NON-COOPERATIVE LOCALIZATION OF BINARY SENSORS

Petar M. Djurić,[†] Mahesh Vemula,[†] Mónica F. Bugallo,[†] and Joaquín Míguez[‡]

[†] Department of Electrical and Computer Engineering
Stony Brook University
Stony Brook, NY, 11794-2350

[‡] Departamento de Teoría de la Señal y Comunicaciones
Universidad Carlos III de Madrid
Madrid, Spain

djuric, vema, monica @ece.sunysb.edu, jmiguez@ieee.org

ABSTRACT

In this paper we address the problem of non-cooperative localization of binary sensors in a field where the sensors are randomly deployed. The localization is based on the sensors' responses to radio-frequency signals transmitted by a beacon from a number of known locations in the field. The beacon travels along a known path in the sensor field, stops at predefined locations, emits a signal querying for sensors and finally collects measurements that represent responses to the query. We propose a method that uses the collected measurements for estimation of the unknown locations of the sensors. We also provide Cramér-Rao bounds of the estimates, and we demonstrate the performance of the method by computer simulations.

1. INTRODUCTION

Wireless sensor networks have been in the focus of intense research lately [8]. Many advances have led to their application to problems that involve detection, tracking, and classification in many areas including environmental monitoring, industrial diagnostics, power grids, water distribution, and battlefield awareness [8].

There are many important problems related to sensor deployment, and one of them is determining the location of the sensors that comprise the network. Indeed, without knowledge of the sensor locations, much of the information that they provide may not be very useful. There are many methods for sensor localization, of which one large class is based on signal measurements and their statistical models. These methods exploit the signals' time-of-arrivals [5], time difference-of-arrivals [8], angle-of-arrivals [2], or received signal strengths [5, 7], and they all vary in complexity and accuracy. Another way of classifying the localization methods is by grouping them into cooperative and non-cooperative methods. The former use information from neighboring sensors and reference nodes, and they do not rely on a central processor for localization, whereas the latter use a central processor and measurements only from individual sensors. Detailed reviews of localization algorithms can be found in [4, 6].

In many applications of wireless sensor networks, it is desirable that the used sensors are low-cost and energy efficient. The former requirement would allow for use of thousands of sensors and the latter would put restrictions on the amount of communication allowed to each sensor. If the sensors transmit only binary information, we call them binary sensors [1, 3]. In the simplest implementation, the binary sensors sense signals (radio-frequency (RF), acoustic, light signals, etc.) from their vicinity, and they only become active by transmitting a signal (that is usually a signal identifying them) if the strength of the sensed signal is above a certain threshold. Based on such ID transmissions, much can be deduced about the sensed phenomenon. In this paper, we study the problem of non-cooperative localization of binary sensors.

Our approach is based on the concept of received signal strength. A beacon is moved in the sensor field and transmits RF signals from various locations. If the sensors sense the transmitted signal they respond to the beacon with their RF ID. In Figure 1, we show a trajectory of a beacon which emits an RF signal with some fixed power when it is in locations r_1, r_2, $\cdots$, r_6 (denoted by the small circles along the trajectory). The binary sensor and its range are drawn by a slightly larger circle and the shaded circular area around it, respectively. The range of the sensor is defined as the portion of the field from which the sensor can sense the beacon when it transmits an RF signal with some fixed power. The received power by the binary sensor is denoted by $y_1, y_2, \cdots, y_6$. In general, when the beacon is within the range of the sensor, the received power is above a predefined threshold γ, and the sensor becomes active, by sending its ID to the beacon. Otherwise, it is in a standby mode. Based on the received signals from the sensor, the beacon has to estimate the sensor's location in the field.

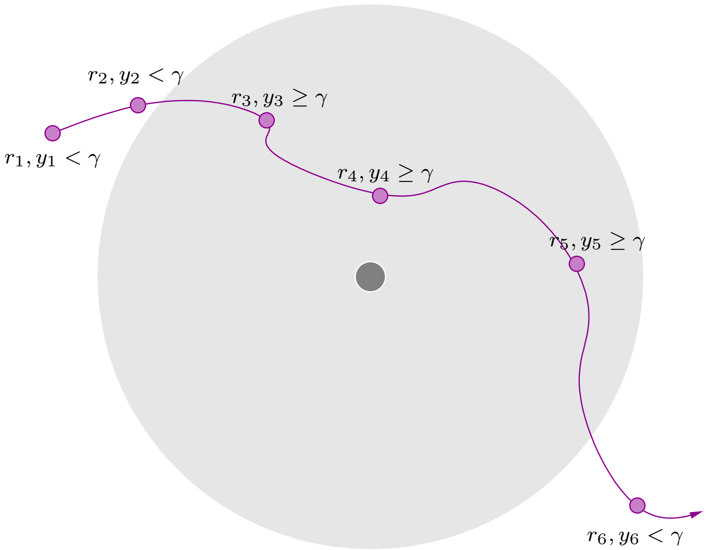

Fig. 1. A beacon's trajectory and the received powers by a binary sensor. The small dark circle in the middle represents the binary sensor, and the shaded circle is its range. The y_ms are the received powers by the sensor when the beacon is at location $\mathbf{r}_m$. When the power y_m is larger than the threshold γ, the sensor responds to the beacon.

In the sequel we first formally state the problem and then we provide the estimator for the locations of the binary sensors. Subsequently, we show the Cramér-Rao bound of the estimates and demonstrate the performance of the proposed method by computer simulations.

2. PROBLEM FORMULATION

Binary sensors are deployed in a sensor field whose locations are unknown and denoted by $\mathbf{x}_n = (x_{1,n}\ x_{2,n})^\top$, $n = 1, 2, \cdots, N$.[1] In the same field, a beacon transmits RF signals with known powers from known locations with coordinates $\mathbf{r}_m = (r_{1,m}, r_{2,m})^\top$, $m = 1, 2, \cdots, M$. The transmitted signals are sensed by the binary sensors, and the received power by the n-th sensor when the beacon is at location $\mathbf{r}_m$ is modeled by [5]

$$y_{mn} \sim \mathcal{N}(\nu_m(\mathbf{x}_n), \sigma_y^2) \tag{1}$$

where

$$\nu_m(\mathbf{x}_n) = \Psi_0 + 10\alpha_{mn} \log_{10} \frac{d_0}{|\mathbf{r}_m - \mathbf{x}_n|} \tag{2}$$

where y_{mn} is the received power, α_{mn} is the "path-loss" between the beacon at $\mathbf{r}_m$ and the sensor at $\mathbf{x}_n$ and which is known and takes values in the range (2,4), Ψ_0 is the known received power in dBm from the beacon at a distance d_0, $|\mathbf{r}_m - \mathbf{x}_n|$ is the distance between the beacon and the n-th sensor given by

$$|\mathbf{r}_m - \mathbf{x}_n| = \sqrt{(r_{1,m} - x_{1,n})^2 + (r_{2,m} - x_{2,n})^2}$$

and σ_y^2 is the variance of the shadowing. The observed measurements y_{mn}, $n = 1, 2, \cdots, N$, $m = 1, 2, \cdots, M$ are assumed independent and identically distributed. If the sensed signals are above a predefined threshold γ, the sensors transmit their ID back to the beacon, which measures the received power and whose model is

$$z_{nm} \sim \mathcal{N}(\eta_m(\mathbf{x}_n), \sigma_z^2), \quad y_{mn} \geq \gamma \tag{3}$$

where

$$\eta_{nm}(\mathbf{x}_n) = \Omega_0 + 10\alpha_{nm} \log_{10} \frac{d_0}{|\mathbf{r}_m - \mathbf{x}_n|} \tag{4}$$

where Ω_0 is the received power in dBm from the sensor at a known distance d_0, the path-loss $\alpha_{nm} = \alpha_{mn} = \alpha$, and σ_z^2 is variance of the shadowing.

Note that here we assume that the thresholds at all the sensors are the same, but this restriction can readily be removed. Another assumption is that the power of the signal sent by the n-th sensor can be measured without interference from other sensors. Based on the received powers z_{nm}, $n = 1, 2, \cdots, N$, $m = 1, 2, \cdots, M$, the objective is to locate the binary sensors as accurately as possible.

Without loss of generality, we focus on the location of one sensor, and therefore from now on we simplify the notation by dropping the subscripts n. So, we assume that the sensor which has to be located has coordinates (x_1, x_2), and that M signals from M different locations are transmitted by the beacon. Based on the responses z_m, $m = 1, 2, \cdots, M$, we aim at estimating the coordinates (x_1, x_2).

3. PROPOSED SOLUTION

We express the solution in the form of the posterior density

$$p(\mathbf{x}|\mathbf{z}) \propto p(\mathbf{z}|\mathbf{x})p(\mathbf{x}) \tag{5}$$

[1] Note that we assume that the field is two-dimensional. The theory and the proposed method can be extended to a three-dimensional field without any difficulty.

where $\mathbf{z} = (z_1 \; z_2 \; \cdots \; z_M)^\top$, $p(\mathbf{z}|\mathbf{x})$ is the likelihood and $p(\mathbf{x})$ is the prior of $\mathbf{x}$. We assume uniform prior for $\mathbf{x}$, and we seek the maximum a posteriori solution (in this case, it is identical to the maximum likelihood estimate), which is given by

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} \{p(\mathbf{z}|\mathbf{x})\}. \tag{6}$$

For the likelihood, we can write

$$p(\mathbf{z}|\mathbf{x}) = \prod_{m=1}^{M} p(z_m|\mathbf{x}). \tag{7}$$

For the received power of the sensor, we have

$$p(y_m|\mathbf{r}_m) = \mathcal{N}\left(\nu_m(\mathbf{x}), \sigma_y^2\right) \tag{8}$$

where

$$\nu_m(\mathbf{x}) = \Psi_0 + 10\alpha_m \log_{10} \frac{d_0}{|\mathbf{r}_m - \mathbf{x}|}. \tag{9}$$

Finally, the received power z_m by the beacon is distributed as a mixture Gaussian, i.e,

$$\begin{aligned} p(z_m|\mathbf{x}) &= p(z_m|\mathbf{x}, y_m \geq \gamma)\; P_{sm}(\mathbf{x}) \\ &+ p(z_m|\mathbf{x}, y_m < \gamma)\; P_{0m}(\mathbf{x}) \end{aligned} \tag{10}$$

where

$$P_{sm}(\mathbf{x}) = P(y_m \geq \gamma|\mathbf{x}) \tag{11}$$

$$P_{0m}(\mathbf{x}) = P(y_m < \gamma|\mathbf{x}) \tag{12}$$

where the distribution of y_m depends on $\mathbf{x}$. Also,

$$p(z_m|\mathbf{x}, y_m \geq \gamma) = \mathcal{N}(\eta_{sm}(\mathbf{x}), \sigma_z^2) \tag{13}$$

$$p(z_m|\mathbf{x}, y_m < \gamma) = \mathcal{N}(\eta_{0m}, \sigma_z^2) \tag{14}$$

$$P_{sm}(\mathbf{x}) = \int_\gamma^\infty \mathcal{N}(\nu_m(\mathbf{x}), \sigma_y^2) dy_m \tag{15}$$

$$P_{0m}(\mathbf{x}) = 1 - P_{sm}(\mathbf{x}) \tag{16}$$

and

$$\eta_{sm}(\mathbf{x}) = \Omega_0 + 10\alpha_m \log_{10} \frac{d_0}{|\mathbf{r}_m - \mathbf{x}|}. \tag{17}$$

The mean η_{0m} is known and is much smaller than $\eta_{sm}(\mathbf{x})$, whereas the mean $\nu_m(\mathbf{x})$ is defined by (9).

We find $\mathbf{x}$ that maximizes (7) by using a numerical method.

4. CRAMÉR-RAO BOUNDS

As in the previous section, we focus on the localization of a single sensor.

Recall that the likelihood function is given by (7) with M being the total number of measurements taken by the mobile beacon. The Cramér-Rao theorem states that the covariance of an unbiased estimator for unknown parameters is bounded by the inverse of the Fisher Information Matrix $\mathbf{I}(\mathbf{x})$ whose elements are given by

$$[\mathbf{I}(\mathbf{x})]_{ij} = E\left[\frac{\partial \ln p(\mathbf{z}|\mathbf{x})}{\partial x_i} \frac{\partial \ln p(\mathbf{z}|\mathbf{x})}{\partial x_j}\right] \tag{18}$$

where $i, j = 1, 2$, and x_1 and x_2 are our unknown parameters.

We note that

$$\frac{\partial \ln p(z_m|\mathbf{x})}{\partial x_i} = \frac{1}{p(z_m|\mathbf{x})} \frac{\partial p(z_m|\mathbf{x})}{\partial x_i} \tag{19}$$

and

$$\begin{aligned} \frac{\partial p(z_m \mid \mathbf{x})}{\partial x_i} &= P_{sm}(\mathbf{x}) \frac{\partial p(z_m|\mathbf{x}, y_m \geq \gamma)}{\partial x_i} \\ &+ p(z_m|\mathbf{x}, y_m \geq \gamma) \frac{\partial P_{sm}(\mathbf{x})}{\partial x_i} \\ &+ p(z_m|\mathbf{x}, y_m < \gamma) \frac{\partial P_{0m}(\mathbf{x})}{\partial x_i}. \end{aligned} \tag{20}$$

It is not difficult to show that the expressions needed to compute (20) (in addition to those of (13), (14), and (15)) are given as follows. First,

$$\begin{aligned} \frac{\partial p(z_m|\mathbf{x}, y_m \geq \gamma)}{\partial x_i} &= \frac{1}{\sqrt{2\pi\sigma_z^2}} \exp\left(-\frac{(z_m - \eta_{sm}(\mathbf{x}))^2}{2\sigma_z^2}\right) \\ &\times \frac{z_m - \eta_{sm}(\mathbf{x})}{\sigma_z} \frac{\partial \eta_{sm}(\mathbf{x})}{\partial x_i} \end{aligned} \tag{21}$$

where

$$\frac{\partial \eta_{sm}(\mathbf{x})}{\partial x_i} = \frac{10\alpha_m}{\ln 10} \frac{(r_{i,m} - x_i)}{(r_{1,m} - x_1)^2 + (r_{2,m} - x_2)^2}. \tag{22}$$

Second, since

$$\begin{aligned} P_{sm}(\mathbf{x}) &= \int_\gamma^\infty \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left(-\frac{(y_m - \nu_m(\mathbf{x}))^2}{2\sigma_y^2}\right) dy_m \\ &= \int_{\frac{\gamma - \nu_m(\mathbf{x})}{\sigma_y}}^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y_m^2}{2}\right) dy_m \end{aligned} \tag{23}$$

the derivative of the integral with respect to x_i yields

$$\frac{\partial P_{sm}(\mathbf{x})}{\partial x_i} = \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left(-\frac{(\gamma - \nu_m(\mathbf{x}))^2}{2\sigma_y^2}\right) \frac{\partial \nu_m(\mathbf{x})}{\partial x_i} \tag{24}$$

where

$$\frac{\partial \nu_m(\mathbf{x})}{\partial x_i} = \frac{\partial \eta_{sm}(\mathbf{x})}{\partial x_i}. \tag{25}$$

Finally,

$$\frac{\partial P_{0m}(\mathbf{x})}{\partial x_i} = -\frac{\partial P_{sm}(\mathbf{x})}{\partial x_i}. \tag{26}$$

Now we have all the terms that are needed for the computation of the Cramér-Rao bounds. The actual computation of the expectation is done by Monte Carlo integration. Namely,

$$\begin{aligned}[\mathbf{I}(\mathbf{x})]_{ij} &= E\left[\frac{\partial \ln p(\mathbf{z}|\mathbf{x})}{\partial x_i}\frac{\partial \ln p(\mathbf{z}|\mathbf{x})}{\partial x_j}\right] \\ &= E\left(\sum_{m=1}^{M}\frac{\partial \ln p(z_m|\mathbf{x})}{\partial x_i}\frac{\partial \ln p(z_m|\mathbf{x})}{\partial x_j}\right) \\ &\approx \frac{1}{L}\sum_{l=1}^{L}\sum_{m=1}^{M}\frac{\partial \ln p(z_m^{(l)}|\mathbf{x})}{\partial x_i}\frac{\partial \ln p(z_m^{(l)}|\mathbf{x})}{\partial x_j}\end{aligned} \tag{27}$$

where $z_m^{(l)} \sim p(z_m|\mathbf{x})$ is the $l-$th drawn observation at location m, and L is the total number of drawn vectors $\mathbf{z}$.

We computed the Cramér-Rao bounds for various parameters of the system. We chose a two dimensional field with dimensions $100\,m \times 50\,m$. The beacon moved along a known trajectory which in our case was a uniform grid with a specified interspacing. The system parameters were set to $\Psi_0 = -44.56\,dBm$, $\Omega_0 = -50.58\,dBm$, $\alpha = 3$, $\sigma_y^2 = 3$, and $\gamma = -69.56\,dBm$.

First, the bounds were obtained after the beacon traversed the sensor field on a uniform grid with interspacings of $2m$. As expected, the bound has the largest values in the corners of the field (see Figure (2)). It should be noted, however, that if sensors are located there, their location can be determined from prior information about the coordinates of the corners.

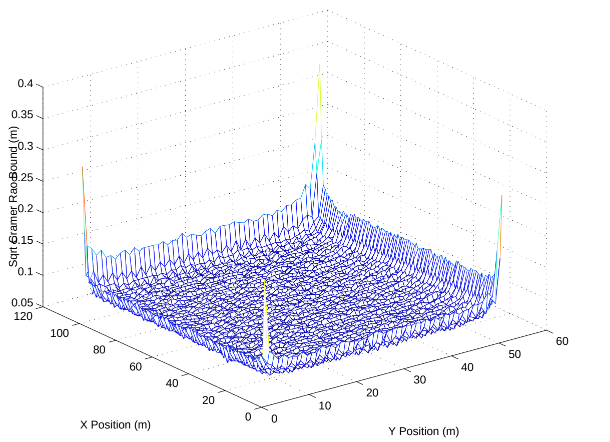

Fig. 2. The Cramér-Rao bound for determining the position of the sensor.

In Figures 3 and 4, we show the variation of the Cramér-Rao bounds for various 'free' system parameters. In Figure 3, we can see the dependence of the Cramér-Rao bound at a particular position with respect to the interspace distance in the mobile beacons trajectory. In Figure 4, we observe the Cramér-Rao bound as a function of various thresholds and various interspace distances. The relation between the thresholds and the sensing ranges denoted by ρ is $\gamma = \Psi_0 - 10\alpha\log_{10}(\rho)$.

We should note that with small sensing ranges, the sensors may be unable to sense the mobile beacon in their vicinity and therefore would not report their presence to the beacon. Also, when the interspacing in the beacon trajectory exceeds the sensors sensing range, the uncertainty of detecting the sensors increases too, and the Cramér-Rao bounds grow considerably. From Figure 3, for example, we see that when $\rho = 15\,m$ and the interspacing gets close to $15\,m$, the Cramér-Rao bound inflates quickly with the further increase of the interspacing.

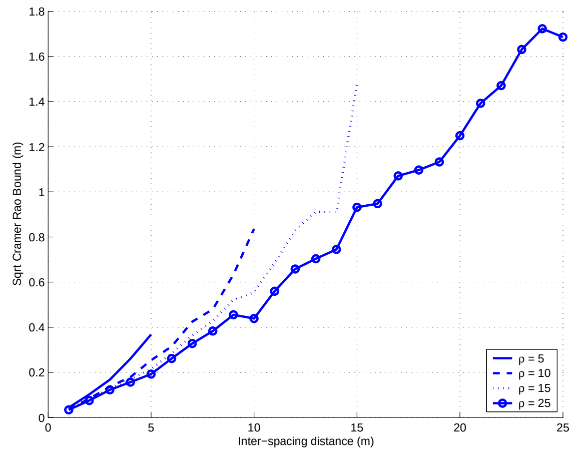

Fig. 3. The Cramér-Rao bound as a function of interspace distance of the beacon trajectory.

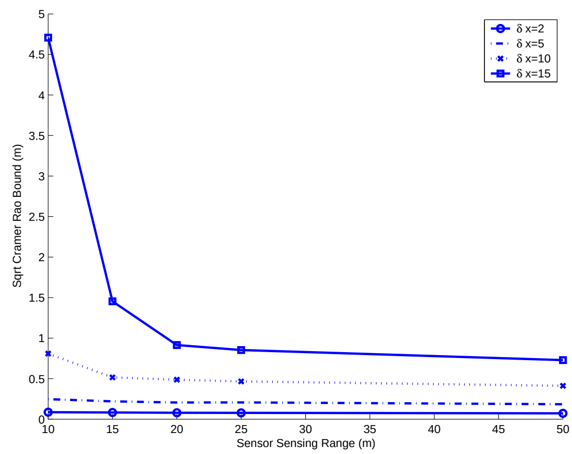

Fig. 4. The Cramér-Rao bound as a function of the sensing range for interspace distances of 2, 5, 10, and $15\,m$ of the beacon trajectories.

5. SIMULATION RESULTS

We also verified the performance of our method by computer experiments. Again, the field was two-dimensional with dimensions $100\,m \times 50\,m$, and the beacon moved along a known trajectory on a uniform grid in the field with interspacing of $2\,m$. The actual sensor position was at $x_1 = 40.2\,m, x_2 = 25.2\,m$. We ran the above algorithm after one complete collection of the readings. The parameters in the experiment were set as follows: $\Psi_0 = -49.07\,dBm$ $\Omega_0 = -55.09\,dBm$, $\alpha = 2.5$, $\sigma_y^2 = 3$, $\eta_{0m} = -150\,dBm$, and $\gamma = -79.07\,dBm$.

Figure 5 shows the logarithm of the posterior function over the entire field, and Figure 6 presents its contour. The maximum of the a posteriori density is obtained at $(40\,m, 26\,m)$.

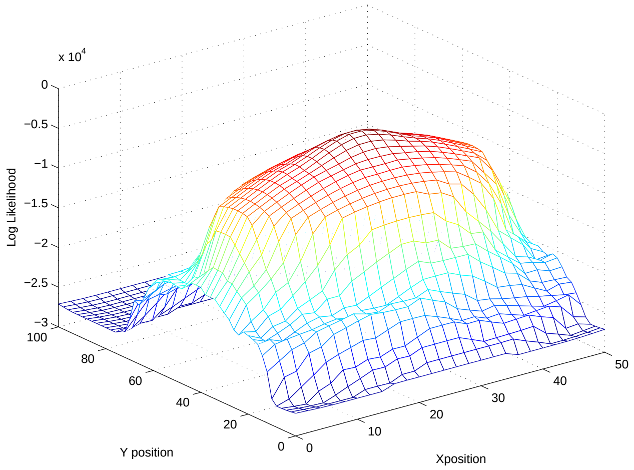

Fig. 5. Logarithm of the posterior of the sensor position.

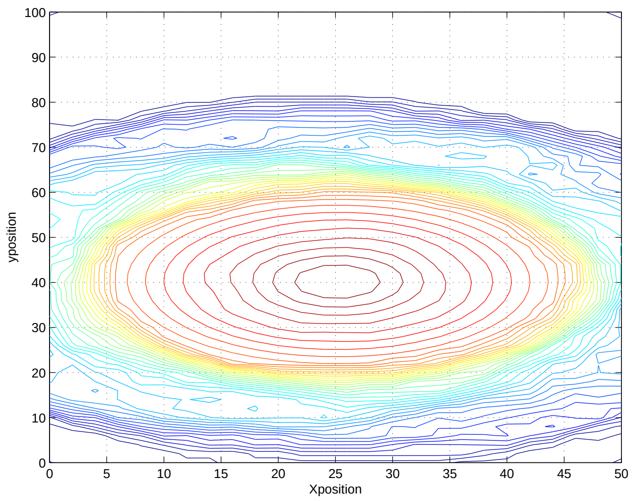

Fig. 6. Contour diagram of the logarithm of the posterior.

6. CONCLUSIONS

We addressed the problem of localization of binary sensors in a field, where the sensors are randomly deployed and do not cooperate during their operation. The localization is achieved by collecting measurements that represent transmitted signals from the sensors responding to queries of the beacon. The proposed method maximizes the posterior distributions of the locations of the sensors and obtains the estimate by numerical computations. We also derived the Cramér-Rao bounds of the estimates, and studied them as functions of the interspacings of the measurement locations on the beacon's trajectory as well as the ranges of the sensors. Finally, we demonstrated the performance of the method by computer simulations.

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