...,K$, $j = 1,...,K$.

Due to lack of we bring only an intuitive proof for this condition. The formal proof will be included in the full version of this paper.

We consider a GCDMA system and examine the effect of the d th symbol transmitted by the k th transmitter. We wish to determine whether it is preferred that the k,d symbol is transmitted with probability 1 or with smaller probability and higher energy. For this purpose Fig. 2 depicts a typical ASIR as a function of the k,d symbol energy ($E_{k,d}$), when all other system parameters remain constant. The figure depicts two curves. The solid line (marked $\rho_{k,d}$) shows the ASIR of the k,d symbol, i.e., the desired symbol ASIR, which is typically linear with $E_{k,d}$. The dashed line (marked $\rho_{j,a}$) shows the ASIR of another symbol for which the k,d symbol is an interference ($j \neq k$ or $j = k, a \neq d$). The ASIR of such symbol is always positive and bounded from above by $\rho_{j,a}(0)$ Typically this ASIR is also monotonic decreasing and convex. First, consider continuous transmission at the allowed average symbol energy ($S_j^{\mathbf{av}}T_c$). The ASIRs achieved in this case are marked with x marks. Next, consider impulsive transmission of this symbol, i.e., according to some probability, the k,d symbol energy is either zero or $S_j^{\mathbf{pk}}T_c$. The ASIRs achieved in each of the cases is marked with circles, while the averaging according to the transmission probability is marked with squares. It is easy to see that if all ASIR curves are convex, then theorem 1 is a simple application of Jansen's inequality.

Using the condition of proposition 1 we wish to demonstrate the existence of a probability controlled optimal system. Again due to the lack of space we will only consider the degenerated scenario in which the RAKE receivers of all users have only a single finger ($L_k = 1$).

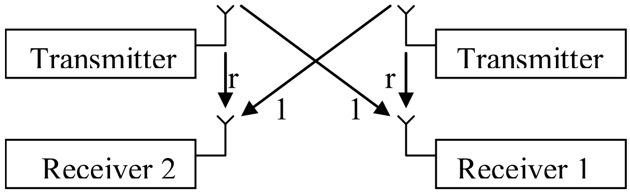

Fig. 3. Two users symmetric interference channel.

Corollary 1: a GCDMA system using single finger RAKE receivers is probability controlled optimal.
Proof of Corollary 1: see appendix B.

IV. Discussion

Nearly every existing CDMA system employ probability control to reduce the mutual interference between the users. The results of the previous section show the importance of probability control as a substitution or at least an addition to the existing power control.

Although the results presented are very definite, we should note that the average SIR is only one of the three popular performance measures for communication systems. The other two performance measures are the uncoded bit error rate (BER) [7],[6], and the achievable data rate [13]. While the proof given here can be valid for the achievable rate at the low single to noise regime, it is not valid in general for the achievable rate nor for the BER. Since none of this performance measures can accurately predict the performance of a practical communication system, we cannot say that every communication system will be optimized by replacing the power control mechanism with a probability control mechanism. However, the results for the average SIR are convincing enough to say that any CDMA system should consider the addition of a probability control mechanism.

In order to demonstrate the advantage of probability control over power control we performed several simulations of an ultra-wideband (UWB) DS-CDMA system [14]. This CDMA system is characterized by a chip rate of 1.3GHz, a center frequency of 3.9GHz and a root raised cosine (RRC) pulse shape with roll-off factor of 0.5. The simulated system differs from the reference system only in the use of random ternary spreading sequences. The simulations examine the 2 users symmetric interference channel depicted in Fig. 3. In this channel, receiver 1 only tries to decode the signal transmitted from transmitter 1, in the presence of the interference from transmitter 2. Due to the system topography, the channel gain of the interfering user is larger than the channel gain of the desired user. We denote by interference ratio (r) the ratio between the channel gain of the interfering user and the channel gain of the desired user.

For the first simulation we assume that the channel is an additive white Gaussian noise channel (AWGN), and we are interested in equal ASIR for both users. To achieve equal ASIR we use identical parameters to both users (i.e., the transmission probability and chip energy). The interference

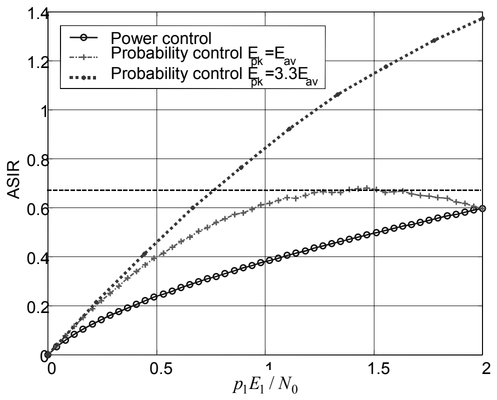

Fig. 4. Average users SIR as a function of the average ratio of the chip energy to N_0 in a 2 user symmetric AWGN interference channel ($N=1$, $r=30$, $p_1=p_2$, $E_1=E_2$).

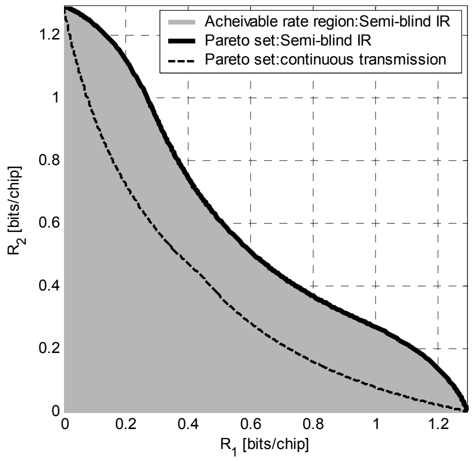

Fig. 5. Achievable ASIR in a 2 user NLOS interference channel ($N=6$, $r=10$, $S_k^{\mathbf{av}}\bar{H}T_c/N_0=2$).

ratio is 30 and the spreading factor is 1. Fig. 4 depicts the users ASIR as a function of the average ratio of the chip energy to N_0 (p_1E_1/N_0). The lower curve (marked by circles) is the ASIR achieved using power control, i.e., both users use continuous transmission ($p_1=p_2=1$), and the system controls the chip energies $E_1=E_2$. This curve is monotonically increasing, and reaches a maximum of $\rho_1=\rho_2=0.6$ at $E_k/N_0=S_k^{\mathbf{av}}T_c/N_0=2$.

The figure depicts the ASIR of two different probability controlled systems. In the first we set $S_k^{\mathbf{pk}}=S_k^{\mathbf{av}}$. In this case the average transmitted power reaches its maximum only in continuous transmission ($p_k=1$), and decreases with any decrease of the transmission probability. The ASIR for this case (marked with crosses) is higher than the ASIR achieved with power control, and achieves a maximal value of $\rho_1=\rho_2=0.68$. This may look as a small improvement, but it comes with the additional benefit of lower transmission power ($p_kE_k/N_0=1.47$).

A more significant increase in the achievable ASIR is achieved if we allow the system a higher peak power. The dotted curve in Fig. 4 depicts the ASIR when $S_k^{\mathbf{pk}}=3.3S_k^{\mathbf{av}}$. In this case the optimal working point use all available average transmission power, and achieves a much higher ASIR of $\rho_1=\rho_2=1.37$, more than twice the ASIR achieved with power control.

If we remove the constraint that both users achieve equal ASIR, we receive many pairs (ρ_1,ρ_2) of ASIR achievable by both users. All of these achievable ASIR pairs for the probability control system with $S_k^{\mathbf{pk}}=3.3S_k^{\mathbf{av}}$ are marked by the gray area in Fig. 5. In this figure the x-axis shows the ASIR achieved by user 1 while the y-axis shows the ASIR achieved by user 2. The solid line bordering the gray area is the set of dominating points, i.e., the rate pairs achieved by states in the Pareto set. Using probability control with lower peak power ($S_k^{\mathbf{pk}}=S_k^{\mathbf{av}}$) achieves a lower ASIR for both users (depicted by a dash-dotted line), but the least performance are achieved as expected using power control (depicted by a dotted line).

V. CONCLUSION

In this paper we introduced the concept of probability control for the optimization of the average SIR achieved by a CDMA system. We showed a sufficient condition for a system to be probability controlled optimal, i.e., to achieve the optimal performance using probability control instead of power control. We also demonstrated that a CDMA system over an AWGN channel is probability controlled optimal.

Although the optimization of the average SIR does not guarantee the optimal performance for any communication system, the results shown in this paper reveal the potential of probability control to increase system performance. In fact, the results obtained for average SIR optimization are so definite that we can easily state that any CDMA system design must consider the implementation of a probability control mechanism.

The actual implementation of probability control algorithm requires additional research, for the development and analysis of probability control algorithms. Moreover, further research is required in order to solve the optimization problem for other optimization criteria such as BER and achievable rate.

APPENDIX A – THE NOISE PLUS INTERFERENCE CORRELATION MATRIX

The vector $\mathbf{u}_{k,d}$ from (4) is given by:

$$\mathbf{u}_{k,d} = \sum_{j\neq k}\sum_{a}\sqrt{E_{j,a}}s_j[a]\mathbf{r}_{k,j,a,d} + \sum_{a\neq d}\sqrt{E_{k,a}}s_k[a]\mathbf{r}_{k,k,a,d} + \tilde{\mathbf{n}}_{k,d} \quad , \tag{12}$$

where the l^{th} term of $\mathbf{r}_{k,j,a,d}$ is given by:

$$\left(\mathbf{r}_{k,j,a,d}\right)_l = \sum_{q=0}^{N_j-1}\sum_{i=0}^{N_k-1} c_{j,a,q}c_{k,d,i} \cdot \int_{-\infty}^{\infty} h_{kj}(\zeta)R_{\text{p}}\left(\tau_k(l)-\zeta+(dN_k+i-aN_j-q)T_{\text{c}}\right)d\zeta \quad ,\tag{13}$$

and:

$$R_{\text{p}}(\tau) = \int_{-\infty}^{\infty} p(t)p(t+\tau)d\tau \, , \tag{14}$$

$$\left(\tilde{\mathbf{n}}_{k,d}\right)_l = \sum_{i=0}^{N_k-1} c_{k,d,i} \cdot \int_{-\infty}^{\infty} n_k(t)p(t-\tau_k(l)-(dN_k+i)T_{\text{c}})dt \, . \tag{15}$$

The noise plus interference correlation matrix of the vector $\mathbf{u}_{k,d}$ ($\mathbf{R}_{k,d} = E\left[\mathbf{u}_{k,d}\mathbf{u}_{k,d}^T\right]$) is given by (6), where:

$$\left(\mathbf{R}_{\mathbf{n}}\right)_{l,m} = \sum_{q=0}^{N_k-1}\sum_{i=0}^{N_k-1} c_{k,d,q}c_{k,d,i} \cdot R_p\left(\tau_k(m)-\tau_k(l)+(i-q)T_{\text{c}}\right) \, . \tag{16}$$

APPENDIX B – MF-RAKE IS PROBABILITY CONTROLLED OPTIMAL

In the case of a single RAKE finger, the ASIR of the RAKE receiver, (8), simplifies to:

$$\rho_k = E\left[\frac{\mu_{k,d}^2}{R_{k,d}}\right], \tag{17}$$

Where we use plain symbols to emphasize that this are scalars and not vectors and matrices. Using proposition 1, in order to show that the system is probability controlled optimal we need to prove that the second derivative of the ASIR with respect to any symbol energy $E_{j,a}$ is positive. We start by testing the derivative with respect to the desired symbol energy ($j=k$, $a=d$). Note that the only term in (17) which depends on $E_{k,d}$ is $\mu_{k,d}^2$ which is linear with $E_{k,d}$:

$$\frac{\partial^2 \rho_k}{\partial E_{k,d}^2} = 0 \, . \tag{18}$$

Next, for an interfering symbol ($j\neq k$ or $j=k$ and $a\neq d$), the derivative of the ASIR with respect to $E_{j,a}$ is given by:

$$\frac{\partial \rho_k}{\partial E_{j,a}} = -E\left[\frac{\mu_{k,d}^2}{R_{k,d}^2}\frac{\partial R_{k,d}}{\partial E_{j,a}}\right] = -E\left[\frac{\mu_{k,d}^2 r_{k,j,a,d}^2}{R_{k,d}^2}\right], \tag{19}$$

where we used:

$$\frac{\partial R_{k,d}}{\partial E_{ja}} = r_{k,j,a,d}^2 \, . \tag{20}$$

The second derivative is given by:

$$\frac{\partial^2 \rho_k}{\partial E_{j,a}^2} = 2E\left[\frac{\mu_{k,d}^2 r_{k,j,a,d}^4}{R_{k,d}^3}\right] \geq 0 \, . \tag{21}$$

Though $\partial^2 \rho_k / \partial E_{j,a}^2$ is positive for any k, j, a, which concludes the proof.

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