A Partially Adaptive Beamforming with Slepian-Based Quiescent Response

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Abstract— **We propose and investigate a partially adaptive beamformer that employs Slepian sequences in both the quiescent weight vector and the signal blocking matrix. This technique maintains robustness in the sense that it preserves low sidelobe levels under conditions of low training data support, signal steering vector mismatch, and moving interference. The adaptive degrees of freedom are chosen based on the properties of Slepian sequences, in order to mitigate the effects of signal self-nulling. Numerical comparisons with adaptive beamformers with Slepian-based and Chebyshev-based quiescent responses support the efficacy of this method.**

I. Introduction

The minimum variance quiescent response (MVQR) beamforming [1], which does not necessarily employ a signal steering vector as the quiescent weight vector, is a modification of minimum variance distortionless response (MVDR) beamforming. In MVQR beamforming, the signal steering vector may be tapered in order to maintain a robust directional quiescent response with low sidelobes in white noise environments, while preserving the ability to adaptively place nulls in the direction of interferers. Techniques of quiescent weight vector design have been developed to make the beampatterns more robust under conditions of limited sample-support and non-stationary environments, where the signal steering vector may be mismatched and directional interferers posseses rapidly time-varying properties [4-7].

However, when the signal is relatively strong, a fully adaptive MVQR beamformer may suffer serious degradation of the shape of main beam. An adaptive null in the main beam produces unacceptable patterns with a severe reduction of the gain in the desired look direction.

In this paper we investigate the following modifications of MVQR beamforming: (1) We design the quiescent weight vector to be the signal steering vector tapered by the zeroth Slepian, or Discrete Prolate Spheroidal, Sequence (DPSS) [2,3]. (2) We make the beamforming partially adaptive, in the generalized sidelobe canceller (GSC) structure [8], by employing a subset of subdominant Slepian sequences in the signal blocking matrix.

The rationale for this choice of quiescent weight vector is that for a uniform linear array (ULA), the zeroth Slepian sequence maximizes the concentration of energy within a specified band of spatial frequencies around the look direction. Similar sequences can be obtained for non-ULA geometries, but the Slepian sequences are especially well studied [2,3]. As will be explained below, we obtain additional performance improvement by reducing the adaptive degrees of freedom, which also helps to reduce the effect of main beam nulling. The advantages of this technique are its improvement of robustness with respect to low sample support or signal and interference uncertainty, while mitigating the effect of mainbeam nulling. Its implementation can exploit a significant literature on the properties and calculation of Slepian sequences, such as the original series of papers by D. Slepian. We validate the expected advantages of this technique with numerical studies of output signal-to-interference plus noise ratio (SINR) versus signal-to-noise ratio (SNR) and the number of adaptive degrees of freedom in the signal blocking matrix.

The paper is organized as follows. Section II presents the partially adaptive MVQR beamforming based on Slepian sequences. Numerical evaluations are presented in section III, and a conclusion is provided in Section IV.

II. Partially Adaptive MVQR Beamforming with Slepian Sequences

Consider an array of N omnidirectional sensors with steering vector $\mathbf{s}(\theta)$. Assume narrow-band signals are emitted from far-field emitters. The desired signal, emitting from angle θ_0, interferers and white noise are assumed to be stationary and spatio-temporally uncorrelated each other. The complex baseband array output vector, at time t, can be denoted by $\mathbf{x}(t)$. The Sample Matrix Inverse (SMI) version of the adaptive MVQR weights is

$$\mathbf{w} = \frac{\hat{\mathbf{R}}_x^{-1}\mathbf{w}_q}{\mathbf{w}_q^\dagger \hat{\mathbf{R}}_x^{-1}\mathbf{w}_q}, \tag{1}$$

where $\mathbf{w}_q$ is the desired quiescent weight vector and † is the Hermitian (conjugate transpose) operation. $\hat{\mathbf{R}}_x$ is the sample data covariance matrix $\hat{\mathbf{R}}_x = \frac{1}{M}\sum_{t=1}^{M}\mathbf{x}(t)\mathbf{x}^\dagger(t)$, estimated by M received snapshots.

Denote the covariance of white noise $\mathbf{n}(t)$ that is bandlimited to bandwidth $2W$ as $\mathbf{T} = \sigma_n^2 \cdot \int_{\mu\in\Omega}\mathbf{s}(\mu)\mathbf{s}^\dagger(\mu)d\mu$, where σ_n^2 is the variance of white noise, $\mu = \sin\theta$ determines the spatial frequency, and Ω is a narrow band of spatial frequencies around the main lobe with half fractional bandwidth W. In this

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paper, we consider the uniform linear array (ULA), where the steering vector can be expressed as

$$\mathbf{s}(\mu) = \{1, e^{i2\pi\frac{\delta}{\lambda}\mu}, \cdots, e^{i2\pi(N-1)\frac{\delta}{\lambda}\mu}\}^T, \tag{2}$$

with distance between any two neighboring sensors δ and the wavelength λ. Then the covariance matrix of white noise, that is bandlimited to $[-W, W]$, is a positive-definite, Toeplitz matrix with (m, n)-th element as

$$\mathbf{T}_{mn} = \sigma_n^2 \cdot \frac{\lambda}{\delta} \cdot \frac{\sin[2\pi W'(m-n)]}{\pi(m-n)}. \tag{3}$$

Here $W' = \frac{\delta}{\lambda} W$. The Slepian, or discrete prolate spheroidal, sequences $\{\nu^{(k)}(N, W')\}$, $k = 0, \cdots, N-1$, are eigenvectors of the Toeplitz matrix $\mathbf{T}$, corresponding to the ordered eigenvalues $1 \geq \alpha_0 \geq \alpha_1 \geq \cdots \geq \alpha_{N-1} \geq 0$.

The zeroth Slepian sequence $\nu^{(0)}(N, W')$, or in short $\nu^{(0)}$, is the dominant eigenvector of the Toeplitz matrix $\mathbf{T}$, with the largest eigenvalue α_0. $\nu^{(0)}$ maximizes the energy concentration in the spatial bandwidth $[-W, W]$, or equivalently minimizes the power leakage outside this narrow band in white noise environment. Because of this property, we use the steering vector at the look direction $\mathbf{s}(\theta_0)$, or in short $\mathbf{s}$, tapered by the zeroth Slepian sequence as the quiescent weight vector $\mathbf{w}_q$, which can be calculated as the Hadamard (element-wise) product [9] of $\mathbf{s}$ and $\nu^{(0)}$ as $\mathbf{w}_q = \mathbf{s} \circ \nu^{(0)}$. The quiescent weight vector $\mathbf{w}_q$ can also be denoted as $\mathbf{w}_q = diag(\mathbf{s}) \cdot \nu^{(0)}$.

A disadvantage of this MVQR beamforming is possible nulling of the main beam in the beam pattern, which may occur when the signal-to-noise ratio levels are relatively high. The reasons for this can be seen by investigating the beamformer in its generalized sidelobe canceller form. The MVQR weights are decomposed into two orthogonal components, consisting of the quiescent weight vector and the auxiliary adaptive weight vector:

$$\mathbf{w} = \{\mathbf{I} - \mathbf{B}[\mathbf{B}^\dagger \hat{\mathbf{R}}_x \mathbf{B}]^{-1} \mathbf{B}^\dagger \hat{\mathbf{R}}_x\} \mathbf{w}_q, \tag{4}$$

where $\mathbf{B}$ is the $N \times (N-1)$ signal blocking matrix whose columns are orthogonal to the quiescent weight vector. The choice of $\mathbf{B}$ is not unique. Due to the mutual orthogonality of Slepian sequences, subdominant Slepian sequences can be employed in the signal blocking matrix as $\mathbf{B} = diag(\mathbf{s}) \cdot \{\nu^{(1)}, \cdots, \nu^{(N-1)}\}$.

An important property of the Slepian sequences is that when the length of Slepian sequences $N \to \infty$, for any η satisfying $0 < \eta < 1$,

$$\alpha_k \to \begin{cases} 1, & \text{when} \quad k = 2NW(1-\eta), \\ 0, & \text{when} \quad k = 2NW(1+\eta). \end{cases}$$

It implies that with N large, there are approximately $K \approx \lfloor 2NW \rfloor$ orthogonal sequences $\{\nu^{(k)}\}$ with "large" eigenvalues, that is $\alpha_k \approx 1$ for $k \leq K$ [2,3]. $\lfloor 2NW \rfloor$ denotes the largest integer less than or equal to $2NW$. When N is relative small, it also has a similar "threshold". For example, Fig.1 demonstrates the eigenvalues $\{\alpha_k\}$ of the Slepian sequences $\{\nu^{(k)}(N, W)\}$ with $N = 20$ and $W = 0.06$. When $K = \lfloor 2NW \rfloor = 2$, α_k is close to 0 for $k > K$.

Consequently, although the zeroth Slepian sequence has a highly directional response pointed in the desired signal direction, the second to the K-th Slepian sequences also have highly directional signal responses. If these Slepian sequences are employed in the signal blocking matrix $\mathbf{B}$, the blocking matrix does not efficiently block the signal. In other words, residual components of the desired signal are then passed through the signal blocking matrix $\mathbf{B}$, and result in the desired signal being subtracted in part from the nonadaptive quiescent response. This effect becomes more severe at higher SNR levels, resulting in a null being formed in the mainbeam in the presence of strong signal. Therefore, only a subset of the subdominant Slepian sequences, which correspond to small eigenvalues, should be used in the signal blocking matrix. The new N by $N-L$ signal blocking matrix $\tilde{\mathbf{B}}$, which only employs the last $N-L$ Slepian sequences with the $N-L$ smallest eigenvalues, is $\tilde{\mathbf{B}} = diag(\mathbf{s}) \cdot \{\nu^{(L)}, \cdots, \nu^{(N-1)}\}$, where $L \geq K+1$. Usually $L = \lfloor 2NW \rfloor + 1$ or $\lfloor 2NW \rfloor + 2$ is chosen, depending on the practice.

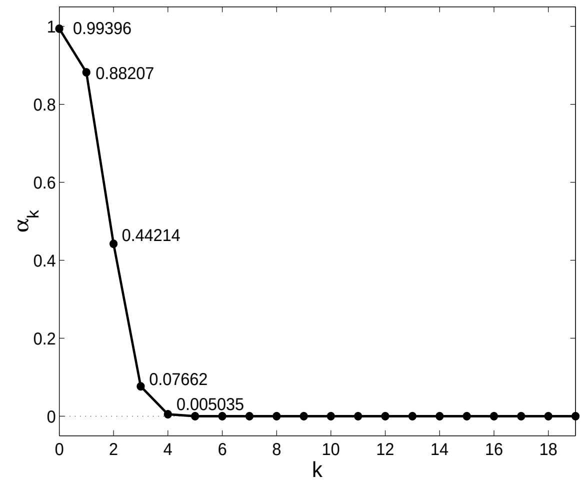

Fig. 1. Eigenvalues $\{\alpha_k\}$ of the Slepian sequences $\{\nu^{(k)}(N, W)\}$ with $N = 20$ and $W = 0.06$.

Another advantage of utilizing only a fraction of the available adaptive degrees of freedom is to reduce the computational load of the adaptive algorithm, especially when N or W is large.

However, reducing the adaptive degrees of freedom even further will tend to degrade the interference cancellation capability. Therefore, the number of adaptive degrees of freedom to employ, $N-L$, or the rank of the signal blocking matrix, should be selected to tradeoff between the performance gain due to reducing main beam nulling and the performance loss from degrading interference cancellation capability.

The unused adaptive degrees of freedom can not be incorporated into the quiescent weight vector design, because the zeroth Slepian sequence is already the optimal choice for maximizing the energy concentration around the desired look direction. Any linear combination of additional Slepian sequences will decrease the energy concentration.

Loading by a diagonal matrix to improve the robustness [10], the SMI version of the partially adaptive MVQR beamforming with Slepian-based quiescent response becomes

$$\tilde{\mathbf{w}} = \{\mathbf{I} - \tilde{\mathbf{B}}[\tilde{\mathbf{B}}^\dagger(\tilde{\mathbf{R}}_x + \beta\mathbf{I})\tilde{\mathbf{B}}]^{-1}\tilde{\mathbf{B}}^\dagger(\hat{\mathbf{R}}_x + \beta\mathbf{I})\}\mathbf{w}_q, \tag{5}$$

where β is the loading factor, and I is an $N \times N$ identity matrix.

III. Numerical Evaluation

In the following two examples, we consider the signal detection problem with low sample support, mismatched signal steering vector and non-stationary interference, in order to illustrate the robustness of partially adaptive MVQR beamforming with Slepian-based quiescent response. We compare the performances of the partially adaptive MVQR beamforming with Slepian-based quiescent response (Partial MVQR-Slepian), with those of other weight design methods as follows:

- *MVDR beamformer (MVDR)*. Its weight vector is given by
$$\mathbf{w} = (\hat{\mathbf{R}}_x + \beta\mathbf{I})^{-1}\mathbf{s}. \quad (6)$$
The scalar to normalize the output power at the desired look direction is ignored here and in the following, because it has no influence on the output signal-to-interference plus noise ratios (SINRs).
- *Slepian-based fully adaptive MVQR beamformer (MVQR-Slepian)*. The weight is given by
$$\mathbf{w} = (\hat{\mathbf{R}}_x + \beta\mathbf{I})^{-1}(\mathbf{s} \circ \nu^{(0)}). \quad (7)$$
- *Chebyshev-based fully adaptive MVQR beamformer (MVQR-Chebyshev)*. The desired quiescent response is based on a Chebyshev weighting. In the following two examples, the Chebyshev weighting has $-30dB$ sidelobes.

All the compared adaptive beamformers are diagonally loaded and have a same loading level. The diagonal loading level is $15dB$ above the noise level.

Another purpose of these two examples is to illustrate how the adaptive degrees of freedom affect the output SINRs.

Consistent with the use of Slepian sequences, we presume a uniform linear array, with $N = 20$ elements, spaced half wavelength apart. The desired signal is located at 0^o, and is assumed to be present in all the data samples. The interferers are located at
$$\theta = 40^o, -25^o, -40^o.$$
All interferers have interference-to-noise ratio (INR) equal to $30dB$. The Slepian sequences have half-fractional bandwidth of $W = 0.06$. 500 realizations were run and averaged for the output SINR measurements in the beam pointed at the signal of the interest.

Example I

In the first series of simulations, we consider the scenario of a low sample support with only N samples.

Fig.2 indicates the effect of changing the adaptive degrees of freedom on the output SINR of the partially adaptive MVQR beamforming with Slepian-based quiescent response. The degrees of freedom are the ranks of the signal blocking matrix, or the numbers of Slepian sequences in the signal blocking matrix. The figure indicates that there does exist an optimal choice of degrees of freedom for a practical scenario. For this experiment, the output SINR can be significantly increased by reducing the adaptive degrees of freedom, or the rank of the signal blocking matrix, from 19 to 16 ($L = \lfloor 2NW \rfloor + 2$). As discussed above, this can be attributed to the effect of removing the dominant projections of a desired signal in the adaptive branch of a generalized sidelobe canceller. By removing these columns, the signal of interest is no longer unexpectedly suppressed by the adaptive weights.

Fig.3 compares the beampatterns of two fully adaptive MVQR beamformers (MVQR-Slepian and MVQR-Chebyshev) with the partially adaptive MVQR beamformer with Slepian-based quiescent response (Partial MVQR-Slepian). The beampatterns are averaged for 500 realizations. For the relatively high SNR level $5dB$, the fully adaptive MVQR beampatterns display nulling of the mainbeam for some realizations, and thus will require additional first and second derivative constraints [11,12], to prevent the resultant suppression of the desired signal, which introduces more complexity to the system. However, for the partially adaptive MVQR beamformers with Slepian-based quiescent response, this mainbeam nulling problem can be resolved easily by only reducing the adaptive degrees of freedom.

Fig.4 compares the output SINRs of the fully and partially adaptive MVQR beamformers with the Slepian-based quiescent response or the Chebyshev-based quiescent response. The partially adaptive MVQR beamformer with Slepian-based quiescent response (Partial MVQR-Slepian) brings an obvious gain (about $2.5 \sim 5$dB to the fully adaptive beamformers when the signal-to-noise ratio is greater than or equal to 0dB ($SNR \geq 0$dB). The difference of the output SINRs at the low SNRs is less pronounced, because the probabilities of signal self-nulling decrease when the desired signal becomes weaker. This numerical result also indicates the robust performance of partially adaptive MVQR beamforming with Slepian-based quiescent response under conditions of low sample support.

Example II

In another series of simulations, we consider the scenario in a non-stationary environment, where the signal steering vector may be mismatched and directional interference posses rapidly time-varying properties. The sample support size is $4N$.

The interferences are assumed to be moving with the time-varying directions $\theta_1(t) = 40^o + 10^o \sin(t/15)$, $\theta_2(t) = -25^o + 10^o \cos(t/5)$, and $\theta_3(t) = -40^o - 15^o \cos(t/15)$, where t is data observation or snapshot index [13]. We simulate steering vector mismatch by modelling the signal response across the array as a random process with a covariance matrix given by $\sigma_s^2[\mathbf{s}\mathbf{s}^\dagger + \xi\mathbf{I}]$, with the variance of the perturbances given by $10 \log \xi/\sigma_s^2 = -5dB$ [14].

Fig.5 indicates the effects of adaptive degrees of freedom on output SINRs. Global maximal output SINR is achieved when the degree of freedom is reduced from 19 to 17 ($L = \lfloor 2NW \rfloor + 1$). This figure also shows the performance loss from degrading interference cancellation capability due to further adaptive degrees reduction, i.e. from 16 to 1. Therefore, the selection of degrees of freedom is to tradeoff between the performance gain by reducing the signal self-nulling and the performance loss by incomplete interference cancellation.

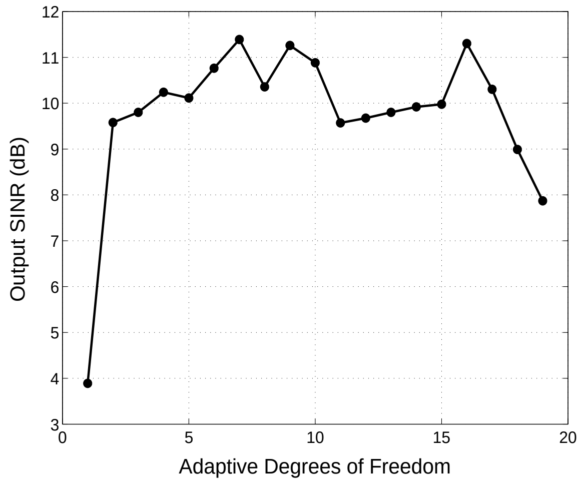

Fig. 2. Output SINR versus the number of adaptive degrees of freedom, with $SNR = 0dB$ and low sample support N.

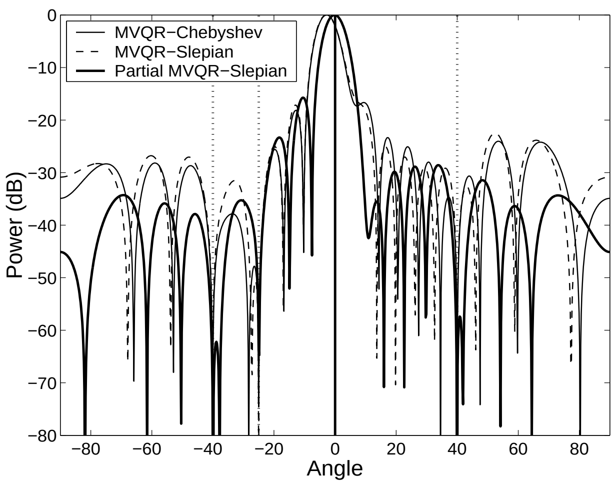

Fig. 3. The number of adaptive degrees of freedom for partially adaptive beamforming methods equals to 17 (reduced from 19). $SNR = 5dB$ and the sample support size is N.

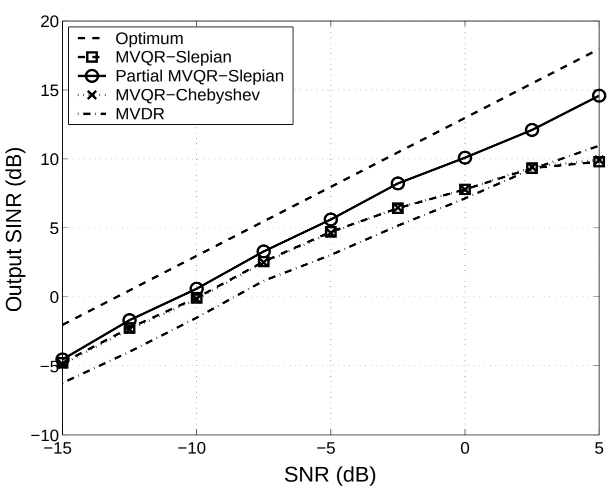

Fig. 4. The number adaptive degrees of freedom for both partially adaptive beamforming methods is 17 (reduced from 19). The sample support size is N.

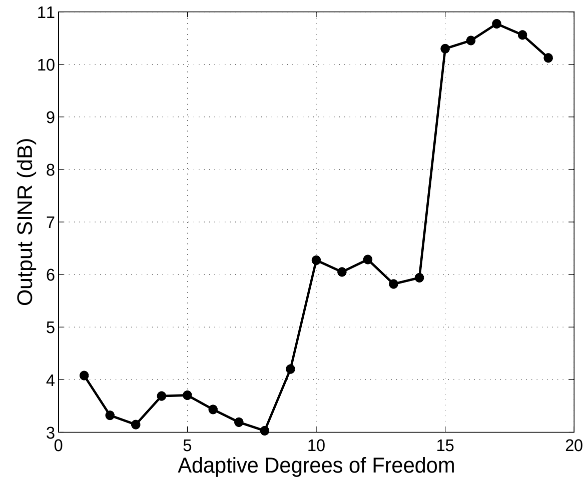

Fig. 5. Output SINR versus the number of adaptive degrees of freedom in a non-stationary environment, with $SNR = 0dB$.

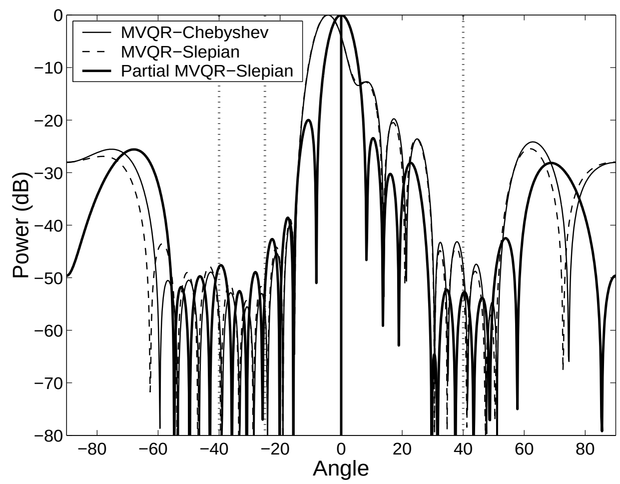

Fig. 6. The number of adaptive degrees of freedom for partially adaptive beamforming methods equals to 17 in a non-stationary environment. $SNR = 5dB$ and the sample support size is $4N$.

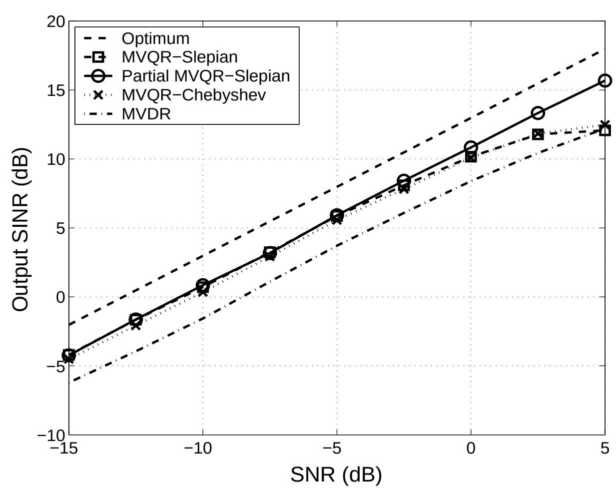

Fig. 7. The number adaptive degrees of freedom for both partially adaptive beamforming methods is 17 (reduced from 19) in a non-stationary environment. The sample support size is $4N$.

Fig.6 compares the beampatterns of fully and partially adaptive MVQR beamformers as Fig.2. As in the results shown above, at the relatively high SNR level 5dB, the fully adaptive MVQR beampatterns also display nulling of the main beam in some realizations. The partially adaptive MVQR beamformer with Slepian-based quiescent response can reduce the probability of signal cancellation very effectively.

Fig.7 illustrate the output SINR performances. The partially adaptive MVQR beamformer with Slepian-based quiescent response (Partial MVQR-Slepian) brings about 2.5dB gain to the fully adaptive beamformers when the signal-to-noise ratio is relatively high, such as $SNR = 5$dB. When SNR is less than 0dB, there is no significant difference between fully and partially adaptive MVQR beamformers. This performance metric also illustrates the robustness of the partially adaptive MVQR beamforming with Slepian-based quiescent response, in the presence of signal steering vector mismatch and non-stationary interference.

IV. CONCLUSION

This paper has described a technique of using a subset of subdominant Slepian sequences in the signal blocking matrix in partially adaptive MVQR beamforming. It maintains robustness in the sense that it preserves low sidelobe levels under conditions of low sample support, signal steering vector mismatched and non-stationary interference. The adaptive degrees of freedom affect both performance gain in reducing the likelihood of main beam nulling and performance loss in interference cancellation. The tradeoff is studied by theoretical analysis and numerical simulations. The adaptive degrees of freedom are chosen based on the properties of Slepian sequences. We recommend choosing the value of L close to, but exceeding, $2NW$.

REFERENCES

[1] H.L. Van Trees, *"Optimum Array Processing Part IV of Detection, Estimation and Modulation Theory"*, pp.670, 2002.

[2] D. Slepian, "Prolate Spheroidal Wave Functions, Fourier Analysis, and Uncertainty-V: The discrete case", *Bell Syst. Tech. J.*, Vol.57, pp.1371-1429, May-June 1978.

[3] D.J. Thomson, "Spectrum Estimation and Harmonic Analysis", *Proceedings of the IEEE*, Vol.70, No.9, Sep 1982.

[4] L.J. Griffiths and K.M. Buckley, "Quiescent Pattern Control in Linearly Constrained Adaptive Arrays", *IEEE Transactions on Accoustics, Speech and Signal Processing*, Vol. ASSP-35, No.7, July 1987.

[5] S.L. Sim and M.H. Er, "An Effective Quiescent Pattern Control Strategy for GSC Structure", *IEEE Signal Processing Letters*, Vol.3, No.8, August 1996.

[6] B.D. Van Veen, "Optimization of Quiescent Response in Partially Adaptive Beamformers", *IEEE Transactions on Acoustics, Speech and Signal Processing*, Vol.38, No.3, March 1990.

[7] B.D. Van Veen, "Minimum Variance Beamforming with Soft Response Constraints", *IEEE Transactions on Signal Processing*, Vol.39, No.9, Sep 1991.

[8] L.J. Griffiths and C.W. Jim "An Alternative Approach to Linearly Constrained Adaptive Beamforming", *IEEE Transactions on Antennas Propagation*, Vol. AP-30, pp.27-34, Jan 1982.

[9] V.V. Prasolov, "Problems and Theorems in Linear Algebra", *American Mathematical Society*, 1994.

[10] B.D. Carlson, "Covariance Matrix Estimation Errors and Diagonal Loading in Adaptive Arrays", *IEEE Trans. on Aerosp. Electron. Syst.*, Vol.24, pp.397-401, July 1998.

[11] K.M. Buckley and L.J. Griffiths, "An Adaptive Generalized Sidelobe Canceller with Derivative Constraints", *IEEE Transactions on Antennas and Propagation*, Vol. AP-34, No.3, March 1986.

[12] C.Y. Tseng and L.J. Griffiths, "A Simple Algorithm to Achieve Desired Patterns for Arbitrary Arrays", *IEEE Transactions on Signal Processing*, Vol.40, No.11, Nov 1992.

[13] S.A. Vorobyov and etc., "Adaptive Beamforming with Joint Robustness against Mismatched Signal Steering Vector and Interference Nonstationarity", *IEEE Signal Processing Letters*, Vol.11, No.2, Feb 2004.

[14] H.C. Robert M. Zeskind and M.M. Owen, "Robust Adaptive Beamforming", *IEEE Transactions on Acoustics, Speech and Signal Processing*, Vol. ASSP-35, No.10, Oct 1987.