TWO NEW APPROACHES TO CHANNEL PREDICTION BASED ON SINUSOIDAL MODELLING

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ABSTRACT

Prediction of communication channels is important, for example to enable adaptive transmission strategies. In this paper, two new predictors are proposed based on sinusoidal modelling. First a Combined LMMSE (given frequency estimates) and LP is proposed. Then an improved LMMSE prediction is proposed by modelling the frequency estimate errors as Gaussian random variables. Both predictions have better performances than the LMMSE prediction (given frequency estimates) in computer simulations.

1. INTRODUCTION

Complex sinusoidal model based radio channel predictions have been investigated by different researchers [1, 2, 3]. This type of channel predictors exploit the underlying signal structure to improve prediction performance. Originally, the sinusoidal model is considered as deterministic as in [1]. Later by modelling the amplitudes of the sinusoid signals as complex Gaussian random variables, an LMMSE (Linear Minimum Mean Square Error Estimate) prediction given frequency estimates is proposed in [2]. In practice, the frequency estimates are subject to errors. It makes a part of the channel signals cannot be represented by the estimated sinusoidal models. This part of channel signal is called residue signal in this paper. Two new approaches are proposed in this paper to mitigate the influence of frequency estimate error on channel prediction. One is using a low order LP to predict the residue signals, which is called Combined LMMSE and LP. The other is named improved LMMSE prediction by modelling the frequency estimate errors as Gaussian random variables. Their performances are evaluated by simulations.

2. SINUSOIDAL MODEL

A flat Rayleigh fading channel in wireless communications can be modelled as superimposed complex sinusoidal signals [1, 2]. Over a short observation interval where the channel parameters can be considered stationary, the time varying channel is then described by

$$x(t) = \sum_{k=1}^{p} s_k e^{j\omega_k t} \tag{1}$$

where p is the the number of contributing paths, s_k are the complex amplitudes and ω_k the normalized doppler frequencies. Observations (successive estimates of a particular channel coefficients) are obtained as

$$y(t) = x(t) + e(t) \tag{2}$$

where $e(t)$ is an additive observation noise assumed to be $\mathcal{N}(0, \sigma_e^2)$ white. In vector form, the N observations can be represented as

$$\begin{aligned} \mathbf{y} &= \mathbf{x} + \mathbf{e} \\ &= \mathbf{A}\mathbf{s} + \mathbf{e} \end{aligned} \tag{3}$$

where $\mathbf{x} = [x(t), \cdots, x(t-N+1)]$, $\mathbf{A} = [\mathbf{a}_1, \cdots, \mathbf{a}_p]$, $\mathbf{a}_k = [e^{j\omega_k t}, e^{j\omega_k (t-1)}, \cdots, e^{j\omega_k (t-N+1)}]^T$.
The complex amplitudes are collected in $\mathbf{s} = [s_1, \cdots, s_p]^T$, and the noise vector is $\mathbf{e} = [e(t), \cdots, e(t-N+1)]^T$.
The value $x(t+L)$ is to be predicted from observations $\mathbf{y}$, where L is the prediction horizon. In practice, a prediction horizon corresponding to a $\lambda/2$ distance travelled by the mobile is considered challenging [4].

3. THE LMMSE CHANNEL PREDICTOR WITH KNOWN FREQUENCIES

Assume the frequency estimates $\hat{\omega} = [\hat{\omega_1}, \cdots, \hat{\omega_p}]$ are given, and model $\mathbf{s}$ as zero-mean random variables with $E[\mathbf{s}\mathbf{s}^H] = \sigma_s^2 \mathbf{I}_N = \mathbf{S}$. The LMMSE estimate of $\mathbf{s}$ is then [2]

$$\hat{\mathbf{s}} = (\hat{\mathbf{A}}^H \hat{\mathbf{A}} + \alpha \mathbf{I})^{-1} \hat{\mathbf{A}}^H \mathbf{y} = \mathbf{R}_{reg}^{-1} \hat{\mathbf{A}}^H \mathbf{y} \tag{4}$$

where $\hat{\mathbf{A}}$ is defined as $\mathbf{A}$, but using the estimated frequencies $\hat{\omega}_k$ instead of the true frequencies, and $\alpha = \frac{\sigma_e^2}{\sigma_s^2} = \frac{1}{SNR}$. This can also be interpreted as a regularized LS estimator [2]. Moreover, the LMMSE prediction of $x(t+L)$ (given that $\hat{\omega}$ is assumed correct) is given by

$$\begin{aligned}\hat{x}(t+L) &= [e^{j\hat{\omega}_1(t+L)}, \cdots, e^{j\hat{\omega}_p(t+L)}]\hat{\mathbf{s}} \\ &= \hat{\mathbf{a}}(L)^H \mathbf{R}_{reg}^{-1} \hat{\mathbf{A}}^H \mathbf{y} \\ &= \mathbf{f}^H \mathbf{y}, \end{aligned} \quad (5)$$

where $\hat{\mathbf{a}}(L)^H = [e^{j\hat{\omega}_1(t+L)}, \cdots, e^{j\hat{\omega}_p(t+L)}]$, and

$$\mathbf{f}^H = \hat{\mathbf{a}}(L)^H \mathbf{R}_{reg}^{-1} \hat{\mathbf{A}}^H \quad (6)$$

is the prediction filter. For Gaussian signals, the LMMSE predictor is also an MMSE predictor.

4. COMBINED LMMSE AND LP

Above, the frequency estimates $\hat{\omega}$ were regarded as exact. In practice, $\hat{\omega}$ is estimated by some method, such as Unitary ESPRIT [5] and subject to errors. The error variance, in theory, is determined by SNR and number of samples [6]. Such a problem was investigated in [7, 8, 9]. Define the residue signal $\epsilon = [\epsilon(t), \cdots, \epsilon(t-N+1)]^T$ as the part of the observations which can not be accurately reconstructed by the estimated sinusoidal model:

$$\epsilon = \mathbf{y} - \hat{\mathbf{x}},$$

where $\hat{\mathbf{x}} = \hat{\mathbf{A}}\hat{\mathbf{s}}$. In the imperfect modelling case, a low order LP can be used to predict the colored residues $\epsilon(t+L)$. So the Combined LMMSE and LP is

$$\hat{x}_{comb}(t+L) = \hat{x}(t+L) + \hat{\epsilon}(t+L). \quad (7)$$

The predictor $\hat{\epsilon}(t+L)$ is computed based on past residues as

$$\hat{\epsilon}(t+L) = \sum_{k=0}^{d-1} \beta_k \epsilon(t-k). \quad (8)$$

The coefficients are computed by solving an LS problem. In principle, the color of the residuals should be taken into account also when estimating the frequencies [10]. However, at the relatively high SNR considered here, this will not substantially change the estimates.

5. LMMSE PREDICTOR WITH FREQUENCY ESTIMATION ERRORS

Another approach to combat the frequency estimate error is motivated by introducing the frequency estimate uncertainty into the LMMSE predictions. We can model ω_k as a random variable with mean $\hat{\omega}_k$ and variance $E[(\Delta\omega_k)^2] = \sigma_{\Delta\omega_k}^2$,

$$\omega_k = \hat{\omega}_k + \Delta\omega_k.$$

The classical form of the LMMSE predictor is given by the Wiener filter [6],

$$\hat{x}(t+L) = \mathbf{R}_{xy}\mathbf{R}_{yy}^{-1}\mathbf{y}, \quad (9)$$

where

$$E\left\{\begin{bmatrix} x(t+L) \\ \mathbf{y} \end{bmatrix} \begin{bmatrix} x(t+L)^H & \mathbf{y}^H \end{bmatrix}\right\} = \begin{bmatrix} \sigma_x^2 & \mathbf{R}_{xy} \\ \mathbf{R}_{yx} & \mathbf{R}_{yy} \end{bmatrix}. \quad (10)$$

The covariance matrix for N observations is

$$\mathbf{R}_{yy} = E\{\mathbf{y}\mathbf{y}^H\} = \mathbf{R}_{xx} + \mathbf{R}_{ee}. \quad (11)$$

Under the assumption that the amplitudes and frequency estimation errors are independent stochastic variables, we can write the (m,n)th element of $\mathbf{R}_{xx}$ as

$$\begin{aligned}[\mathbf{R}_{xx}]_{mn} &= E\{[\mathbf{A}\mathbf{s}\mathbf{s}^H\mathbf{A}^H]_{mn}\} \\ &= \sum_{k=1}^{p} E\{|s_k|^2\} E\{e^{j(\hat{\omega}_k + \Delta\omega_k)(n-m)}\}. \end{aligned} \quad (12)$$

The expectation over the frequency estimation error can be expressed as

$$\begin{aligned} E\{e^{j(\hat{\omega}_k+\Delta\omega_k)(n-m)}\} &= e^{j\hat{\omega}_k(n-m)} E\{e^{j\Delta\omega_k(n-m)}\} \\ &= e^{j\hat{\omega}_k(n-m)} \Phi_{\Delta\omega_k}(n-m) \end{aligned} \quad (13)$$

where $\Phi_{\Delta\omega_k}(\tau)$ is the characteristic function of the frequency estimation error $\Delta\omega_k$. If we assume the frequency errors to be Gaussian distributed, then

$$\Phi_{\Delta\omega_k}(n-m) = e^{-\frac{\sigma_{\Delta\omega_k}^2}{2}(n-m)^2}. \quad (14)$$

The expectation over the ensemble of amplitudes is just the variance

$$E\{|s_k|^2\} = \sigma_s^2. \quad (15)$$

The (m,n)th element of the covariance matrix is thus obtained as

$$[\mathbf{R}_{xx}]_{mn} = \sum_{k=1}^{p} \sigma_s^2 e^{j\hat{\omega}_k(n-m)} e^{-\frac{\sigma_{\Delta\omega_k}^2}{2}(n-m)^2}. \quad (16)$$

For simplicity, let all the frequency errors be IID with $\sigma_{\Delta\omega_k}^2 = \sigma_{\Delta\omega}^2$. We then have

$$[\mathbf{R}_{xx}]_{mn} = \sigma_s^2 \left[\sum_{k=1}^{p} e^{j\hat{\omega}_k(n-m)}\right] e^{-\frac{\sigma_{\Delta\omega}^2}{2}(n-m)^2}. \quad (17)$$

Define the damping matrix $\mathbf{\Gamma}$ by

$$[\mathbf{\Gamma}]_{mn} = e^{-\frac{\sigma^2_{\Delta\omega}}{2}(n-m)^2}, \tag{18}$$

The covariance matrix can then be expressed as

$$\mathbf{R}_{xx} = \sigma_s^2 \hat{\mathbf{A}}\hat{\mathbf{A}}^H \odot \mathbf{\Gamma} \tag{19}$$

where $\odot$ denotes the Hadamard product.
Similarly, the cross covariance $\mathbf{R}_{xy}$ is given by

$$\begin{aligned}\mathbf{R}_{xy} &= E\{\mathbf{a}(L)^H \mathbf{s}\mathbf{s}^H \mathbf{A}^H\} \\ &= \sum_{k=1}^{p} E\{|s_k|^2\} \\ &\quad \cdot E\left\{\left[e^{j(\hat{\omega}_k+\Delta\omega_k)L}, \cdots, e^{j(\hat{\omega}_k+\Delta\omega_k)(L+N-1)}\right]\right\}.\end{aligned} \tag{20}$$

Define the damping vector

$$\gamma = \left[e^{\frac{-\sigma^2_{\Delta\omega}}{2}L^2}, \cdots, e^{\frac{-\sigma^2_{\Delta\omega}}{2}(L+N-1)^2}\right]. \tag{21}$$

The cross covariance is then obtained as

$$\begin{aligned}\mathbf{R}_{xy} &= \sigma_s^2 \sum_{k=1}^{p} \left\{\left[e^{j\hat{\omega}_k L}, \cdots, e^{j\hat{\omega}_k (L+N-1)}\right]\right\} \odot \gamma \\ &= \sigma_s^2 \hat{\mathbf{a}}(L)^H \hat{\mathbf{A}}^H \odot \gamma.\end{aligned} \tag{22}$$

The improved LMMSE predictor is finally obtained as

$$\begin{aligned}\hat{x}(t+L) &= \mathbf{R}_{xy}\mathbf{R}_{yy}^{-1}\mathbf{y} \\ &= (\hat{\mathbf{a}}(L)^H \hat{\mathbf{A}}^H \odot \gamma)(\hat{\mathbf{A}}\hat{\mathbf{A}}^H \odot \mathbf{\Gamma} + \alpha \mathbf{I}_N)^{-1}\mathbf{y}.\end{aligned} \tag{23}$$

Clearly, the influence of old observations is reduced by the damping matrix $\mathbf{\Gamma}$ and the damping vector γ. The damping vector γ is also dependent on the prediction range. The further ahead we are looking, the smaller is the gain of the predictor. The way the frequency error is taken into account can be interpreted as a convolution of the line spectrum of the signal with the error distribution. The filter design is thus done for distributed sources.

The errors in the frequency estimates force the predictor to miss-trust older data, as even a small frequency error can cause a large phase error if one waits long enough. As the special case, when $\sigma^2_{\Delta\omega} = 0$, (23) degenerates to (5). When $\sigma^2_{\Delta\omega} > 0$, (23), although being LMMSE, is not the MMSE due to the nonlinear dependency on the frequency estimates.

6. MEASURES OF PREDICTION PERFORMANCE

Normalized Mean Square Error (NMSE) is widely used to evaluate the performances of channel predictors. First we define the normalized square error in a single realization as

$$e^2_{NSE}(t+L) = \frac{N|x(t+L) - \hat{x}(t+L)|^2}{\mathbf{x}^H\mathbf{x}}.$$

Then the NMSE is the mean value of $e^2_{NSE}(t+L)$ taken over the Monte Carlo simulations. But it was found that NMSE is not suitable for evaluation of sinusoid model based predictors [2]. This is because a small amount of outliers might ruin the whole averaged performance. So the outliers are dropped when the NMSE of their power prediction is larger than 0.04, or in other words, when the prediction error of the complex amplitude is larger than 20% of the Root Mean Square (RMS) channel amplitudes. The measure is called Adjusted NMSE (ANMSE).

7. SIMULATIONS

The simulation parameter setting is as follows:

- SISO scenario;
- Spatial channel sampling interval = 0.1 λ ($\Delta l = 0.1$);
- Prediction horizon = 0.5 λ ($L = 5$);
- Number of sinusoids = 8 ($p = 8$, which is assumed to be known);
- Number of samples = 100 ($N = 100$);
- Number of Monte Carlo simulations = 1000;
- Order of LP is two times the number of sinusoids;
- In the combined method, the sinusoids with $|s_k| > 2\sigma_e$ are predicted by the LMMSE method, the residue is predicted using an LP with order 2 ($d = 2$);
- The frequency error variance is used as a design parameter and is taken to be $\sigma^2_{\Delta\omega} = 1/N^3$. This is somewhat arbitrary, but is motivated by the CRB [6];
- The Unitary-ESPRIT method is used for frequency estimation in the LMMSE prediction.
- Other settings are the same as in [2].

In Fig. 1, the Cumulative Density Function (CDF) of the NMSE are prsented. All sinusoid model based predictors have light tails, but the LP has a heavy tail.

The ANMSE of different predictors are given in Fig. 2. It can be seen that the LMMSE with known frequency and the LP has the best and the worst performance, respectively. The LMMSE with estimated frequency has much better performance than the LP in the investigated scenarios.

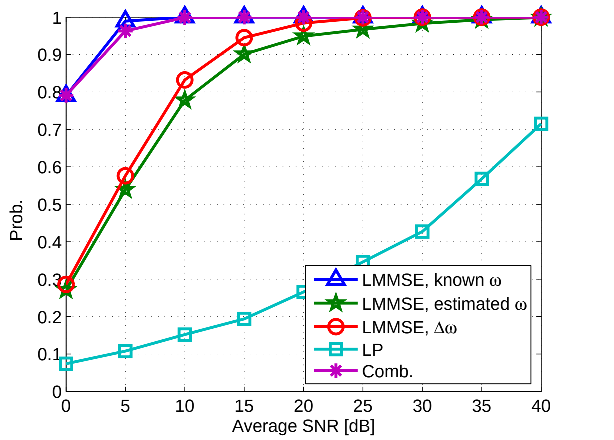

Fig. 1. Probability of channel prediction error less than 20% ($L = 0.5\lambda$, $N = 100$, 8 Rays)

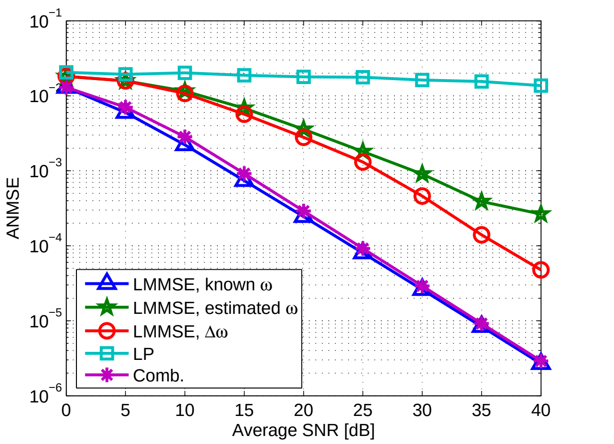

Fig. 2. Adjusted NMSE of channel prediction ($L = 0.5\lambda$, $N = 100$, 8 Rays)

By modelling the frequency error as Gaussian random variables, a slight improvement is observed. Finally the combined method has extremely good performance, which approaches the performance of the LMMSE with known frequency with the increase of SNR.

Note that, in these simulations, for a given carrier frequency, f, and velocity, v, the spatial sampling interval can be easily converted into time sampling interval as $\Delta t = \frac{\Delta l \cdot c}{f \cdot v}$, where c is the speed of light.

8. CONCLUSIONS

Frequency estimate errors give rise to colored residual signals in sinusoidal modelling based channel predictions. Two new approaches are proposed to mitigate the influence of the frequency estimate error. One is using a low order LP to predict the residual signals. The second is to model the frequency estimates as Gaussian random variables, which contributes exponential damping weights on the observations. Both methods have better performance than the "classical" LMMSE method. For high SNR the performance of the Combined LMMSE and LP approaches that of LMMSE prediction with exact frequencies.

9. REFERENCES

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