PHD FILTERING FOR TRACKING AN UNKNOWN NUMBER OF SOURCES USING AN ARRAY OF SENSORS

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ABSTRACT

In this paper, direction of arrival (DOA) tracking of an unknown number of sources in a highly non-stationary environment is considered. Conventional DOA estimation techniques, such as MUSIC, fail when the stationarity assumption is violated. Furthermore, the time-varying number of sources makes the problem even more challenging. Recently, a particle filtering approach, which propagates the approximate posterior of the target states and then adopts a reversible jump Markov chain Monte Carlo (RJMCMC) diversity step to resolve the number of targets, was proposed. However, this algorithm is sensitive to incorrect model order initialization.

In this paper, we propose a new algorithm for tracking an unknown number of sources based on the probability hypothesis density (PHD) filter, which propagates only the first moment of the joint posterior distribution of targets in terms of particles, as a computationally efficient alternative to the RJMCMC method. The PHD algorithm provides an automatic way to estimate the number of sources, eliminating the need for a separate model order initialization or update step, which is typically the source of problem in particle-filtering based methods. In addition to the fact that the PHD implementation is simple, simulation results show that, the PHD implementation yields superior performance over the other method.

Keywords: array signal processing, direction of arrival tracking, Bayesian estimation, particle filters, PHD filters.

1. INTRODUCTION

The problem of DOA tracking of multiple moving targets using passive sensor arrays has many applications in communications and signal processing, such as radar, sonar and biomedical engineering to name a few. Numerous DOA estimation techniques that focus on high resolution techniques can be found in the literature [10, 12]. The accuracy of such techniques depends on the accuracy of the temporal averages taken over the samples and the knowledge of the exact number of sources. In practice, many applications require DOA tracking of an unknown number of dynamic sources where the temporal averages are no longer accurate.

Recently, new methods were developed to track (a known number of) moving targets assuming that the targets are piece-wise stationary [2, 4]. However, these methods fail to track rapidly moving targets and they do not address the more general problem of unknown number of sources. Since the state-space model in DOA tracking is non-linear, extended Kalman filtering (EKF) [1] techniques, which consider a Taylor series approximation of the state space, can be used to track the unknown states. However, these techniques are known to fail in many scenarios [8]. In such scenarios, Bayesian techniques like the particle filter, which propagate the posterior of the unknowns in the form of a histogram, provide a better solution.

Bayesian techniques cannot be used to propagate the posterior if the dimension of the model is time-varying (without the additional step to update the model order and re-initialize). Finite set statistics (FISST) provide a solution for the above problem [3, 6]. However, FISST is a computationally expensive method. In [5], a particle filtering approach, which propagates an approximate posterior of the unknown states, followed by an RJMCMC step to resolve the number of targets was proposed. The RJMCMC step adds additional computational load to the algorithm that is already burdened by the need to calculate the (approximate) full posterior of the states.

In this paper, we transform DOA tracking into a multi-target tracking problem where observations from the targets are independent of each other. We propose the use of discrete Fourier transform (DFT) technique to find a coarse estimation of the DOAs to be used as observations from the targets. The problem of data association as well as that of the unknown number of sources are addressed together by using the PHD filter. It propagates the first moment of the multi-target posterior, known as the probability hypothesis density (PHD), which is defined over the state space of one target. The PHD has local maxima at the expected locations of the target states. Hence, target locations can be obtained from the PHD. Further, the PHD holds a nice property that the integral of the PHD over the state space is the expected number of targets. Hence, the PHD filter [7] is found to be a computationally efficient alternative to the FISST method of propagating the full posterior. Further, the PHD formulation does not need any ad-hoc techniques, as in the case of RJMCMC method, to resolve the number of targets.

The rest of this paper is organized as follows. Section 2 formulates the DOA tracking problem. Section 3 introduces the state space model used in our proposed method. Section 4 reviews the PHD filter tracking scheme. Simulation results to demonstrate the performance of the proposed algorithm are given in Section 5.

2. PROBLEM FORMULATION

Consider a uniform linear array of M sensors. At time t, let $\phi_i(t)$, $i = 1, 2, \dots, K(t)$ be the directions of arrival of $K(t)$ *narrow band* sources that are in the *far-field* of the array and let the corresponding amplitudes of the $K(t)$ sources be $a_i(t)$, $i = 1, 2, \dots, K(t)$. The transmission medium is assumed to be *isotropic* and *non-dispersive*.

The *steering vector* corresponding to the kth source is given by

$$\boldsymbol{s}(\phi_k(t)) = [e^{-j(w_0/c)d_1 \sin\phi_k(t)}, e^{-j(w_0/c)d_2 \sin\phi_k(t)}, \dots, e^{-j(w_0/c)d_M \sin\phi_k(t)}]^T \quad (1)$$

where d_m, $m = 1, 2, \dots, M$, is the position of the mth sensor, c is the velocity of propagation, and w_0 is the center frequency of the narrow band sources.

The states to be estimated are assumed to evolve as follows:

$$\phi_i(t) = \phi_i(t-1) + \sigma_v v(t) \quad (2)$$

$$a_i(t) \sim \mathcal{N}(0, \sigma_{a_i}^2) \quad (3)$$

where $v(t)$ is an i.i.d Gaussian noise sequence with zero mean and unity variance while σ_v^2 is the corresponding process noise covariance, and $\sigma_{a_i}^2$ is the amplitude variance.

The array observation vector $\boldsymbol{y}(t)$, which is composed of the incident signals from the $K(t)$ distinct sources embedded in Gaussian noise, is given by

$$\boldsymbol{y}(t) = \sum_{k=1}^{K(t)} \boldsymbol{s}(\phi_k(t)) a_k(t) + \sigma_w \boldsymbol{w}(t) \quad (4)$$

where, $\boldsymbol{w}(t)$ is an i.i.d Gaussian noise vector with zero mean and unit variance while σ_w^2 is the corresponding process noise covariance. The noise variances, σ_v^2 and σ_w^2 are assumed unknown.

Let us introduce a vector of the parameters describing the model

$$\boldsymbol{\theta}_{1:t} \triangleq (\boldsymbol{\phi}_{1:t}, \boldsymbol{a}_{1:t}, K_{1:t}, \sigma_v^2, \sigma_w^2) \quad (5)$$

where $x_{1:t}$ indicates all the elements from $x(1)$ to $x(t)$, $\boldsymbol{\phi}(t) = [\phi_1(t), \phi_2(t), \dots, \phi_{K(t)}(t)]^T$, and $\boldsymbol{a}(t) = [a_1(t), a_2(t), \dots, a_{K(t)}(t)]^T$.

Our objective in this paper is to track the DOAs as well as the amplitudes of the sources, hence, all the other parameters are considered unwanted or nuisance parameters. They could either be estimated or integrated out [5]. One way to get rid of the nuisance parameters is to numerically marginalize the *posterior distribution*, $p(\boldsymbol{\theta}_{1:t}|\boldsymbol{y}_{1:t})$, with respect to those parameters. However, such integration will result in higher computational requirement. In [5], assuming the prior distributions of both of the variances σ_v^2 and σ_w^2 as Gamma distribution, a *maximum a posteriori* (MAP) estimate of the nuisance parameters was presented.

Hence, the unknown states reduce to $\boldsymbol{x}_{1:t}$, where $\boldsymbol{x}(t) = [\boldsymbol{\phi}(t)^T, \boldsymbol{a}(t)^T]^T$.

3. STATE SPACE MODEL

We introduce the following state and observation models to be used in our Bayesian tracking scheme discussed in Section 4.

$$\boldsymbol{x}_i(t) = g(\boldsymbol{x}_i(t-1), \boldsymbol{v}(t)) \quad (6)$$

$$\boldsymbol{y}_i(t) = h(\boldsymbol{x}_i(t), \boldsymbol{w}(t)) \quad (7)$$

where $\boldsymbol{x}_i(t)$ is the state of the ith target and $\boldsymbol{y}_i(t)$ is the array observation due to the ith target.

The state model in (6) evolves as given by (2)–(3). However, the observation model is different from (4). The observations in (7) are assumed to be separable and is independent for targets. However, such observations are not available in this problem. This leads to an approximation discussed below.

Let us derive the likelihood of the observation $\boldsymbol{y}_i(t)$ given the state $\boldsymbol{x}_i(t)$. Considering the unknown parameter of the ith source as $\boldsymbol{x}_i(t) = [\tilde{w}_i(t), a_i(t)]^T$, where $\tilde{w}_i(t) = (w_0 d_0/\ c) \sin\phi_i(t)$, all unknown parameters can be considered as a matrix $\boldsymbol{X}(t) = [\boldsymbol{x}_1(t), \boldsymbol{x}_2(t), \ldots, \boldsymbol{x}_{K(t)}(t)]$.

The joint probability density function of $\boldsymbol{y}(t)$, given the unknown parameters $\boldsymbol{X}(t)$, is given by[1]

$$f(\boldsymbol{y}|\boldsymbol{X}) = \left(\frac{1}{\pi\sigma_w}\right)^M \exp\Big[-\frac{1}{\sigma_w^2}\sum_{m=1}^{M}\big((a_m-\mu_m)^2 + (b_m-\nu_m)^2\big)\Big] \quad (8)$$

where a_m and b_m are the real and imaginary parts of the observation y_m of the mth sensor, $\mu_m = \sum_{i=1}^{K} a_i \cos(\tilde{w}_i m)$, and $\nu_m = \sum_{i=1}^{K} a_i \sin(\tilde{w}_i m)$.

The above probability could be further simplified as (see [9])

$$f(\boldsymbol{y}|\boldsymbol{X}) = \left(\frac{1}{\pi\sigma_w^2}\right)^M \exp\Big[\frac{M}{\sigma_w^2}(L_1 + L_2 + L_3)\Big] \quad (9)$$

where $L_1 = -\frac{1}{M}\sum_{m=1}^{M}\boldsymbol{y}_m^2$, $L_2 = \sum_{i=1}^{K}\{2a_i\mathrm{Re}\big[A(\tilde{w}_i)\big]\}$, and $L_3 = -\frac{1}{M}\sum_{i\neq k}\sum_k a_i a_k \sum_m \cos(m(\tilde{w}_i - \tilde{w}_k))$. Further, in L_2, $A(\tilde{w}_i)$ is the DFT of the observation of the array given by

$$A(\tilde{w}_i) = \frac{1}{M}\sum_{m=1}^{M} y_m \exp(-j\tilde{w}_i m) \quad (10)$$

Since we are interested in $f(\boldsymbol{y}_i|\boldsymbol{x}_i), \quad i = 1, 2, \ldots, K$, the probability in (9) should be marginalized with respect to $\boldsymbol{x}_j$ and $\boldsymbol{y}_j$, where $j \neq i$, to get the required likelihood. However, such marginalization will require huge computation. Hence, it is desired to look for some approximations.

By neglecting the term L_3 in (9), we get an approximate likelihood function $\tilde{f}(\boldsymbol{y}|\boldsymbol{X}) = \left(\frac{1}{\pi\sigma_w^2}\right)^M \exp\Big[\frac{M}{\sigma_w^2}(L_1 + L_2)\Big]$. Assigning $L = L_1 + L_2$, we notice further that L can be written in the following format: $L = \sum_{i=1}^{K}\bar{L}(a_i, \tilde{w}_i, y_{m,i})$, where, $y_{m,i}$ is the observation at the mth sensor due to the ith source. Hence, the approximate likelihood function can be written as $\tilde{f}(\boldsymbol{y}|\boldsymbol{X}) = \tilde{f}(\boldsymbol{y}|\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_K) = C\prod_{i=1}^{K}\bar{f}(\boldsymbol{y}_i|\boldsymbol{x}_i)$,where C is a constant.

Hence, the approximate likelihood function $\bar{f}(\boldsymbol{y}_i|\boldsymbol{x}_i)$ is given by

$$\bar{f}(\boldsymbol{y}_i|\boldsymbol{x}_i) = \bar{C}\exp\Big[\frac{M}{\sigma_w^2}\Big(-\frac{1}{M}\sum_{m=1}^{M} y_{m,i}^2 + 2a_i\mathrm{Re}\big[A_i(\tilde{w}_i)\big]\Big)\Big] \quad (11)$$

where, $A_i(\tilde{w}_i) = \frac{1}{M}\sum_{m=1}^{M} y_{m,i}\exp(-j\tilde{w}_i m)$ is the DFT of the array observation due to the ith source, assuming equal power of the sources, $y_{m,i}^2$ is approximately given as $y_{m,i}^2 = (y_m^2/K)$, and $\bar{C}$ is a constant.

In practice, the above likelihood can be very easily calculated as follows: The DFT spectrum of the array observation $\boldsymbol{y}$ contains K peaks corresponding to the K sources. We separate these peaks and use them as the DFTs of the observations due to the individual sources. The extracted peak represents spectrum $A_i(\tilde{w})$, where $\tilde{w}$ ranges from $(w_0 d_0/c)\sin(-\pi)$ to $(w_0 d_0/c)\sin(\pi)$. In this case, we do not know which peak corresponds to which source. This problem is well known in the target tracking community as the *data association problem*. Here, the PHD filter will perform data association as well. For a detailed study of the PHD filter, the reader is referred to [7]. For the ith observation, the PHD filter requires to know the likelihood of that observation given any $\tilde{w}_j$, which can be obtained from (11) using $A_i(\tilde{w}_j)$.

Despite the fact that this method may pick some ripples in the DFT spectrum as potential sources, the method works well because the PHD filter accounts for the ripples as a false alarm.

4. PHD FILTER

The set of tracked targets at time t can be considered as a random set [3, 11], $\mathrm{X}_t = \{\boldsymbol{x}_1(t), \boldsymbol{x}_2(t), \ldots, \boldsymbol{x}_{K(t)}(t)\}$. Similarly, the set of observations received at time t can also be considered a random set, $\mathrm{Y}_t = \{\boldsymbol{y}_1(t), \boldsymbol{y}_2(t), \ldots, \boldsymbol{y}_{N_{\mathrm{Y}(t)}}(t)\}$. A certain outcome of the random set X_t is denoted as x_t and that of Y_t is denoted as y_t.

All information of the unknown state of the target can be deduced from the *posterior distribution* [3, 11] $f_{\mathrm{X}_t|\mathrm{Y}_{1:t}}(\mathrm{x}_t|\mathrm{y}_{1:t})$. For large number of targets, the computational complexity of estimating such posterior distribution is very high. However, the full posterior $f_{\mathrm{X}_t|\mathrm{Y}_{1:t}}(\mathrm{x}_t|\mathrm{y}_{1:t})$ can be approximately recovered from the first moment of this distribution, the probability hypothesis density (PHD), given by [3, 11]

$$D_{\mathrm{x}_t|\mathrm{Y}_{1:t}}(\mathrm{x}_t|\mathrm{y}_{1:t}) = \int f_{\mathrm{X}_t|\mathrm{Y}_{1:t}}(\{\mathrm{x}_t\}\cup Z|\mathrm{y}_{1:t})\delta Z \quad (12)$$

which is defined over the state space Φ of one target which is much smaller compared to the state space of $K(t)$ targets.

As mentioned earlier, the PHD possesses a nice property to resolve the number of targets — the integral of the PHD over Φ gives the expected number of targets. We follow the particle filter implementation of propagating the PHD given in [11].

[1]From here on, the time index t is suppressed in this section to simplify the notation.

5. SIMULATION RESULTS

In this section, a simulation example is presented to demonstrate the performance of the proposed algorithm. The true DOAs are generated using a first order random walk model with a variance of σ_v^2. The number of true targets is fixed at 2. Table 1 gives the values of the parameters used in the simulation. The receiver array is uniform linear and consists of $M = 8$ elements.

Table 1. Parameters used in the simulation

σ_v^2	σ_w^2	$\boldsymbol{\phi}(1)$	$\boldsymbol{a}(1)$	σ_a^2
5deg.^{2}	0.01	$[-20^0, 40^0]^T$	$[1, 1]^T$	0.0707

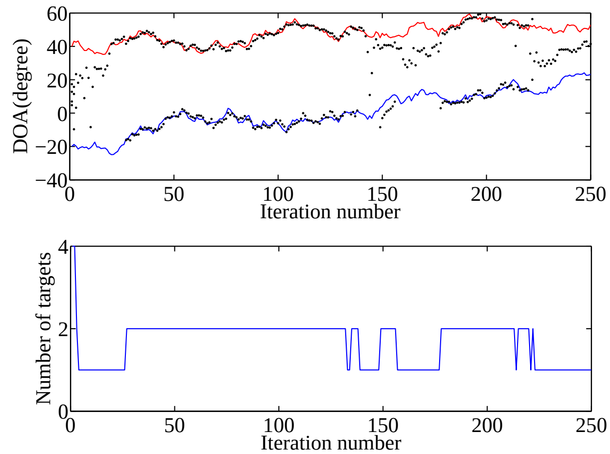

Fig. 1. Top: Sequential DOA estimation of the RJMCMC particle filter [5] versus iteration number; the continuous line shows the true value and the dots show the estimated value. Bottom: Corresponding number of detected signals versus iteration number using the particle filter. Actual number of sources is 2 for the entire simulation.

5.1. First example: Tracking an unknown number of targets where the number of targets remains fixed

Figure 1 (top) shows the estimated DOAs of the RJMCMC method against the iteration number. This method uses $N = 100$ particles and the number of targets was randomly initialized. As shown in the bottom of the figure, at start, it takes almost 25 iterations to converge to the correct number of targets. It can be observed from the figure that when the targets get closer, the estimated number of targets falls to one and the DOA estimation becomes inaccurate. That is, the targets are unresolved.

Figure 2 (top) shows the estimated DOAs of the PHD filter against the iteration number. The PHD filter also uses 100 particles. As shown in the bottom of the figure, the PHD filter instantly estimates the number of targets correctly and it also can separate tracks of very closely spaced targets when the other filter fails. Further, in contrast to the RJMCMC method, PHD filter does not need any initialization for the number of targets.

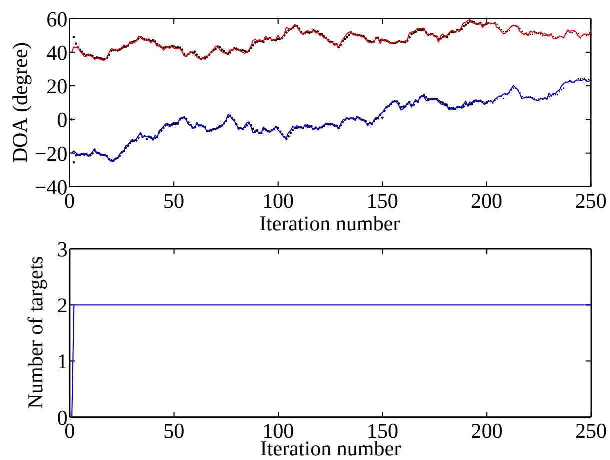

Fig. 2. Top: Sequential DOA estimation of the PHD filter versus iteration number. The true target states are the same as in Figure 1; the continuous line shows the true value and the dots show the estimated value. Bottom: Corresponding number of detected signal versus iteration number using the PHD filter.

5.2. Second example: Tracking an unknown number of targets where the number of targets vary with time

In this example, there is only one target from the start to iteration 50. Another target enters at iteration 51 and the first one leaves at iteration 151.

Figure 3 (top) shows the estimated DOAs of the RJMCMC method against the iteration number. The number of targets was randomly initialized. As shown in the bottom of the figure, it takes almost 25 iterations to converge to the correct number of targets. When the second target enters, it is quickly identified. However,

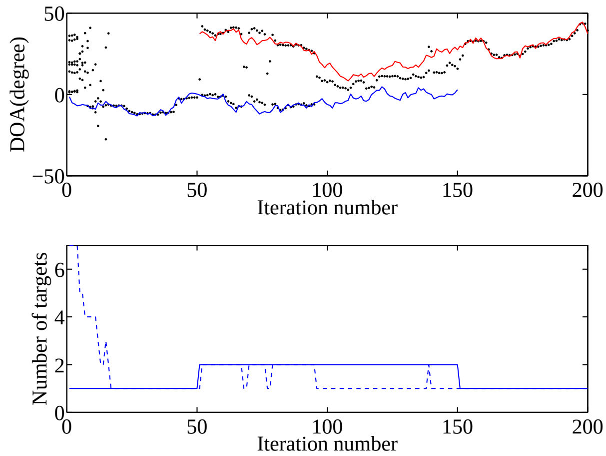

Fig. 3. Top: Sequential DOA estimation of the RJMCMC particle filter [5] versus iteration number. The continuous line shows the true value and the dots show the estimated value. Bottom: Corresponding number of detected signal versus iteration number using the particle filter. The solid line shows the actual number of targets and the dashed line shows the estimated number of targets.

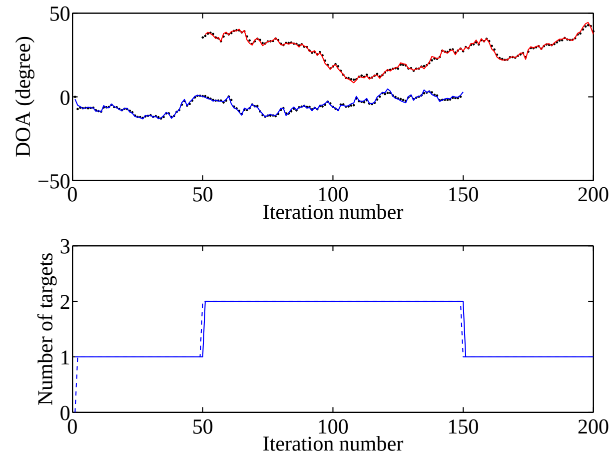

Fig. 4. Top: Sequential DOA estimation of the PHD filter versus iteration number. The continuous line shows the true value and the dots show the estimated value. Bottom: Corresponding number of detected signal versus iteration number using the PHD filter. The solid line shows the actual number of targets and the dashed line shows the estimated number of targets.

when the targets becomes closer, the estimated number of targets becomes one and the DOA estimation becomes inaccurate.

Figure 4 (top) shows the estimated DOAs of the PHD filter against the iteration number. The bottom of the figure shows that the proposed method instantly tracks the number of targets initially as well as when the number of targets changes.

6. CONCLUSIONS

A PHD filter based algorithm has been proposed for tracking the DOAs of time-varying number of targets. The proposed method employs DFT techniques to find a coarse estimate of the DOAs using one snapshot of the array observation and then uses them as observations for the PHD filter. The PHD filter then produces accurate estimates of the DOAs as well as the estimated number of targets. Simulation results show that the proposed method is able to estimate the number of targets instantly and that it is able to track closely-spaced targets better compared to the RJMCMC method.

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