BLIND MIMO SYSTEM ESTIMATION BASED ON PARAFAC DECOMPOSITION OF HOS TENSORS

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ABSTRACT

We present a novel frequency domain approach for the identification of a multiple-input multiple-output (MIMO) system driven by white, mutually independent unobservable inputs. Samples of the system frequency response are obtained based on Parallel Factorization (PARAFAC) of three-way tensors constructed based on third-order cross-spectra of the system outputs. The main difficulties in frequency domain methods are frequency dependent permutation and filtering ambiguities. We show that the information available in the higher-order spectra allows for the ambiguities to be resolved up to a constant scaling and permutation, and a linear phase ambiguity. The advantages of the proposed approach is that, it does not require exact channel length information, need no phase unwrapping, and unlike the majority of existing methods, need no pre-whitening of the system outputs.

keywords-**PARAFAC decomposition, high-order statistics, blind channel estimation.**

1. INTRODUCTION

We consider a linear time-invariant (LTI) multiple-input multiple-output (MIMO) system driven by unobservable inputs. Our goal is to identify the system function based on the system outputs. MIMO models arise frequently in speech processing, multi-access communication, multi track digital magnetic recording, and biomedical applications.

The case of a memoryless (or scalar) system excited by white inputs has been studied under the name of independent component analysis (ICA) [14]. Among the possible approaches for identification convolutive MIMO systems, frequency domain methods [7, 11, 16], offer certain advantages over time domain ones [4, 17]; they do not require system length information, and also, their formulation can take advantage of existing results for the scalar MIMO problem. Indeed, in the frequency domain, at each frequency, the convolutive problem is transformed into a scalar one. However, an additional step is required to resolve the frequency-dependent permutation, scaling and phase ambiguities.

Most of the blind convolutive MIMO system identification methods in the literature exploit either second-order statistics (SOS) or higher-order statistics (HOS). SOS based methods[8, 16], as opposed to HOS based ones, do not require long data in order to obtain good estimation, and involve low complexity. All these SOS methods achieve blind system identification by exploiting available channel diversity. Furthermore, they apply to narrowband signals only. On the other hand, HOS[4, 7, 11, 17] based methods provide system information without requiring channel diversity, and also can deal with white inputs as long as they are non-Gaussian.

However, there are some limitations of the HOS based methods. In [7], in order for joint diagonalization to be applicable, the system frequency response needs to be a unitary matrix. To guarantee this, a prewhitening operation was applied to the system output. Similar prewhitening is employed in the majority of HOS-based blind MIMO estimation methods [8, 13, 11]. However, this is a sensitive process as it tends to lengthen the global system response, and as a result increase complexity and estimation errors.

The need for whitening can be obviated by another decomposition that does not require unitary matrices. One such approach is the PARAFAC decomposition, which is a low rank decomposition of three- or higher-way arrays. It was first developed in [1] in order to overcome the rotation problem, which exists in two dimensional arrays, and later generalized in [2, 3]. The PARAFAC decomposition can be thought as an extension of singular value decomposition to multi-way arrays, where uniqueness is guaranteed even if the non-diagonal matrices involved are non-unitary.

In this paper, we show how the PARAFAC ideas can be used in the frequency domain framework of [7] to avoid the need for prewhitening. The decomposition is applied on the tensor formed based on HOS of the system output to get the channel estimate at certain frequency, which can be used later to get the channel estimate at all frequency by iteration.

2. PROBLEM FORMULATION AND PARAFAC BACKGROUND

2.1. Background on PARALLEL FACTORIZATION

Singular Value Decomposition (SVD) expresses a matrix (2-way array) as a superposition of rank one matrices. PARAFAC is a generalization of SVD for multi-way arrays. Let us consider a 3-way tensor $\mathcal{X}$ with dimensions $J \times I \times K$, elements $[x_{i,j,k}]_{1\leq j\leq J, 1\leq i\leq I, 1\leq k\leq K}$, and the F-component decomposition [10]:

$$x_{i,j,k} = \sum_{f=1}^{F} a_{i,f} b_{j,f} c_{k,f} \tag{1}$$

In a compact form, $\mathcal{X}$ can be expressed in terms of its slices $\mathbf{X}_i$ $(J \times K)$, $i = 1, ..., I$ as:

$$\mathbf{X}_i = \mathbf{B}\mathbf{D}_i[\mathbf{A}]\mathbf{C}^T \tag{2}$$

where $\mathbf{A}$ is a $I \times F$ matrix with entries $a_{i,f}$; $\mathbf{B}$ is a $J \times F$ matrix with entries $b_{j,f}$; $\mathbf{C}$ is a $K \times F$ matrix with entries $c_{k,f}$.

Definition 1: The matrix $\mathbf{A}$ has k-rank equal to k_A, if every set of k_A columns of $\mathbf{A}$ are linearly independent, while there is at least one set of $(k_A + 1)$ columns that are linearly dependent [3, 10].

Theorem 1: [2],[3],[12] *Let $\mathcal{X}$ be a tensor whose slice $\mathbf{X}_i$ is given as in (2). $\mathcal{X}$ can be decomposed into $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ uniquely up to permutation and scaling ambiguities if*

$$k_A + k_B + k_{C^T} \geq 2F + 2. \tag{3}$$

There exist several algorithms for decomposing tensor $\mathcal{X}$ [10]. We here consider the COMplex parallel FACtor analysis (COMFAC) method [10], [15]. The main steps of this method are basically, first compressing the array, then initializing and fitting of the PARAFAC model on that compressed array, and finally decomposing and refining the solution in the raw data space.

2.2. Problem Formulation

Let us consider a N_i-input N_o-output LTI system with $N_o \geq N_i$.

$$\mathbf{x}(k) = \sum_{l=0}^{L-1} \mathbf{h}(l)\mathbf{s}(k-l) + \mathbf{n}(k) \tag{4}$$

where $\mathbf{s}(n)$ is a N_i by 1 source vector; $\mathbf{x}(n)$ is N_o by 1 observation vector; and $\mathbf{n}(n)$ is the observation noise. $\mathbf{h}(l)$ is the FIR MIMO system impulse response matrix whose (i,j) element is denoted by $\{h_{ij}(n)\}_{1\leq i\leq N_o,\ 1\leq j\leq N_i}$, where L is the length of the longest $h_{ij}(k)$.

Let $\mathbf{H}(k)$ be an $N_o \times N_i$ matrix defined as the N-point Discrete Fourier Transform of $\mathbf{h}(n)$, $\mathbf{H}(k) = \sum_{n=0}^{L-1} \mathbf{h}(n)e^{-j\frac{2\pi}{N}kn},\ k = 0, ..., N-1$ where $N > L$.

Our goal is to estimate $\mathbf{H}(k)$ based on the system output. Since the inherent ambiguities the problem contains, at best we can find $\hat{\mathbf{H}}(k)$ such that:

$$\hat{\mathbf{H}}(k) = \mathbf{H}(k)\mathbf{P}\mathbf{\Lambda}e^{j\frac{2\pi}{N}k\mathbf{M}} \tag{5}$$

where $\mathbf{P}$ is a column permutation matrix, $\mathbf{\Lambda}$ a constant diagonal matrix and $\mathbf{M}$ diagonal matrix with integer elements. We will refer to these ambiguities as *trivial ambiguities*. If such a system estimate were used to recover the inputs, it would yield each input within a scalar ambiguity, and a circular shift. Also, the inputs would be recovered in some unknown order.

We next provide a list of assumptions to be used throughout this paper.

A1) Each $s_i(.), i = 1, ..., N_i$ is a zero mean, non-symmetrically distributed, independent identically distributed (i.i.d.), stationary process with nonzero skewness, i.e., $\gamma^3_{s_i} = Cum[s_i(n), s_i^*(n), s_p(n)]$. The s_i's are mutually independent.

A2) The matrix $\mathbf{H}(k)$ is invertible for all $k = 0, ..., N-1$.

A3) The k-rank of $\mathbf{H}(k)$ satisfies: $2k_H + k_{H^T} \geq 2N_i + 2$ for every k.

A4) $n_i(.), i = 1, ..., N_o$ are zero mean Gaussian stationary random processes with variance σ_n^2, mutually independent and independent of the inputs.

Assumption (A1) guarantees that there is a nonzero third order cumulant of the system output. Assumption (A2) requires that the channel matrix is full column rank. Multipath/delay channel taps can be assumed independent [12], thus assumption (A2) will be satisfied for such channels. A full rank matrix is also full k-rank. Thus, under (A2), $\mathbf{H}(k)$ will also be full k-rank. In that case, condition (A3) is equivalent to $3min(N_i, N_o) \geq 2N_i + 2$. For $N_i \leq N_o$ this is satisfied for $N_i \geq 2$. Assumption (A4) is needed in order for the noise to be suppressed in the higher-order cumulant domain.

3. CHANNEL ESTIMATION

The $N \times N$ discrete-frequency cross-bispectrum of the outputs $x_l(k), x_i^*(k), x_j(k)$ is the two-dimensional Discrete Fourier transform of the third order cross-cumulant[9], and equals:

$$C^3_{lij}(k_1,k_2) = \sum_{p=1}^{N_i} \gamma^3_{s_p} H_{lp}(-k_1-k_2)H^*_{ip}(-k_1)H_{jp}(k_2) \tag{6}$$

where $k_1, k_2 = 0, ..., N-1$. For fixed k_1, k_2, $C^3_{lij}(k_1,k_2)$ can be viewed as the (l,i,j)-th element of tensor $\mathcal{C}^3(k_1,k_2)$ ($N_o \times N_o \times N_o$). The l-th slice of that tensor equals:

$$\mathbf{C}^3_l(k_1,k_2) = \mathbf{H}^*(-k_1)\mathbf{\Gamma}^3\mathbf{D}_l[\mathbf{H}(-k_1-k_2)]\mathbf{H}^T(k_2), \tag{7}$$

where $\mathbf{\Gamma}^3 = Diag\{\gamma^3_{s_1}, ..., \gamma^3_{s_{N_i}}\}$.

In the following, we will consider the tensors $\mathcal{C}^3(-m + r\delta, \delta),\ r = 0, 1, 2, .., 2N-1$, for some constant m, δ, and show how they can be used to recover $\mathbf{H}(m - r\delta - 2\delta)$ for each r.

First we apply the PARAFAC decomposition to the tensor $\mathcal{C}^3(-m + r\delta, \delta)$. Under assumption (A3) and via Theorem 1, the tensor can be decomposed into:

$$\begin{aligned} \hat{\mathbf{A}}_0 &\triangleq \mathbf{H}(m-\delta)\mathbf{P}_1\mathbf{\Lambda}_1 \\ \hat{\mathbf{B}}_0 &\triangleq \mathbf{H}^*(m)\mathbf{\Gamma}^3\mathbf{P}_2\mathbf{\Lambda}_2 \\ \hat{\mathbf{C}}_0 &\triangleq \mathbf{H}(\delta)\mathbf{P}_3\mathbf{\Lambda}_3 \end{aligned} \tag{8}$$

where $\mathbf{P}_i$ is a permutation matrix, $\mathbf{\Lambda}_i$ is a complex diagonal matrix. It holds [3]:

$$\mathbf{P}_1 = \mathbf{P}_2 = \mathbf{P}_3 = \mathbf{P}, \qquad \mathbf{\Lambda}_2\mathbf{\Lambda}_1\mathbf{\Lambda}_3 = \mathbf{I} \tag{9}$$

For $r = [1, 2, ...2N-1]$, define:

$$\mathbf{F}_l(r) \triangleq (\mathbf{F}^*(r-1))^{-1}\mathbf{C}^3_l(-m+r\delta,\delta)(\hat{\mathbf{C}}_0^T)^{-1},\ l = 1, ..., N_o \tag{10}$$

where

$$\mathbf{F}(r) \triangleq [diag(\mathbf{F}_1(r)), ..., diag(\mathbf{F}_{N_o}(r))]^T \tag{11}$$

$$\mathbf{F}(0) = \hat{\mathbf{A}}_0 \tag{12}$$

It can be shown (see Appendix I) that:

$$\mathbf{F}(r) = \mathbf{H}(m - r\delta - \delta)\mathbf{P}\mathbf{K}_{((r))_2}e^{j(\mathbf{\Phi}_1 + r\mathbf{\Phi}_2)} \tag{13}$$

where $\mathbf{\Phi}_1, \mathbf{\Phi}_2$ are diagonal matrices, $\mathbf{K}_1, \mathbf{K}_0$ are diagonal matrices with positive elements, and $((.))_2$ denotes modulo 2 operation.

Equation (13) provides $\mathbf{H}(m - r\delta - \delta)$ within a fixed permutation matrix, a diagonal matrix that assumes a different fixed value depending on whether r is odd or even, and a phase diagonal ambiguity, which depends on r. If we could compute $\mathbf{\Phi}_2$ then (13) would give the odd or the even samples of the channel matrix within trivial ambiguities. As long as $N > 2L$ we could then recover the impulse response channel matrix based on either the odd or the even frequency response samples.

The term $\mathbf{\Phi}_2$ can actually be estimated with an acceptable ambiguity. Consider $\mathbf{F}(N+i)$ for some $i \in [0, N-1]$. For N, δ co-prime, it can be easily shown from (13) that:

$$\mathbf{F}^{-1}(N+i)\mathbf{F}(i) = \mathbf{K}^{-1}_{((N+i))_2}\mathbf{K}_{((i))_2}e^{-jN\mathbf{\Phi}_2} \tag{14}$$

Thus, $\mathbf{F}^{-1}(N+i)\mathbf{F}(i)$ is a diagonal matrix. If N is even, the latter matrix has unit modulus.

Let us consider N to be even and co-prime to δ. Under (A1)-(A4), the system matrix can then be estimated for $r = 0, ..., N-1$ as:

$$\begin{aligned} \hat{\mathbf{H}}(m - r\delta - \delta) &\triangleq \mathbf{F}(r)[\mathbf{F}^{-1}(N+i)\mathbf{F}(i)]^{r/N} \quad (15) \\ &= \mathbf{H}(m - r\delta - \delta)\mathbf{P}\mathbf{K}_{((r))_2} e^{j(\mathbf{\Phi}_1 + \frac{2\pi}{N}kr)} \end{aligned}$$

where i is some integer in $[0, N-1]$; $\quad k : integer$. We will refer to eq. (15) as the Single PARAFAC decomposition (SPD) method.

Applying an $N/2$ point IDFT on the even samples of $\hat{\mathbf{H}}(k)$ of (15), we get an upsampled by δ version of $\mathbf{h}$, circularly shifted by k and modulated due to the $m-\delta$ term in 15). Also, there is always a constant column permutation and diagonal scaling ambiguity.

Once we extract the L-samples long segment (modulo N) with the maximum energy based on its absolute value, we can cancel the modulating factor, and compute the amount of circular shift.

4. SIMULATIONS

We next demonstrate the performance of the proposed approach.

We consider 2×2 MIMO channels that are of the form:

$$\begin{aligned} h_{ij}(n) = r_1 \cdot c(0.25(n-10)), 0.11) \\ +r_2 \cdot c(0.25(n-6), 0.11) + r_3 \cdot (0.25(n-8), 0.11) \quad (16) \end{aligned}$$

where $(i, j = 1..2)$,$c(n, \alpha)$ is a raised cosine function with rolloff α, and r_i's independent zero-mean Gaussian random variables. Such channels are bandpass, and model several real channel situations.

The inputs were taken to be i.i.d. single side exponentially distributed. The additive noise processes were white, zero-mean, complex Gaussian with identical variances and independent of the source signals.The cross-polyspectrum was estimated via the indirect class method [9], and the sample cross-cumulant estimate was windowed by a Hamming window of size $L_e \times L_e$, where L_e is an upper bound for the channel length ($L_e > L$). The data length used to obtain the cross-cumulant estimates is denoted by T.

The PARAFAC decomposition was performed using the MATLAB code downloaded from
$http : //www.ece.umn.edu\ /users/nikos/public\_html/ 3SPICE/code.html.$

In order to minimize accumulating errors due to the inversion of $\hat{\mathbf{C}}_\mathbf{0}$ in (10), we chose δ from the high energy region of the MIMO output power-spectrum.

In the simulations we took $m = \delta$ so that we did not have to account for the modulation of the channel estimated.

The estimation for performed for M_c independent input runs. As performance index for the estimated cross-channel h_{ij} we used the overall normalized mean-square error (ONMSE), i.e.,

$$ONMSE_{ij} \triangleq \frac{1}{N_i N_o} \sum_{i=1}^{N_o} \sum_{j=1}^{N_i} \frac{\frac{1}{M_c}\sum_{l=1}^{M_c}\sum_{k=0}^{L_e-1}(\hat{h}_{ij}(k) - h_{ij}(k))^2}{\sum_{k=0}^{L_e-1}(h_{ij}(k))^2}. \quad (17)$$

where $\hat{h}_{ij}(k)$ denotes the cross-channel estimate.

Before we computed the ONMSE, the estimate $\hat{\mathbf{h}}$ was aligned with the true channel by taking correlation between $\hat{\mathbf{h}}$ and the upsampled true channel $\mathbf{h}$ with various amounts of circular shift.

One could also use $\hat{\mathbf{H}}(k)$ of (13) to decouple the input signals, leaving a phase ambiguity in each input. Then one can use a SISO equalizer (for example a simplification of [4]) to recover each input. Then by taking correlation between the recovered inputs and the system outputs, one can get the estimated channel. This approach was followed in [7] to avoid phase estimation as it was a sensitive step.

The ONMSE performance of the proposed approach for one channel described by (4) is given in Fig. 1. The data length, T and the extended channel length Le were varied and used $N = 128, m = \delta = -11, M_c = 50$.

A lower bound was also generated for the same channel assuming that the input is known and obtaining the channel estimate by cross-correlating input and output. The figure shows that estimation improves as T increases. Also, although the error is smaller when $L_e = L = 6$, there is no significant difference with that for $L_e = 10$, thus indicating that the proposed method does not depend critically on channel length information.

We also provide comparison results between the proposed method against the methods of [7], and [4]. To make the comparison independent of the channel, we tested the performance of the two methods for 50 2×2 channels of length $L = 6$, generated based on (4) by varying the r_is randomly for each channel.

For each channel we performed 50 Monte Carlo runs. We took $L_e = 10, T = 8000$ in both methods. In both methods the output cross-cumulants were estimated using the same parameters. We use $N = 128, m = \delta$ for the SPD method, and δ is chosen from the peak of the power-spectra.

In Fig. 2, we show the cumulative probability function for 50 random channel runs (50 MC runs each) for (I) lower bound; (II) the proposed method; (III) The method of [7] with SISO; (IV) The proposed method with SISO; (V) the time domain method of [4]. We can see the proposed SPD method outperform the method in [7], while the method of [4] has the largest error variance.

5. CONCLUSION

We present a novel frequency domain approach for the identification of a multiple-input multiple-output (MIMO) system driven by white, mutually independent unobservable inputs. The proposed SPD method, requires only one PARAFAC decomposition. The frequency response samples are then obtained via a recursive scheme. Comparisons with the method of [7] that requires prewhitening indicated that the proposed approach achieves lower ONMSE values.

Appendix I

$$\begin{aligned} \mathbf{F}_l(1) &\triangleq (\hat{\mathbf{A}}_0^*)^{-1}\mathbf{C}_l^3(-m+\delta, \delta)(\hat{\mathbf{C}}_0^T)^{-1} \\ &= (\mathbf{\Lambda}_{10}^*)^{-1}\mathbf{P}^T\mathbf{\Gamma}^3\mathbf{D}_l[\mathbf{H}(m-2\delta)]\mathbf{P}\Lambda_{30}^{-1} \\ &= (\mathbf{\Lambda}_{10}^*)^{-1}\mathbf{D}_l[\mathbf{H}(m-2\delta)\mathbf{P}]\mathbf{\Gamma}_p^3\mathbf{\Lambda}_{30}^{-1} \\ &= \mathbf{D}_l[\mathbf{H}(m-2\delta)\mathbf{P}]|\mathbf{\Lambda}_{20}|\mathbf{\Gamma}_p^3 e^{j(2\mathbf{\Phi}_{10}+\mathbf{\Phi}_{20})} \quad (18) \end{aligned}$$

It can be seen that $\mathbf{F}_l(1)$ is a diagonal matrix.

Based on $\mathbf{C}_l^3(-m+\delta, \delta)$, for $l = 1, ..., N_o$, and placing the diagonal elements of $\mathbf{F}_l(1)$ at the l^{th} row of $\mathbf{F}(1)$ we get:

$$\mathbf{F}(1) = \mathbf{H}(m-2\delta)\mathbf{P}|\mathbf{\Lambda}_{20}|\mathbf{\Gamma}_p^3 e^{j(2\mathbf{\Phi}_{10}+\mathbf{\Phi}_{20})} \quad (19)$$

Similarly, It holds:

$$\mathbf{F}(r) = \mathbf{H}(m - r\delta - \delta)\mathbf{P}\mathbf{K}_r e^{j((r+1)\mathbf{\Phi}_{10} + r(\mathbf{\Phi}_{\Gamma_p^3} + \mathbf{\Phi}_{20}))}, \quad (20)$$

which can be written as:

$$\mathbf{F}(r) = \mathbf{H}(m - r\delta - \delta)\mathbf{P}\mathbf{K}_r e^{j(\mathbf{\Phi}_1 + r\mathbf{\Phi}_2)}, \quad (21)$$

where

$$\mathbf{K}_r = \begin{cases} |\mathbf{\Lambda}_{20}|.|\mathbf{\Gamma}_p^3| & for\ r\ odd \\ |\mathbf{\Lambda}_{10}| & for\ r\ even \end{cases} \quad (22)$$

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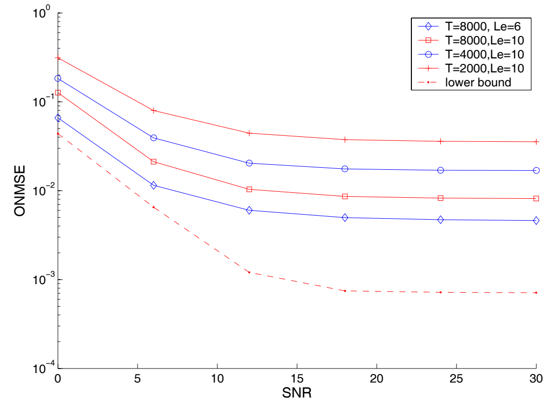

Fig. 1. ONMSE vs SNR for SPD method with different T and Le

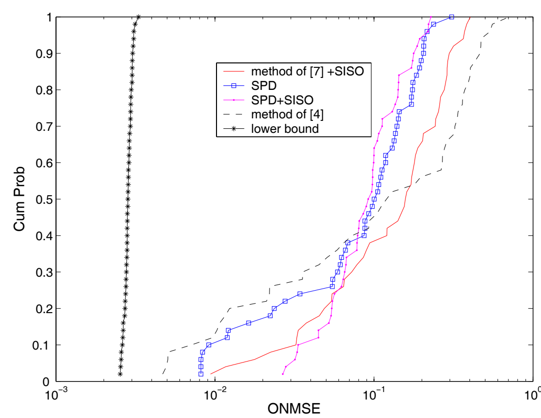

Fig. 2. Cumulative distribution of the ONMSEs obtained by the SPD approach and the method of [7]