A NOVEL BLIND DECONVOLUTION METHOD VIA MAXIMUM ENTROPY

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ABSTRACT

We propose a new closed form (approximated) expression for the conditional expectation that is based on Maximum Entropy. This expression does not rely on the knowledge of the convolutional noise power nor imposes any restrictions on the probability distribution of the unobserved input sequence and is suitable for the general case of real and complex source signals. In addition, we propose a set of algebraic linear equations for the Lagrange multipliers related to the blind deconvolution problem that can be easily computed in a non iterative approach. Our new derivation leads to a new blind deconvolution algorithm with improved equalization performance compared with Godard's equalizer.

1. INTRODUCTION

We consider a blind deconvolution problem in which we observe the output $y(n)$ of an unknown possibly a nonminimum phase linear system corrupted with noise (see Fig.1) from which we want to recover its input using an adjustable linear filter (equalizer). The observer does not know the exact sequence $x(n)$ transmitted through the channel. All he knows about the random variables $x(n)$ can be summarized in the form of some mathematical constraints such as statistical characteristics for instance.

The problem of blind deconvolution arises comprehensively in various applications such as digital communications, seismic signal processing, speech modeling and synthesis, ultrasonic nondestructive evaluation, and image restoration [9], [3]. A partial list of references, with emphasis on the problem of channel equalization in data communication, is given by [14], [6], [2], [8], [4] and [5].

Blind deconvolution algorithms are essentially adaptive filtering algorithms designed such that they do not require the external supply of a desired response to generate the error signal in the output of the adaptive equalization filter [11]. The algorithm itself generates an estimate of the desired response by applying a nonlinear transformation to sequences involved in the adaptation process [11]. The Bussgang algorithms, is one of the three important families of blind equalization algorithms, where the nonlinearity is in the output of the adaptive equalization filter [11]. The nonlinearity is designed to minimize a cost function that is implicity based on higher order statistics (HOS) according to one approach [6], [14], or calculated directly according to Bayes rules [1], [2], [4], [5], [8]. The main difference between the Bussgang type algorithms lies in the choice of the memoryless nonlinearity [11]. Obviously, the performance of such kind of blind equalizer depends strongly on the memoryless nonlinearity. In this paper, we consider the second approach where the nonlinearity ($T\left(z\left(n\right)\right)$ in Fig.1) is calculated directly according to Bayes rules. According to [11], the conditional expectation ($E[x(n)/z(n)]$, where $E(\cdot)$ stands for the expectation operation) may be the best estimate of $T(z(n))$. In the literature, the conditional expectation was derived for non-Gaussian sources by Bellini ([1], [2]), Fiori ([4], [5]) and Haykin [8]. However, Bellini's and Haykin's expression for the conditional expectation [1], [2], [8] rely on the knowledge of the convolution noise power where in fact, a suitable estimation for this power is quite difficult [4]. Moreover, an a priori optimal estimate for the convolutional noise power does not exist since this power changes with time due to the adaptation progress [4]. Recently, a new expression for the conditional expectation was derived by Fiori ([4], [5]). This expression does not rely on the knowledge of the convolution noise power and is suitable for uniformly distributed real source signals. However, all the mentioned expressions for the conditional expectation ([1], [2], [4], [5], [8]) are suitable only for uniformly distributed real source signals, thus they can not cope with a source having a general probability density function (pdf) shape. The limitation of uniformly distributed source signals was recently removed by [13]. In [13], a new approximated expression for the conditional expectation (for the real and two independent quadrature carrier case) was developed, which does not rely on the knowledge of the convolution noise power nor imposes any restrictions on the probability distribution of the (unobserved) input sequence.

But still, up to now, there exist no general closed expression for the conditional expectation suitable for the general case of source signals, such as the case where the source signal is complex (where the imaginary and real part are dependent) and has a pdf resembling the Gaussian. It should be pointed out, that HOS blind equalization methods provide poor equalization performance when the source signal pdf resembles the Gaussian. Recently [17], the vector constant modulus algorithm (VCMA) was introduced as an extension of the constant modulus algorithm (CMA) [6], which can equalize data from shaped sources having nearly Gaussian marginal distributions. However, VCMA assumes that the source shaping is accomplished by choosing signal points uniformly distributed in a 2N-dimensional sphere and transmitting them as a sequence of N complex values. Thus it is not a general solution for source signals having a pdf resembling the Gaussian. Therefore a general closed expression for the conditional expectation that can handle with the general case of source signals is necessary.

In this paper we present a new closed approximated expression for the conditional expectation based on the Maximum Entropy. This expression does not rely on the knowledge of the convolutional noise power nor imposes any restrictions on the probability distribution of the unobserved input sequence and is suitable for the general case of real and complex source signals. In addition, we propose a set of algebraic linear equations for the Lagrange multipliers related to the blind deconvolution problem. Based on our new findings, a new blind deconvolution algorithm is proposed. Computer simulations show that our new proposed algorithm has improved equalization performance compared to [6] and [16].

The organization of the work described in this paper is as follows. Section II contains the system under consideration. In Section III we present our new closed approximated expression for the conditional expectation and our non iterative expression for the Lagrange multipliers related to the blind deconvolution problem. In Section IV we present our new proposed blind deconvolution algorithm and in Section V we present our simulation results. Finally, some conclusions are drawn in Section VI.

2. SYSTEM DESCRIPTION

The system under consideration is illustrated in Fig. 1, where we make the following assumptions:
1. The input sequence $x(n)$ consists of zero mean real or complex random variables with an arbitrary probability distribution.
2. The unknown channel $h(n)$ is a possibly nonminimum phase linear time-invariant filter in which the transfer function has no "deep zeros", namely, the zeros lie sufficiently far from the unit circle.
3. The equalizer $c(n)$ is a tap-delay line.
4. The noise $w(n)$ is an additive Gaussian white noise.
5. The function $T[\cdot]$ is a memoryless nonlinear function.
The transmitted sequence $x(n)$ is transmitted through the channel $h(n)$ and is corrupted with noise $w(n)$. Therefore, the equalizer's input sequence $y(n)$ may be written as:

$$y(n) = x(n) * h(n) + w(n) \tag{1}$$

where "$*$" denotes the convolution operation. Since $c(n)$ is unknown, we assume that some initial guess $c_g(n)$ has been selected for the impulse response of the equalizer. We denote:

$$c_g\left(n\right) * h\left(n\right) = \delta\left(n\right) + \xi\left(n\right) \tag{2}$$

where $\xi(n)$ stands for the difference (error) between the ideal value $c(n)$ and the guess $c_g(n)$. Convolving $c_g(n)$ with the received sequence $y(n)$ and using (1) and (2), we obtain:

$$z\left(n\right) = x\left(n\right) + p\left(n\right) + \tilde{w}\left(n\right) \tag{3}$$

where $p(n)$ is the convolutional noise, namely, the residual intersymbol interference (ISI) arising from the difference between $c_g(n)$ and $c(n)$ and $\tilde{w}\left(n\right) = w\left(n\right) * c_g\left(n\right)$. Next we consider the adaptation mechanism of the equalizer. We define some estimator of $x(n)$, $d(n)$ which is produced by the function $T(z(n))$. Thus the error signal is:

$$\tilde{e}\left(n\right) = T\left(z\left(n\right)\right) - z\left(n\right). \tag{4}$$

This error is fed into the adaptive mechanism which updates the equalizer's taps. As we have mentioned in the introduction, the best estimate of $x(n)$ is the conditional expectation $E[x(n)/z(n)]$ [11]. But, in order to derive the conditional expectation $E[x(n)/z(n)]$, the source input probability density function $f_x(x)$ has to be known. However, in our case, the source input probability density function $f_x(x)$ is not known. In the following section, we derive a closed approximated expression for the conditional expectation without the knowledge of the source input probability density function $f_x(x)$.

3. A NEW CLOSED FORM EXPRESSION FOR THE CONDITIONAL EXPECTATION

In this section we present a systematic approach for obtaining the conditional expectation. For simplicity we denote: $E[x(n)/z(n)] = E[x/z]$ and do the following assumptions: 1. The convolutional noise $p(n)$, is a zero mean, white Gaussian process with variance $\sigma_p^2 = E[pp^*]$. 2. No noise is added. 3. The convolutional noise $p(n)$ and the source signal are independent. 4. σ_p^2 is a small number.
Theorem: The approximated conditional expectation is de-

fined by:

$$E[x/z] \cong \frac{z + \frac{(\sigma_z^2-\sigma_x^2)}{4Q(z_1,z_2)}\left[G_{x_1x_1}(z_1,z_2)+G_{x_2x_2}(z_1,z_2)\right]}{1+\frac{(\sigma_z^2-\sigma_x^2)}{4Q(z_1,z_2)}\left[Q_{x_1x_1}(z_1,z_2)+Q_{x_2x_2}(z_1,z_2)\right]} \tag{5}$$

where:

$$G_{x_1x_1}(z_1,z_2) =$$
$$\left(\frac{\partial^2}{\partial x_1^2}\left((x_1+jx_2)\exp\left(\sum_{k_1+k_2=2}^{K}\lambda_{k_1,k_2}x_1^{k_1}x_2^{k_2}\right)\right)\right)_{x=z}$$
$$G_{x_2x_2}(z_1,z_2) =$$
$$\left(\frac{\partial^2}{\partial x_2^2}\left((x_1+jx_2)\exp\left(\sum_{k_1+k_2=2}^{K}\lambda_{k_1,k_2}x_1^{k_1}x_2^{k_2}\right)\right)\right)_{x=z} \tag{6}$$

$$Q_{x_1x_1}(z_1,z_2) = \left(\frac{\partial^2}{\partial x_1^2}\left(\exp\left(\sum_{k_1+k_2=2}^{K}\lambda_{k_1,k_2}x_1^{k_1}x_2^{k_2}\right)\right)\right)_{x=z}$$
$$Q_{x_2x_2}(z_1,z_2) = \left(\frac{\partial^2}{\partial x_2^2}\left(\exp\left(\sum_{k_1+k_2=2}^{K}\lambda_{k_1,k_2}x_1^{k_1}x_2^{k_2}\right)\right)\right)_{x=z} \tag{7}$$

$$Q(z_1,z_2) = \exp\left(\sum_{k_1+k_2=2}^{K}\lambda_{k_1,k_2}z_1^{k_1}z_2^{k_2}\right) \tag{8}$$

The various Lagrange multipliers λ_{k_1,k_2} are obtained by the following equation:

$$k_1(k_1-1)E\left[x_2^{k_2}x_1^{k_1-2}\right] + 2\lambda_{k_1,k_2}k_1^2E\left[x_2^{2k_2}x_1^{2k_1-2}\right] + \sum_{l_1+l_2=2,\ l_1\neq k_1,l_2\neq k_2}^{K}2\lambda_{l_1,l_2}k_1l_1E\left[x_2^{k_2+l_2}x_1^{k_1+l_1-2}\right] + k_2(k_2-1)E\left[x_1^{k_1}x_2^{k_2-2}\right] + 2\lambda_{k_1,k_2}k_2^2E\left[x_1^{2k_1}x_2^{2k_2-2}\right] + \sum_{l_1+l_2=2,\ l_1\neq k_1,l_2\neq k_2}^{K}2\lambda_{l_1,l_2}k_2l_2E\left[x_1^{k_1+l_1}x_2^{k_2+l_2-2}\right] = 0 \tag{9}$$

where $x = x_1 + jx_2$, $z = z_1 + jz_2$, σ_x^2 and σ_z^2 are the source and equalized output signal variances respectively. The variance of the equalized output signal (σ_z^2) is derived according to:

$$\langle z(n)z^*(n)\rangle_n = (1-\beta)\langle z(n)z^*(n)\rangle_{n-1} + \beta\left(z(n)z^*(n)\right) \tag{10}$$

where $()^*$ means the conjugate of $()$, $\langle\rangle$ stands for the estimated expectation and β is a positive stepsize parameter.

Comments:
Assumptions 1 and 3 were also made in [1], [8] and in [11]. It should be noted that the described model for the convolutional noise $p(n)$ is applicable during the latter stages of the process where the process is close to optimality. According to [8], in the early stages of the iterative deconvolution process, the ISI is typically large with the result that the data sequence and the convolutional noise are strongly correlated and the convolutional noise sequence is more uniform than Gaussian [7]. Evidently our new proposed estimator (conditional expectation) is valid for the latter stages of the process where the "eye diagram" is already open. However, as is shown by simulation in this paper, our derivation can be successfully extended also to the early stages where the "eye diagram" is still close.

Proof:
The conditional expectation is defined by Bayes rule as:

$$E[x/z] = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f_{z/x}(z/x) f_{x_1,x_2}(x_1,x_2)dx_1dx_2}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{z/x}(z/x) f_{x_1,x_2}(x_1,x_2)dx_1dx_2} \tag{11}$$

where $f_{z/x}(z/x)$ is assumed to be:

$$f_{z/x}(z/x) = \frac{1}{\pi\sigma_p^2}\exp\left(-\frac{|z-x|^2}{\sigma_p^2}\right). \tag{12}$$

Since all we know about the source signal $x(n)$ can be summarized in the form of some mathematical constraints such as statistical characteristics (moments constraints or equations of moments), we turn to Maximum Entropy techniques for obtaining a proper approximation for the source input pdf as we have done in [13]. The Maximum Entropy approach is a flexible and powerful tool for density approximation. According to [10], one can find most of the important probability distributions (the uniform law, the exponential distribution, the normal distribution, the Cauchy distribution, etc.) by choosing suitable mathematical constraints for the probability density. Following [10], we choose $\hat{f}_{x_1,x_2}(x_1,x_2)$:

$$\hat{f}_{x_1,x_2}(x_1,x_2) = \exp\left(\sum_{k_1+k_2=0}^{K}\lambda_{k_1,k_2}x_1^{k_1}x_2^{k_2}\right) \tag{13}$$

for the approximation of the source input pdf $f_{x_1,x_2}(x_1,x_2)$, where x_1, x_2 are the real and imaginary part of $x(n)$ and λ_{k_1,k_2} are the Lagrange multipliers. In order to determine λ_{k_1,k_2}, we can use the moments m_{j_1,j_2}, where we just have to solve the system:

$$m_{j_1,j_2} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1^{j_1}x_2^{j_2}\hat{f}_{x_1,x_2}(x_1,x_2)dx_1dx_2. \tag{14}$$

However, solving this system is not easy and in general there is no analytical solution for the various Lagrange multipliers when $K > 4$. We substitute (13) into (11) and obtain:

$$E\left[x/z\right] \simeq \frac{\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_1\left(x_1,x_2\right)\exp\left(-\frac{\Psi\left(x_1,x_2\right)}{\rho}\right)dx_1dx_2}{\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_0\left(x_1,x_2\right)\exp\left(-\frac{\Psi\left(x_1,x_2\right)}{\rho}\right)dx_1dx_2} \tag{15}$$

where

$$\Psi\left(x_1,x_2\right) = \left|z-x\right|^2; \qquad \rho=\sigma_p^2 \tag{16}$$

$$\begin{aligned} g_1\left(x_1,x_2\right) &= x g_0\left(x_1,x_2\right) \\ g_0\left(x_1,x_2\right) &= \exp\left(\sum_{k_1+k_2=2}^{K}\lambda_{k_1,k_2}x_1^{k_1}x_2^{k_2}\right) \end{aligned} \tag{17}$$

Next, we use Laplace's integral method [12] for solving (15). The Laplace's method is a general technique for obtaining the asymptotic behavior as $\rho \rightarrow 0$ of integrals in which the large parameter $\frac{1}{\rho}$ appears in the exponent. The main idea of Laplace's method is: if the real continuous function $\Psi(x_1,x_2)$ has its minimum at (x_{01},x_{02}) which is between infinity and minus infinity, then it is only the immediate neighborhood of $x_1=x_{01}, x_2=x_{02}$ that contributes to the full asymptotic expansion of the integral for large $\frac{1}{\rho}$. Therefore, according to [12] we may write:

$$\begin{aligned} &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_1\left(x_1,x_2\right)\exp\left(-\frac{\Psi\left(x_1,x_2\right)}{\rho}\right)dx_1dx_2 \simeq \\ &A\Bigg(g_1\left(x_{01},x_{02}\right)+\frac{\rho}{2}\frac{\left(\frac{\partial^2}{\partial x_1^2}g_1\left(x_1,x_2\right)\right)_{(x_{01},x_{02})}}{\left(\frac{\partial^2}{\partial x_1^2}\Psi\left(x_1,x_2\right)\right)_{(x_{01},x_{02})}}+ \\ &\frac{\rho}{2}\frac{\left(\frac{\partial^2}{\partial x_2^2}g_1\left(x_1,x_2\right)\right)_{(x_{01},x_{02})}}{\left(\frac{\partial^2}{\partial x_2^2}\Psi\left(x_1,x_2\right)\right)_{(x_{01},x_{02})}}+O\left(\rho^2\right)\Bigg) \end{aligned} \tag{18}$$

where

$$\begin{aligned} A &= \exp\left(-\frac{\Psi\left(x_{01},x_{02}\right)}{\rho}\right)\frac{2\pi\rho}{\sqrt{a_1}\sqrt{a_2}} \\ a_1 &= \left(\frac{\partial^2}{\partial x_1^2}\left(\Psi\left(x_1,x_2\right)\right)\right)_{(x_{01},x_{02})} \\ a_2 &= \left(\frac{\partial^2}{\partial x_2^2}\left(\Psi\left(x_1,x_2\right)\right)\right)_{(x_{01},x_{02})} \end{aligned} \tag{19}$$

and $O(x)$ is defined as $lim_{x\rightarrow 0}\frac{O(x)}{x}=r_{const}$ where "r_{const}" is a constant. The functions $a1, a2$ and x_{01}, x_{02} are defined via:

$$\begin{aligned} &\tfrac{\partial}{\partial x_1}\left(\Psi\left(x_1,x_2\right)\right)=0;\ \rightarrow x_{01}=z_1 \\ &\tfrac{\partial}{\partial x_2}\left(\Psi\left(x_1,x_2\right)\right)=0;\ \rightarrow x_{02}=z_2 \\ &\tfrac{\partial^2}{\partial x_1^2}\left(\Psi\left(x_1,x_2\right)\right)=2;\ \rightarrow a_1=2 \\ &\tfrac{\partial^2}{\partial x_2^2}\left(\Psi\left(x_1,x_2\right)\right)=2;\ \rightarrow a_2=2 \end{aligned} \tag{20}$$

Note that the integral from the denominator in (15) can be carried out by switching $g_1\left(x_1,x_2\right)$ in (18) with $g_0\left(x_1,x_2\right)$. Next we divide the numerator and denominator of (15) by $g_0\left(x_{01},x_{02}\right)$ and use (18) together with (19), (20) and the relation of $\sigma_p^2=\sigma_z^2-\sigma_x^2$ to obtain (5). Next we turn to evaluate the Lagrange multipliers. According to our new proposed approach, the Lagrange multipliers are obtained through the MSE expression where the MSE is defined as:

$$MSE=E\left[\left(E\left[x/z\right]-x\right)\left(E\left[x/z\right]-x\right)^*\right] \tag{21}$$

By using (5), (3), assumptions 2 and 3 and assuming sufficiently low convolutional noise power, we may write:

$$E\left[x/z\right]-x \simeq \frac{p(n)}{1+\frac{Q_{x_1,x_1}\left(z_1,z_2\right)}{2Q\left(z_1,z_2\right)}\frac{\sigma_p^2}{2}+\frac{Q_{x_2,x_2}\left(z_1,z_2\right)}{2Q\left(z_1,z_2\right)}\frac{\sigma_p^2}{2}} \tag{22}$$

Note that (22) is an expression having random variables on both the numerator and denominator. Thus, deriving the approximated MSE is not an easy task. In the following we adopt the technique applied in [13] for evaluating the approximated MSE. Thus for the case of sufficiently low convolutional noise power and $\left|\sigma_p^2 E\left[\frac{Q_{x_1,x_1}(z_1,z_2)}{4Q(z_1,z_2)}+\frac{Q_{x_2,x_2}(z_1,z_2)}{4Q(z_1,z_2)}\right]\right| \ll 1$, we obtain:

$$MSE \simeq \sigma_p^2\left[1-\sigma_p^2\left(E\left[\frac{Q_{x_1,x_1}(z_1,z_2)}{2Q(z_1,z_2)}+\frac{Q_{x_2,x_2}(z_1,z_2)}{2Q(z_1,z_2)}\right]\right)\right] \tag{23}$$

Next, we differentiate (23) subject to the Lagrange multipliers and set the obtained expression to zero:

$$\frac{\partial}{\partial\lambda_{k_1,k_2}}E\left[\frac{Q_{x_1,x_1}\left(z_1,z_2\right)}{2Q\left(z_1,z_2\right)}+\frac{Q_{x_2,x_2}\left(z_1,z_2\right)}{2Q\left(z_1,z_2\right)}\right]=0 \tag{24}$$

Now, solving (24) leads to (9). This completes the *Proof.*

4. MAXIMUM ENTROPY BLIND DECONVOLUTION ALGORITHM

In this section we present our proposed blind deconvolution algorithm. The taps of the proposed blind equalizer are updated according to:

$$\begin{aligned} &c_l\left(n+1\right)=c_l\left(n\right)-\mu W y^*\left(n-l\right) \qquad \text{with} \\ &W=\left[E[x/z]\frac{z^*(n)E[x/z]}{\left\langle z(n)z^*(n)\right\rangle_n}-z(n)\right] \end{aligned} \tag{25}$$

where μ is a positive stepsize parameter, l stands for the l-th tap equalizer, $E[x/z]$ and $\langle z(n)z^*(n)\rangle_n$ are defined in (5) and (10) respectively.

5. SIMULATION

We tested the performance of our proposed blind equalization method via simulation and compared them to the simulated results obtained by Godard's [6] algorithm. We have also compared our proposed equalization method via simulation with the super-exponential algorithm [16] (which is a batch algorithm) where the equalizers' coefficients in our proposed method were updated every sample period during the gathering of the data samples to a batch. The equalizer taps for Godard's algorithm [6] were updated according to:

$$c_l\left(n+1\right) = c_l\left(n\right) - \mu_G\left(\left|z\left[n\right]\right|^2 - \frac{E\left[\left|x[n]\right|^4\right]}{E\left[\left|x[n]\right|^2\right]}\right) z\left[n\right] y^*\left(n-l\right) \tag{26}$$

where μ_G is the step-size parameter. The equalizer taps for algorithm [16] were updated according to:

$$c' = \hat{R}^{-1}\hat{d}; \qquad c'' = c'\sqrt{\frac{1}{\left(c'\right)^+ \hat{R}c'}} \tag{27}$$

where c'' is the vector of taps after iteration and $\hat{R}$ is the $L \times L$ (L is the equalizer length) matrix whose elements are : $\hat{R}_{nm} = \dfrac{c\hat{u}m\left(y_{t-m}; y^*_{t-n}\right)}{cum\left(x_t; x^*_t\right)}$ and $\hat{d}$ is the $L \times 1$ vector whose elements are:

$$\hat{d}_n = \frac{\frac{1}{N}\sum\limits_{t=1}^{N}\left|z_t\right|^2 z_t y^*_{t-n} - \frac{2}{N}\sum\limits_{t=1}^{N}\left|z_t\right|^2 \cdot \frac{1}{N}\sum\limits_{t=1}^{N} z_t y^*_{t-n}}{E\left[\left|x_t\right|^4\right] - 2E\left[\left|x_t\right|^2\right]} - \frac{\frac{1}{N}\sum\limits_{t=1}^{N} z_t^2 \cdot \frac{1}{N}\sum\limits_{t=1}^{N} z^*_t y^*_{t-n}}{E\left[\left|x_t\right|^4\right] - 2E\left[\left|x_t\right|^2\right]} \tag{28}$$

where N is the number of available data samples and $c\hat{u}m()$ denotes the estimate of $cum()$ (where $cum()$ means cumulant of ()), obtained by approximating ensemble averages with empirical averages. In the following, we denote algorithm [16] as Supex and "MAXENT" as our proposed algorithm. Two types of sources were used. The first one is a V29 source based on the V29 standard, in which the $x(n)$ admit the 16 equiprobable values: $1+j, 1-j, -1-j, -1+j, 3+3j, 3-3j, -3-3j, -3+3j, 3, -3, 3j, -3j, 5, -5, 5j, -5j$. The second source is a "wild source", in which the $x(n)$ is the discrete random variable:$1+j, 1-j, -1-j, -1+j$ with probability equal to $\frac{4}{36}$ of each of them, $3+3j, 3-3j, -3-3j, -3+3j, 3, -3, 3j, -3j$ with probability equal to $\frac{2}{36}$ of each of them and $5, -5, 5j, -5j$ with probability equal to $\frac{1}{36}$ of each of them. The source input signals were transmitted through a nonminimum phase channel described in [15]: $h_n = \{0 \;\text{ for }\; n < 0;\; -0.4 \;\text{ for }\; n = 0;\; 0.84 \cdot 0.4^{n-1} \;\text{ for }\; n > 0\}$. In our simulation, we substituted $\sigma_z^2 = \sigma_x^2$ for initialization. The simulated results are shown in Fig.2, Fig.3 and Fig.4. As may be seen from these figures, our new proposed blind equalization method has improved equalization performance compared to [6] and [16]. As expected, this performance gets better when the source input pdf tends to resemble the Gaussian pdf.

6. CONCLUSION

In this paper we proposed a new blind deconvolution method based on a new closed approximated expression for the conditional expectation based on Maximum Entropy. This expression does not relay on the knowledge of the convolutional noise power nor imposes any restrictions on the probability distribution of the unobserved input sequence and is suitable for the general case of complex source signals. In addition, efficient and non iterative approximated Lagrange multipliers were derived for the blind deconvolution problem.

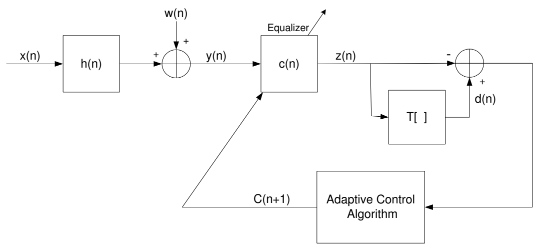

Fig. 1. Baseband communication system.

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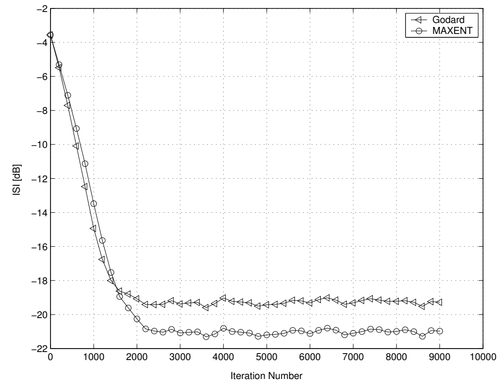

Fig. 2. Equalization performance comparison for a V29 source input with SNR=30 [dB], taplength $= 13$, $\mu_G = 2.5 \cdot 10^{-5}$, $\mu = 3.5 \cdot 10^{-4}$, $\beta = 7.1 \cdot 10^{-5}$.

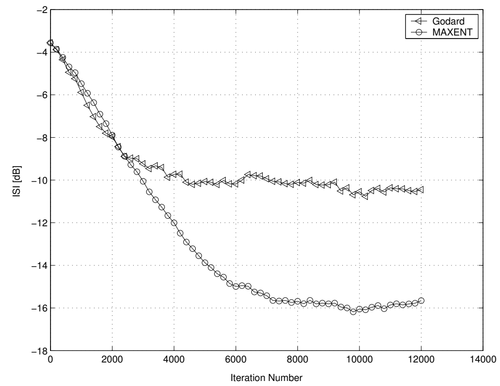

Fig. 3. Equalization performance comparison for a "wild source" input with SNR=30 [dB], taplength $= 13$, $\mu_G = 5 \cdot 10^{-5}$, $\mu = 3.5 \cdot 10^{-4}$, $\beta = 2 \cdot 10^{-5}$.

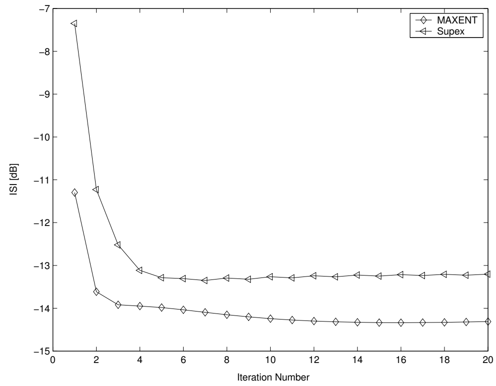

Fig. 4. Equalization performance comparison for a "wild source" input with SNR=30 [dB], taplength $= 13$, $\mu = 3.5 \cdot 10^{-4}$, $\beta = 2 \cdot 10^{-5}$ and $N = 3600$.

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