KALMAN FILTER FOR CHANNEL TRACKING IN WIRELESS MC-CDMA SYSTEMS

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ABSTRACT

This paper presents a Kalman filtering channel estimator for wireless multicarreir code-division multiple access (MC-CDMA) communication systems. Joint Kalman filter and Kalman filter per-subcarrier schemes are employed to track the time-varying channel. Pilot symbols are equally-placed in the training subcarriers for channel estimation. The proposed pilot-aided channel tracking scheme presents better performance than the decision-directed method in the case of high fading rate. The robustness of the Kalman filter is also studied when the fading rate is unknown. Numerical simulations illustrate the performance of the proposed pilot-aided channel estimator.

1. INTRODUCTION

MC-CDMA [1] was recently proposed as an efficient multicarrier transmission scheme for supporting multiple access communications, which combines code-division multiple access (CDMA) and orthogonal frequency division multiplexing (OFDM) techniques. It recently receives considerable attention because of its advantages in frequency diversity, multipath fading resilience and etc [2]-[3]. In MC-CDMA, the effective signature waveform is the multiplication of the given spreading code and the unknown channel coefficients associated with each subcarrier. Reliable channel estimation is then critical for the detection of the information symbols.

Kalman filtering-based methods have been proposed to perform channel estimation in mobile OFDM systems [4] and MIMO-OFDM systems [5]. In [6], the Kalman-based channel estimator was investigated for MC-CDMA systems. In these approaches, training symbols are transmitted at the training mode, then the receiver switches to the decision-directed mode. The decision-directed receiver works quite well when the fading rate of the channel is low. As the channel fading becomes faster, its performance will degrade significantly due to the feedback of erroneous detected symbols [6]. Pilot-symbol-aided parameter estimation is another type of scheme. A minimum mean square error (MMSE) channel estimator was proposed in [7] for wireless OFDM systems using two-dimensional interpolation between the pilots. The MMSE channel estimator needs the channel statistics and the signal-to-noise ratio (SNR). The maximum likelihood (ML) channel estimator was presented in [8], which shows that the equally-spaced placement of the pilots is optimal in the presence of noise. In this paper, we propose a pilot-aided channel estimator based on the Kalman filter, where the pilots are inserted in the training subcarriers at each MC-CDMA symbol for tracking fast fading channels.

The rest of this paper is organized as follows: Section II briefly describes the MC-CDMA system model. The channel tracking scheme based on the Kalman filtering is presented in Section III. In Section IV, numerical results are given. Finally, conclusions are drawn in Section V.

2. SYSTEM MODEL

Consider a $K-$user MC-CDMA system for downlink transmission. Fig. 1 shows the structure of an MC-CDMA transmitter. The original serial data stream of the kth user is first converted into P parallel data sequences $\mathbf{b}_k(\tau)=[b_{k,0}(\tau),\cdots,b_{k,P-1}(\tau)]$ at the τth time. Each S/P converted output spreads with the user's spreading sequence $\mathbf{c}_k=[c_{k,0},\ldots,c_{k,M-1}]^T$. The data chips after spreading are S/P converted into M parallel subcarriers. In order to achieve the maximum frequency diversity, the data bit $b_{k,p}$ are transmitted on subcarriers with frequencies of $f_1+(p+mP)\cdot\Delta f$, $m=0,\ldots,M-1$. The resulting chips in total $N=PM$ are expressed as $\mathbf{u}_k(\tau)=[b_{k,0}(\tau)c_{k,0},\ldots,$

$b_{k,P-1}(\tau)c_{k,0},\ldots,b_{k,0}(\tau)c_{k,M-1},\ldots,b_{k,P-1}(\tau)c_{k,M-1}]^T$. The $N\times 1$ data vector $\mathbf{u}_k(\tau)$ is then modulated by an inverse fast Fourier transform (IFFT). The $(u,v)th$ element of the

IFFT matrix $\mathbf{F}_1$ is $\frac{1}{N}\exp(j2\pi\frac{uv}{N})$.

The kth user's signal propagates through a frequency-selective fading channel with L paths. A first-order Gauss-Markov model can adequately characterize the dynamics of the time-varying channel [9]. The channel is represented by the following first-order Gauss-Markov process

$$\mathbf{h}(\tau) = a\mathbf{h}(\tau-1) + \mathbf{w}(\tau), \quad (1)$$

where $\mathbf{h}[\tau] = [h_0[\tau], \cdots, h_{L-1}[\tau]]^T$ is a complex Gaussian random process with zero mean and variance σ_h^2, the parameter a is the fading correlation coefficient that characterizes the degree of time variation. The value of a is related to the 3-dB frequency f_d of the corresponding Doppler power spectrum as $a = \exp(-w_d T_s)$ ($w_d = 2\pi f_d$) [7], and $\mathbf{w}[\tau]$ is the driving noise with zero mean and variance $\sigma_w^2 = (1-a^2)\sigma_h^2$.

To combat inter-symbol interference (ISI) caused by multipath fading, a cyclic prefix of Ng samples is added to an MC-CDMA symbol. When $Ng \geq L-1$, the effect of ISI can be eliminated. At the receiver, the signal is sampled at a rate $(N+N_g)/T_s$. The samples corresponding to the cyclic prefix are then discarded. Finally, a fast Fourier transform (FFT) of size N is performed at the receiver. The discrete-time MC-CDMA signal in the frequency-domain can then be obtained in matrix notation as

$$\mathbf{y}(\tau) = \sum_{k=0}^{K-1} \sqrt{P}\mathbf{U}_k(\tau)\mathbf{g}(\tau) + \mathbf{n}(\tau), \quad (3)$$

where P is the chip energy, $\mathbf{U}_k(\tau) = diag\{\mathbf{u}_k(\tau)\}$ is a diagonal matrix, the diagonal elements of which are the transmitted data block of the kth user. $\mathbf{g} = [g_0, \ldots, g_{N-1}]^T$ represents the channel frequency response, given as

$$\mathbf{g}(\tau) = \mathbf{F}_L\mathbf{h}(\tau) \quad (4)$$

where $\mathbf{F}_L \in \mathbf{C}^{N\times L}$ is the FFT matrix, and the elements of $\mathbf{g}$ is denoted as $g_n = \sum_{l=0}^{L-1} h_l \exp(-j2\pi\frac{nl}{N})$, $n = 0, \ldots, N-1$. Finally, $\mathbf{n}(\tau)$ is the complex additive white Gaussian noise after the FFT, with zero mean and variance σ_n^2.

3. KALMAN FILTERING-BASED CHANNEL ESTIMATOR AND MMSE DETECTION

Here we propose to insert pilots in MC-CDMA blocks to track the time-varying channel. In [8], it is shown that in the presence of noise, equally-spaced placement of the

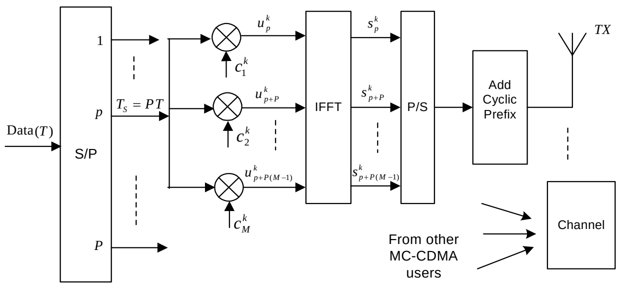

Fig. 1. MC-CDMA transmitter diagram.

pilot symbols is optimal. Therefore, N_p ($N_p \geq L+1$) out of N subcarriers are chosen as the training subcarriers from the set $\{1, 1+N_p, \cdots, 1+[N/N_p]\}$. The pilot symbols denoted as $\mathbf{d}_p = [d_p^0, \cdots, d_p^{N_p}]^T$, are sent over the N_p training subcarriers in each MC-CDMA block.

3.1. Kalma filtering channel estimators

In the paper, two types of channel estimators based on the Kalman filtering are presented, the named joint Kalman filtering channel estimator and per-subcarrier Kalman filtering channel estimator. The former gives the optimal linear estimate of the channel, while the latter has a simpler computation.

Joint Kalman filtering channel estimator:

Considering the channel model given in (1), the time-varying channel and the received data vector for the N_p training subcarriers satisfy the following state-space model

$$\begin{aligned} \mathbf{g}_p(\tau) &= \mathbf{F}_{p,L}\mathbf{h}(\tau), \\ \mathbf{y}_p(\tau) &= \mathbf{D}_p(\tau)\mathbf{g}_p(\tau) + \mathbf{n}_p(\tau), \end{aligned} \quad (5)$$

where $\mathbf{D}_p(\tau) = diag\{\mathbf{d}_p(\tau)\}$ is the pilot matrix, the rows of the matrix $\mathbf{F}_{p,L} \in \mathbf{C}^{N_p \times L}$ are from the FFT matrix $\mathbf{F}_L$ corresponding to the training subcarriers, and $\mathbf{n}_p(\tau)$ is white Gaussian noise with zero mean and variance σ_n^2.

The state-space model of (5) allows the use of the Kalman filter to track the channel in the time domain. The algorithm is given below [10].

1. Initialize the Kalman filter with $\mathbf{h}(0) = 0_{L\times 1}$ and $\mathbf{M}(0) = \delta\mathbf{I}_L$, where δ is a small positive constant.
2. Perform the Kalman filter update as the following

$$\tilde{\mathbf{D}}_p(\tau)) = \mathbf{D}_p(\tau))\mathbf{F}_{p,L},$$

$$\mathbf{M}(\tau) = a\mathbf{M}(\tau-1)a^* + \sigma_w^2\mathbf{I}_L,$$

$$\mathbf{K}(\tau) = \mathbf{M}(\tau)\tilde{\mathbf{D}}_p^H(\tau)\cdot(\tilde{\mathbf{D}}_p(\tau)\mathbf{M}(\tau)\tilde{\mathbf{D}}_p^H(\tau)+\sigma_n^2\mathbf{I}_{N_p})^{-1},$$
$$\mathbf{h}(\tau) = a\mathbf{h}(\tau-1)+\mathbf{K}(\tau)(\mathbf{y}_p(\tau)-\tilde{\mathbf{D}}_p(\tau)a\mathbf{h}(\tau-1)),$$
$$\mathbf{M}(\tau) = (\mathbf{I}_L-\mathbf{K}(\tau)\tilde{\mathbf{D}}_p(\tau))\cdot\mathbf{M}(\tau).$$

The channel gains at each subcarrier can then be obtained as $\mathbf{g}(\tau)=\mathbf{F}_L\mathbf{h}(\tau)$. The joint Kalman filter gives the optimal linear estimate of the channel. However, it involves an inverse operation of a $L\times L$ matrix in each update.

Per-subcarrier Kalman filtering channel estimator:
A simpler Kalman-filtering channel estimator is to operate at a per-subcarrier criterion. The channel model in the frequency domain is represented as

$$\mathbf{g}_p(\tau) = a\mathbf{g}_p(\tau-1)+\mathbf{w}_p(\tau), \quad \mathbf{w}_p(\tau)=\mathbf{F}_{p,L}\mathbf{w}(\tau). \tag{6}$$

Therefore, the state-space model for the *nth* subcarrier can be obtained as ($n=0,1,\cdots,N_p$)

$$\begin{aligned} g_p^n(\tau) &= a g_p^n(\tau-1)+w_p^n(\tau),\\ y_p^n(\tau) &= d_p^n(\tau)g_p^n(\tau)+n_p^n(\tau). \end{aligned} \tag{7}$$

The Kalman filtering algorithm is then given as:

1. Initialize the Kalman filter with $\mathbf{g}_p(0)=0_{N_p\times 1}$ and $\mathbf{M}(0)=[\delta,\cdots,\delta]_{N_p\times 1}$.
2. For each n ($n=0,1,\cdots,N_p$), perform the Kalman filter update according to

$$M_n(\tau) = aM_n(\tau-1)a^*+\sigma_w^2,$$
$$K_n(\tau) = M_n(\tau)d_p^{n*}(\tau)/(d_p^n(\tau)M_n(\tau)d_p^n(\tau)+\sigma_n^2),$$
$$g_p^n(\tau) = a g_p^n(\tau-1)+K_n(\tau)(y_p^n(\tau)-d_p^n(\tau)a g_p^n(\tau-1),$$
$$M_n(\tau) = (1-K_n(\tau)d_p^n(\tau))\cdot M_n(\tau).$$

The channel gain at each subcarrier can be estimated as

$$\mathbf{g}(\tau) = \mathbf{F}_L\mathbf{F}_{p,L}^H\mathbf{g}_p(\tau)/N_p. \tag{8}$$

3.2. MMSE detection
Once the channel state information has been estimated, the minimum mean squared-error (MMSE) detector can be employed to recover the data symbols. Referred to the matrix form of the system model in (3), the received data vector corresponding to the *pth* data stream can be represented as the following ($p=1,\cdots,P$)

$$\mathbf{y}_d^p(\tau) = \sqrt{P}\mathbf{G}_d^p(\tau)\mathbf{C}\mathbf{b}^p(\tau)+\mathbf{n}_d^p(\tau), \tag{9}$$

where $\mathbf{y}_d^p(\tau)$ is an $M\times 1$ vector denoting the received data vector, $\mathbf{G}_d^p(\tau)=diag\{\mathbf{g}_d^p\}$ with $\mathbf{g}_d^p$ denoting the channel gains, $\mathbf{b}^p(\tau)=[b_0^p(\tau),\cdots,b_{K-1}^p(\tau)]^T$ represents the data vector of the users corresponding to the transmitted *pth* data stream, respectively, and $\mathbf{C}=[\mathbf{c}_0,\cdots,\mathbf{c}_{K-1}]_{M\times K}$ is the code matrix of the users which satisfies $\mathbf{C}\mathbf{C}^T=\mathbf{I}_M$.

Considering the *pth* data stream, the optimization criterion is to find a matrix such that

$$\mathbf{V}_o(\tau) = \arg\min_{\mathbf{V}(\tau)} E\{\left\|\mathbf{b}^p(\tau)-\mathbf{V}(\tau)\mathbf{y}_d^p(\tau)\right\|^2\}. \tag{10}$$

The data vector estimate is then obtained as the following,

$$\hat{\mathbf{b}}(\tau) = \mathrm{sgn}[\Re\{\mathbf{V}_o(\tau)\mathbf{y}_d^p(\tau)\}]. \tag{11}$$

The Wiener solution of (10) is given by

$$\mathbf{V}_o(\tau) = \mathbf{R}_{\mathbf{by}}(\tau)\mathbf{R}_{\mathbf{yy}}^{-1}(\tau), \tag{12}$$

where

$$\mathbf{R}_{\mathbf{by}}(\tau) = E\{\mathbf{b}(\tau)\mathbf{y}_d^{pH}(\tau)\} = \sqrt{P}\mathbf{C}^T\mathbf{G}_d^{pH}(\tau),$$
$$\mathbf{R}_{\mathbf{yy}}(\tau) = E\{\mathbf{y}_d^p(\tau)\mathbf{y}_d^{pH}(\tau)\} = P\mathbf{G}_d^p(\tau)\mathbf{C}\mathbf{C}^T\mathbf{G}_d^{pH}(\tau)+\sigma_n^2\mathbf{I}.$$

It is noticed that $\mathbf{R}_{\mathbf{yy}}(\tau)$ is a diagonal matrix since $\mathbf{G}_d^p(\tau)$ is diagonal and $\mathbf{C}$ is unitary orthogonal. Hence, the MMSE detector in (12) has a simple form because the inverse of $\mathbf{R}_{\mathbf{yy}}(\tau)$ only involves the inverse of its diagonal elements.

4. NUMERICAL RESULTS

In this section, numerical results are presented to illustrate the performance of the proposed MC-CDMA detectors. The signal-noise ratio (SNR) is defined to be the average received bit energy to noise ratio P/σ_n^2. A frequency-selective channel is considered with $L=6$ paths. The multipath intensity profile decays exponentially, and the total channel power is normalized to 1. We set $M=8$ and $P=16$. Therefore, the total number of subcarriers is $N=128$. Consequently, the original data sequence is first serial/parallel converted into 16 parallel data streams, then each data symbol after S/P spreads with a Walsh code of length $M=8$. The subcarrier spacing is $\Delta f=10$ kHz, and the time duration at the subcarrier is $T_s=100\mu s$. An additional guard tones $N_g=8$ is added to prevent ISI due to channel frequency selectivity. There are 8 simultaneous users in the channel, and the first user is assumed the desired one. The pilot symbols of length $N_p=8$ are placed at the training subcarriers for each MC-CDMA symbol, which means 6.25% of the MC-CDMA symbols are used for channel estimation. The fading rate under consideration is in the range of $1\times10^{-3}\le w_dT_s\le 5\times10^{-2}$.

Fig. 2 depicts the normalized channel estimation error of the joint Kalman filter and Kalman filter per-subcarrier schemes under different fading rates. The normalized channel estimation error is measured by $\|\mathbf{g}-\hat{\mathbf{g}}\|^2/\|\mathbf{g}\|^2$. The simulation results are averaged over 150 trials. The bit-error ratio (BER) performance is illustrated in Fig. 3. It shows that the joint Kalman filtering approach presents similar performance to the Kalman filter per-subcarrier method at low fading rates ($w_d T_s \le 1\times 10^{-2}$). While the former performs better as the channel fading becomes faster ($w_d T_s$ increases).

Next, we study the performance of the pilot-aided channel estimator and the decision-directed method proposed in [6], both using the joint Kalman filtering technique. For the decision-directed receiver, the first 10 MC-CDMA symbols are used as the training symbols, then the receiver switches to the decision-directed mode. For a fair comparison, 10 training symbols are sent every 160 MC-CDMA symbols. Fig. 4 shows the performance of the pilot-aided and decision-directed approaches. It shows that when the fading rate is low, the two methods present similar performance; when the fading rate becomes high ($w_d T_s \ge 1\times 10^{-2}$), the pilot-aided approach demonstrates much better performance than the decision-directed method.

In the two cases mentioned above, we assume that the fading correlation coefficient a is known to the receiver. Fig. 5 illustrates the BER performance of the joint Kalman filter estimator when the prior knowledge of a is available or not (let $a = 0.998$ when it is unknown). It shows that the Kalman filter estimator is not sensitive to the parameter a under the fading rates considered.

5. CONCLUSIONS

A pilot-aided channel tracking scheme based on the Kalman filter is presented for MC-CDMA communication systems. The pilot symbols are transmitted at the equally-spaced training subcarriers. The joint Kalman filter channel estimator has better performance than the Kalman filter per-subcarrier method with larger computational comple-xity. When the fading rate is high, the pilot-aided channel tracking scheme presents better performance than the decision-directed method. The robustness of the Kalman filter is also investigated. Simulation results were presented to illustrate the performance of the proposed detector.

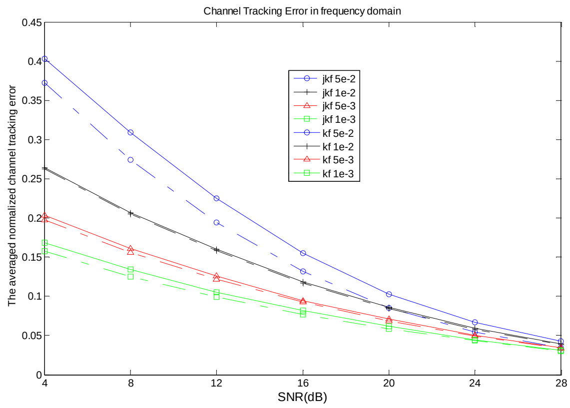

Fig. 2. The normalized channel estimation error of the joint Kalman filter and Kalman filter per-subcarrier methods.

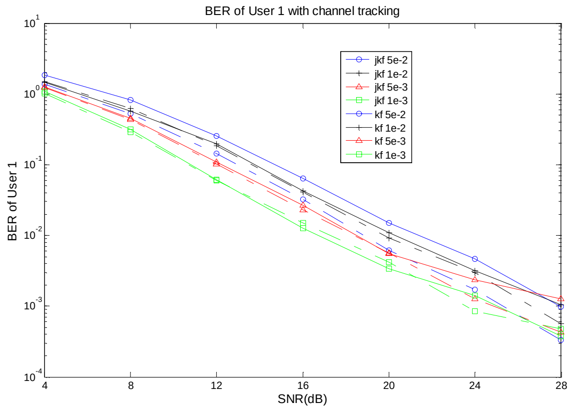

Fig. 3. BER of the first user for the joint Kalman filter and Kalman filter per-subcarrier methods.

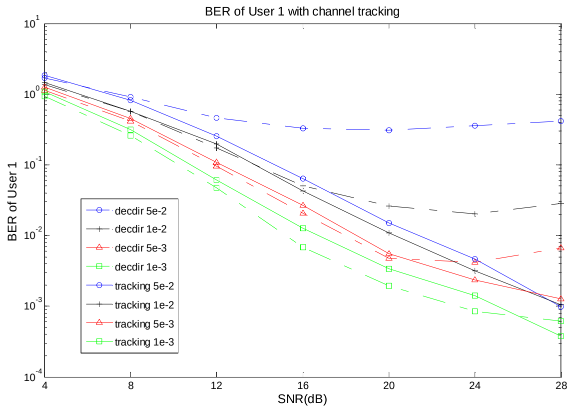

Fig. 4. Performance of the pilot-aided and decision-directed methods using the joint Kalman filter channel estimator.

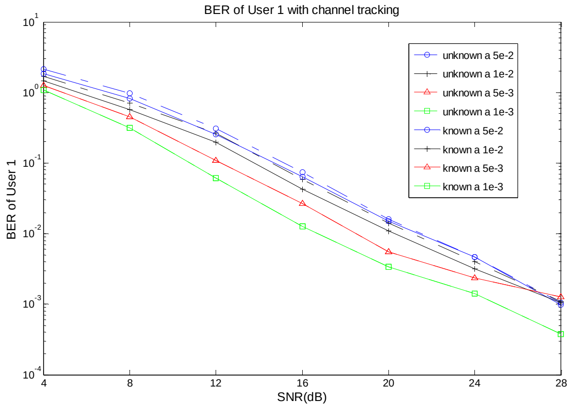

Fig. 5. Performance of the joint Kalman filter channel estimator when the parameter a is known and unknown.

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